

FREDERICK FAGUNDES ALVES

**INVESTMENT AND HUMAN CAPITAL  
EXTERNALITIES: KNOWLEDGE SPILLOVERS**

Tese apresentada à Universidade Federal de Viçosa, como parte das exigências do Programa de Pós-Graduação em Economia Aplicada, para obtenção do título de *Doctor Scientiae*.

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Leonardo Bornacki de Mattos  
(Orientador)

To my father, my mother and my sister.

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I thank all those who accompany me, guided me, encouraged me, gave me strength, and especially those who did not let me fall in moments of fatigue.

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# Abstract

ALVES, Frederick Fagundes, D.Sc., Univesidade Federal de Viçosa, September, 2017. **Investment and Human Capital Externalities: Knowledge Spillovers.** Adviser: Leonardo Bornacki de Mattos.

The more general concept of human capital includes any characteristic associated with the individual to provide a productivity differential. However, more often, this concept is reduced to education and experience of individuals. Much of the research begins with the assumption that there are human capital externalities and that these effects is positive. But, few are the authors that measure this externality and do not show how a social planner would act to internalize the distortions caused by the action of the market, while maximizing the welfare of those involved in this economy. The general goal is investigate size and sources of externalities in human capital accumulation and Pareto inefficiency generate in the competitive equilibrium. More specifically aims to: Analyzing decisions of households of investing in human capital and trade-offs faced by these agents; Investigating complementarity between physical and human capital in technology and its implications for human capital investment; Examining the Pareto inefficiency created by externalities in human capital; Comparing Pareto inefficiency with “the first best” set of allocations through a Social Planner Problem. It is important to study the effects generated by human capital because the impacts that these effects can generate economic growth and increasing welfare agents. Thus, it is important to measure what the optimal level of investment in human capital that will bring growth to a long-run equilibrium point (steady state) and, at the same time, generate public policy implications. To do this, we used the four countries called “Four Asian Tigers” (South Korea, Singapore, Hong Kong and Taiwan) in this study. These countries were chosen because they made a leap in development generated by the high investment in education since 1960, increasing the levels of human capital and GDP per capita in these countries. In the theoretical model we assume that there are three generations in each period: children, adult and old. It is assumed that just adult agents make the economic decisions. The adult chooses consumption level, future capital for retirement purposes, and if he will educate his children or not. If the children go to school, they will become skilled workers next period. The old agents just consume the lifetime savings in form of capital. The main theoretical conclusion is that benevolent social planner maximizes a social welfare function and internalizes the external effects. The social planner in Pareto optimal, choose a higher level of human capital than in competitive equilibrium, to internalize the beneficial effects of the human capital externality. As the number of skilled workers increases, the marginal productivity of skilled labor tends to reduce and the marginal productivity of unskilled labor increases. In empirical results,

we have in the competitive equilibrium model, the reduction of the externality generates reduction of the skill premium given by the increase of the number of skilled workers. In the model with subsidies, the reduction of the externality generates increase of the skill premium, increase of the utility and increase of welfare.

# Resumo

ALVES, Frederick Fagundes, D.Sc., Univesidade Federal de Viçosa, setembro de 2017. **Investment and Human Capital Externalities: Knowledge Spillovers**. Orientador: Leonardo Bornacki de Mattos.

O conceito mais geral de capital humano inclui qualquer característica associada ao indivíduo para fornecer um nível diferente de produtividade. No entanto, mais frequentemente, esse conceito é restringido à educação e à experiência dos indivíduos. Grande parte das pesquisas iniciam com o pressuposto de que há externalidades de capital humano e que essas externalidades são positivas. Mas, poucos são os autores que quantificam essa externalidade e não mostram como um planejador atuaria para internalizar as distorções causadas pela ação do mercado, ao mesmo tempo em que maximiza o bem-estar dos agentes envolvidos nesta economia. O objetivo geral é investigar o tamanho e as fontes de externalidades na acumulação de capital humano e a ineficiência de Pareto gerada no equilíbrio competitivo. Mais especificamente visa: Analisar as decisões das famílias de investir em capital humano e os trade-offs enfrentados por esses agentes; Investigar a complementaridade entre capital físico e capital humano e suas implicações para o investimento em capital humano; Examinar a ineficiência de Pareto criada por externalidades de capital humano; Comparar a ineficiência de Pareto com o conjunto de alocações eficientes através de um problema de planejador social. É importante estudar os efeitos gerados pelo capital humano, pois os impactos desses efeitos podem gerar crescimento econômico e aumento do bem-estar dos agentes. Assim, é importante medir o nível ideal de investimento em capital humano que levará o crescimento para um ponto de equilíbrio de longo prazo (estado estacionário) e, ao mesmo tempo, gerar implicações de políticas públicas. Para tanto, utiliza-se como estudo os quatro países chamados "Tigres Asiáticos" (Coreia do Sul, Cingapura, Hong Kong e Taiwan). A escolha desses países se justifica devido ao salto de desenvolvimento dado pelo alto investimento em educação feito a partir de 1960, elevando cada vez mais os níveis de capital humano e do PIB per capita desses países. No modelo teórico, assume-se que há três gerações em cada período: crianças, adultos e idosos. Assume-se que apenas os agentes adultos tomam as decisões econômicas. O adulto escolhe o nível de consumo, capital futuro para fins de aposentadoria, e se ele deseja educar seus filhos ou não. Se as crianças vão à escola, elas se tornarão trabalhadores qualificados no próximo período. Os agentes idosos apenas consomem o que foi poupado em forma de capital. A principal conclusão teórica é que um planejador social benevolente maximiza uma função de bem-estar social e internaliza os efeitos externos. O planejador social é Pareto ótimo e escolhe um nível mais elevado de capital humano do que em equilíbrio competitivo para internalizar os efeitos benéficos da externalidade de capital humano.

Como o número de trabalhadores qualificados aumenta, a produtividade marginal do trabalho qualificado tende a reduzir e a produtividade marginal do trabalho não qualificado aumenta. Como resultado empírico tem-se que no modelo com equilíbrio competitivo, a redução da externalidade gera redução do prêmio pela qualificação dado pelo aumento do número de trabalhadores qualificados. No modelo com subsídios, a redução da externalidade gera aumento do prêmio pela qualificação, aumenta a utilidade e o bem-estar.

# 1 INTRODUCTION

Most authors who study human capital say that there is externality of knowledge due to interactions among economic agents. However, these authors do not show how a social planner would act to internalize the externalities caused by the action of the market.

[Lucas \(1988\)](#) has been the pioneer dealing formally with the external effects generated by the concentration of human capital in a given locality, from a long-run growth model. In this model, the motor of growth is the accumulation of human capital that contributes to increasing the productivity of all factors of production.

As a microeconomic way, the paper of [Mincer \(1958\)](#) became a motivation to study the investment in education as an economic decision related to the labor market, remuneration of workers throughout the production cycle and causes of income inequality.

Empirically, to quantify human capital externalities, [Rauch \(1993\)](#) used a regression based on the Mincerian model of income determination and added a variable for human capital aggregate of cities. [Rauch \(1993\)](#) did not observe the bias caused by the omission of variables and endogeneity of the average. Although the results were statistically significant, they were unreliable that the increase from one year to the level of schooling raised the wages of workers in cities, on average, 2.8%.

One of the main problems reported in the literature on the use of Mincerian equations is that there is a selection bias. This bias is because the wage is influenced by not only observable characteristics such as years of education and other controls, but also unobservable variables such as skills and other innate characteristics that are correlated to education.

In an attempt to correct the bias of the paper of [Rauch \(1993\)](#), [Acemoglu & Angrist \(2001\)](#) used instrumental variables to estimate the effect of average schooling level of the states. Despite the difficulty in finding variables that suit the conditions, [Acemoglu & Angrist \(2001\)](#) replaced the variable correlated with the error by another correlated with the explanatory variable, but did not correlate with the error. However, these authors found few evidences of external returns in human capital for the states of the United States of America (USA).

[Moretti \(2004\)](#) also used instrumental variables to correct the endogeneity instead the average schooling. The author calculated the externalities by education groups and used the proportion of population with higher education. As results, [Moretti \(2004\)](#) shows that an increase of one percentage point in the proportion of graduates increases the city's wage level, approximately, 1.6% and 1.9% for workers with and without secondary

education, respectively; and, 0.4% and 1.2% for workers with and without complete higher education, respectively.

Ciccone & Peri (2006) emphasize that the Mincerian approach adopted by Rauch (1993) confuses wage changes with human capital externalities. They propose an alternative approach to identify this type of externality maintaining a constant educational distribution. The authors used the same variables that Moretti (2004) and Acemoglu & Angrist (2001) for two different years (1970 and 1990) in cities and states of the USA. Ciccone & Peri (2006) found consistent estimates of demand curve for human capital is negatively sloped. However, the results are, in general, statistically insignificant for both the state level and at the municipal level.

The studies cited presented different results that reveal the existence of distinct external returns for different educational levels, and there is not a contradiction between these papers. The instrumental variables used correlate with different educational tracks, which may have influenced the results of econometric models.

The emphasis has been the impact of education on labor income. However, according to Barbosa Filho & Pessoa (2010) and Acemoglu, Gallego & Robinson (2014), education may be result in positive externalities for society that are not being considered in a wage equation.

Education can increase the proximity of people, which facilitates communication, reduces crime, policy awareness increases, decreases fertility rate, and other factors that may cause social return rate of education is greater than the private return rate ((ACEMOGLU; ANGRIST, 2001); (MORETTI, 2004); (Barbosa Filho; PESSOA, 2010)).

According to Barbosa Filho & Pessoa (2010), many of these factors can have an impact on the growth of aggregate output. However, the impact of education on growth, from the wage equations and the labor market with micro data do not seem to be possible to empirically test and capture all these channels properly.

Some analytical simplifications studies of the real world can lead to poor solutions, and even unreliable. Therefore, differently from previous work, this research will present a simulation model that tries to capture a more broadly and dynamics great of the factors that lead to investment in human capital. This research is also an attempt to verify the Pareto efficiency of the level of investment in order to analyze whether there are or not knowledge externalities.

Faced of many failed attempts to capture the externality of human capital, the main contributions of this research are: i) Externality placed directly in the aggregate production function; ii) Trade-off between capital stock and labor directly inserted in the overlapping generations model; iii) Functional weight measurement that generates the human capital externality that improves the countries sampled; iv) Measurement of how

public policies internalize the human capital externality.

Gradstein & Justman (2002) developed a dynamic model in which transactions productivity between two individuals depends on the “social distance” between them. The type of education chosen by the parents affects this “social distance”. The closer to “common culture” is the choice, the greater son productivity. In this environment, public policy can be made so that there is an improvement of Pareto where social differences are reduced gradually, which encourages growth.

It should be noted that although scholars highlight the existence of positive externalities of human capital, the vast majority just does theoretically entering this assumption in their models. However, this study intends to verify that there really is a spillover of knowledge through simulation. In that complex relations are established between many variables and observing the dynamic phenomena that arise from these interactions, which would hardly be achieved through purely theoretical analysis.

Initially, we intend to apply the model developed for the territories that make up the bloc called Four Asian Tigers (Hong Kong, Taiwan, South Korea and Singapore). These economies have in common a strong policy of investing in human capital held between 1960 and 1990 and since then reach high educational and technological levels, which directly affects economic growth due to increased productivity. Analyzing the human capital index per capita, South Korea ranked 10th, Singapore ranked 13th, Hong Kong ranked 33rd and Taiwan ranked 34th, in a total of 182 countries in 2014 according to the Penn World Table 9.0 database.

The “Tigers” invested in the education system at all levels, including the university system, ensuring that every child to attend primary school and high school. In general, these advances in education allowed high levels of literacy and cognitive skills. During the 1960s, the “Tigers” were relatively poor, having abundance of cheap labor, and took that advantage by uniting it with the educational reform creating a low-cost workforce, but very productive.

## 1.1 The problem and its importance

The points of social returns and private returns to education observed in the form of higher wages are closely related to economic growth. The impact of education on labor income represents a real gain for society, it follows that policies that raise the educational level will have a strong impact on the long-run income, and therefore on growth. In this case, the social return to education will be higher than the private return. Moreover, as suggested by the endogenous growth models, it is possible that increased schooling has a permanent impact on the growth rate (Barbosa Filho; PESSOA, 2010).

Theories of economic growth focus on the importance of human capital in the growth of countries. This human capital generates spillovers (positive externalities) that can lead to long run economic growth. Then, the search problem is to measure the human capital externality and the economic growth by simulation of the theoretical model developed in this research.

Much of the researches begin with the assumption that there are human capital externalities and that this effect is positive. However, these authors do not show how a social planner would act to internalize the distortions caused by the action of the market, while maximizing the welfare of those involved in this economy.

It is important to study the effects generated by human capital because the impacts (direct and indirect) that these effects can generate economic growth and increasing welfare agents. The hypothesis is that the results of a market economy cannot be Pareto optimal because the spillovers of knowledge constitute a form of externality.

According to [Lall \(2005\)](#) and [Freeman \(1989\)](#), in a market economy there may be underinvestment in knowledge because of the high costs, high risks, long periods of learning and diffuse externalities. Thus, it is important to measure what the optimal level of investment in human capital that will bring growth to a long-run equilibrium point (steady state) and, at the same time, indicate appropriate public policy.

## 1.2 Hypothesis

As a hypothesis, we have that in the competitive equilibrium there are high costs for education and investment risk of agents. These externalities reduce the welfare of agents and investment in education. The beneficial effects of externalities are internalized with the introduction of the Social Planner and there is an increase in the utility of the agent who decides to invest in education.

## 1.3 Objectives

Therefore, this research has as general goal to investigate size and sources of externalities in human capital accumulation and Pareto inefficiency generate in the competitive equilibrium.

### 1.3.1 Specifics Objectives

More specifically aims to:

- Analyzing decisions of households of investing in human capital and trade-offs faced by these agents;

- Investigating complementarity between physical and human capital in technology and its implications for human capital investment;
- Examining the Pareto inefficiency created by externalities in human capital;
- Comparing Pareto inefficiency with “the first best” set of allocations through a Social Planner Problem;
- Simulating the theoretical model developed and analyze the quantitative results of agents to invest in education.

In addition, this introduction, this thesis presented in section 2 a theoretical discussion about the externality concept of human capital based on the main researchers in this field. Section 3 shows the four Asian Tigers (South Korea, Singapore, Hong Kong and Taiwan). In section 4, we show the theoretical model with and without social planner that we will use to get the search results. Then, in section 5 is defined, as the simulation method will be used in this thesis. We show the simulation results in the competitive equilibrium and benevolent government in section 6. Section 7 is the conclusion.

## 2 ECONOMIC GROWTH AND HUMAN CAPITAL EXTERNALITY

The importance of education as one of the factors that generate economic growth has been widely discussed by its link to productivity in the theory of human capital. In the late 1950s and early 1960s, the education is gaining importance in the economic growth literature through the work done by Solow that indicated a growth of output higher than that attributed to the increase of capital and labor factors.

The growth differential between the observed data for the United States economy and the exposed by Solow was called Solow residual and could be attributed to two factors: technical progress and other variables not captured by this neoclassical growth model. From the Solow model, [Mincer \(1958\)](#) uses the higher average education of the population and the employee's experience, as able to explain this difference of output and income.

[Mincer \(1958\)](#) tried to explain how the remuneration of labor is influenced by the human capital stock from its most microeconomic view of the return of education and based on the discussion of Adam Smith about the compensatory wage differentials. According to the author, this remuneration should increase as higher levels of education and experience. Therefore, worker's remuneration is equal to the marginal economic cooperation for the firm, thus the human capital stock is recognized by the labor market as capable of indicating a higher level of productivity, resulting in higher remuneration.

The mincerian presentation represents a seminal attempt to measure econometrically the empirical concept that the individual income distribution is a function of the skills and knowledge acquired. Mincer aims to clarify the main causes of income inequality trying to explain investment in human capital resulting from an economical choice. One possibility was that the skills being able to explain this difference, although these differences show that all individuals have the same level of skill and wage differentials due to the higher level of education compensate the private costs of educating the wage distribution should be asymmetric.

The difficulty of explaining the asymmetrical distribution of income as a result only of the variable ability of each individual point to an explanation based on a factor correlated with income, in this case human capital, measured from the education and individual experience. Another factor that may explain the differential between wages is the mobility in the labor market that is related to the acquired knowledge and level of experience.

[Mincer \(1958\)](#) uses the logarithm of income, years of education and work experience

to estimate the return to education. A major problem in the use of the equation developed by Mincer is the existence of a selection bias in estimating the income gain in the logarithm of the salary for each additional year of worker education. This bias is due salary not only influenced by observable variables such as years of education, but also unobservable variables such as ability and other variables that cannot be correlated to education.

The development of research and models that address the economics of education to explain the difference in growth between countries ((SCHULTZ, 1960); (SCHULTZ, 1961)) and individual income (BECKER, 1962) is consolidating rapidly after 1960.

Schultz (1960) is the first author to consider education as an investment in the people. The investment is therefore the formation of human capital providing some degree of productivity that can be quantified economically. Thus, according to Schultz (1961), the increase in income is due to the increase of human capital and the little existing human capital in poor countries was a good limitation that prevented the best use of physical capital investment, becoming a limiting factor growth.

According to the definition of Becker (1962), human capital is any activity that implies a cost on the current period and increase productivity in the future can be analyzed within the investment theory of the structure.

Becker (1962) analyzes the optimal level of investment in education aiming to evaluate public policies given the scarcity of resources. The author claims that the internal rate of return of education is greater than the internal rate of return on physical capital there is evidence of under investment in education. In addition, Becker (1962) indicates the existence of a certain complementarity between ability and education.

The studies of education were also oriented to the labor market through a microeconomic perspective. Becker (1962) shows that the individual decision to study, which takes into consideration costs and private benefits is linked to the remuneration of workers throughout the production cycle, higher turnover in the labor market, greater investment in training, etc. The author also points out that people with higher levels of education are increasingly investing in human capital, and therefore have higher incomes. Inequality in human capital investment among individuals reflects the income of each agent and results in great inequality of earnings.

There will be self-selection to assume that individuals have different skills and that individuals with greater ability have lower costs in acquiring education and/or the gain of educating will be higher for individuals with higher skill. Therefore, the most skilled individuals decide to stay more years in school, while the cost of educating for those less skilled will be higher (Barbosa Filho; PESSOA, 2010).

Many authors as Angrist & Krueger (1991), Card (1999), Conneely & Uusitalo (1998) and Angrist & Krueger (2001), used instrumental variables able to capture the

unobservable characteristics on earnings for each additional year of education to try to control the selection bias the mincerian equation. In this case, the instrumental variables, in addition to controlling auto selection between education and skill, could also reduce the measurement error problem.

Lange & Topel (2006) investigate the externalities associated with education through microdata from the labor market in cities and an adaptation of the mincerian equation. The objective of the authors was to determine the sign and the value of the social return to education. The various papers cited by Lange & Topel (2006) point out that this return is positive, and there is no evidence of negative externalities associated with education. According to the authors, the main difference among the studies is the instrumental variable used to control the average education level of the population as endogenous.

The mincerian formulation tries to explain how education and professional experience interacting in the process of skills training and income from the individual's perspective. The Mincer seminal contribution in the modern economy work, points out the heterogeneity of income as a result of the difference in the stock of human capital input. The author assumes the possibility that investments in human capital are not observable directly, presenting an important measurement methodology from the quantification of wage profiles and the return earned by the market for each increase of this factor. Although over the years there has been a great expansion microdata and estimation techniques, it is possible to deduce that the equation developed by Mincer is still a reference in the determination of wages and earned return for labor market schooling.

Uzawa (1965) emphasizes that technological progress should not be seen as a "manna falling from the sky", but as the result of intentional actions taken by economic agents that use scarce resources to advance the state of technological knowledge from a more macroeconomic framework of the theory of economic growth and human capital. Uzawa (1965) assumed that the technological knowledge should be placed on labor in terms of an aggregate production function.

Lucas (1988) shows one of the first models of endogenous growth in substitution for neoclassical exogenous growth models based on the model Uzawa (1965) and supported by theory exposed by Schultz (1960), Schultz (1961), Becker (1962) and Becker (1964). Lucas (1988) modeled an economy in which technology supports constant returns to scale and that there was the presence of externalities associated with the accumulation of human capital. In this model, human capital accumulation would be the main factor that would generate permanent effects on the growth rate.

Lucas (1988) shows that the acquisition of human capital results in benefits not only for the individual but also for the whole society because the increase in the aggregate level of this factor of production contributes to increased productivity of everyone else.

This model approximates the economic theory of the results found in several empirical studies related to the conditional convergence. Introducing the possibility of multiple steady states, it depends on the initial conditions of the economy and not only of its growth rate.

A differentiate between Becker's perspective and Lucas's perspective of human capital is the first author considers the returns to investment in human capital decreasing, while Lucas assumes that these returns are constant throughout life.

Lucas (1988) shows a concept that until then was not seen, which is a positive externality between agents or spillover of knowledge to try to explain the most relevant factor for the long run economic growth. Education can increase the proximity between people, facilitating communication, reducing crime, enhancing democracy through greater political awareness, reducing fertility rates, among other benefits that may cause the social rate of return to education exceeds the private rate of return (Barbosa Filho; PESSOA, 2010). Some of these effects had been thought by Lucas (1988), to put that much of what we know, we have learned with others in mutual form, and that the externality generated by these relationships are the "engine" of economic growth.

The theory presented by Lucas (1988) indicates that the allocation trajectory of diverse agent initiatives along the time constantly affects their productivity. Therefore, the effect of adding human capital results in effect on the level of current production and allocation activities, affecting their accumulation in the current and future time. Lucas, to include human capital in the production function, indicates the possibility of agents modifying the level of investment in education, choosing how long are dedicated to studies showing the accumulation of other input rather than just physical capital accumulation.

A large literature that endogenized technical progress was initiated from the model developed by Lucas (1988). In this direction, Romer (1990) in their model incorporates an innovation sector with the use of human capital. In the study by Romer (1990), the long run growth rate is given by the speed with which the laboratory research and development (R&D) generate new ideas, which stimulates a permanent impact on growth. The great contribution of Romer (1990) was the explanation of how to construct an economy of three sectors made up of agents of maximizing profit becomes endogenous technological progress.

Economies with high levels of human capital attract investment firms using more advanced technological resources. Moreover, it can only firms to adopt new technological processes if there are skilled workers capable of working with these technologies. Cohen & Levinthal (1989) argue that firms invest in research and development (R&D) not only to continue with new processes and products, but also to develop and maintain their ability to assimilate, absorb and exploit the information available from other firms. Recognition of the role played by R&D suggests that the factors affecting the facility of learning will

affect the incentives of companies to make more investment in knowledge.

Glaeser & Mare (1994) and Henderson (2003) follow the same initial reasoning Schultz (1961) and suggest that areas with low human capital cannot follow technological progress and so have low economic growth. Thus, the interaction among economic agents that are in a more dynamic environment promotes frequent exchanges of experiences among workers that accelerate the learning process.

Two different types of human capital externalities are presented in Acemoglu & Angrist (2001), the pecuniary and non-pecuniary externalities. Non-pecuniary externalities (non-financial), are not defined by price, these can be considered as the exchange of ideas, imitation of new technologies, the system learning by doing, etc.

In contrast, the pecuniary externalities of human capital are given by investing in regions with supply of skilled labor. The higher human capital encourages greater investment firms and increases workers' wages ((ACEMOGLU, 1996); (ACEMOGLU, 1997)). Externalities of human capital, in this case, happen due to increased physical capital chosen by companies in anticipation of the average human capital that workers choose in the future. Since human capital and physical capital are complementary in this configuration, the most educated workforce will lead to a positive relationship between increased investment in physical capital and high wages, due to increased worker productivity.

While studies with more microeconomic perspective on education are facing the labor market and address issues such as: the income of individuals in a particular city/state using microdata; education costs for more and less qualified individuals; the gap between social and private returns, and the externality given by these returns. Macroeconomic perspective of human capital aims to increase the aggregate output and economic growth of countries. When dealing with externalities, this perspective uses the relationship among agents generating some sort of knowledge that can be used by many others. From this perspective, the studies also address the inequality of income among countries, increased productivity, and the complementarity between human capital and physical capital to generate growth.

When we construct the macroeconomic growth model, we base on consumer theory and the firm, using agents that represented the aggregate, since it assumes that all agents make the same decisions. This is a macroeconomic simplification to work with the aggregate rather than an individual analysis. Furthermore, the elevation of the individual gains in education generates benefits for the whole society, as the increase in human capital aggregate level contributes to the increase in productivity of all individuals.

Evidently, the microeconomic perspective of returns on wages and macroeconomic perspective of increases in long run aggregate output are closely related. The impact

of education on labor income represents a real gain for society, it follows that policies that significantly raise the level of education will have a strong impact on the long run income, and consequently they will have impacts on economic growth throughout the dynamic transition. It is also possible that the impact of education on the product is greater than the impact obtained in studies using regression wages. In this case, the social return to education will be higher than the private return. Moreover, as suggested by some endogenous growth models, it is possible that increased schooling has a permanent impact on the growth rate.

### 3 FOUR ASIAN TIGERS

The group of countries called Four Asian Tigers (Hong Kong, South Korea, Singapore and Taiwan) is famous for their fast economic and industrial growth since the 1960s. These countries showed an accelerated process of industrialization based on technological progress from middle of the twentieth century. Thus, the Four Asian Tigers have maintained high growth rates and captured significant shares of the international markets, transforming their economic bases.

After the end of World War II, and Japan's weakening that dominated these countries, there was an end to violent oppression. However, the Japanese rule over these territories left a positive legacy that influenced the improvement of production techniques in these countries (SILVA, 2012). Therefore, Japan served as technical progress of economic model for the countries in the region, and from the end of World War II these countries changed their policies and start to invest heavily in education, labor training, hiring outside professionals, and acquisition of new technologies.

Much of the growth in these countries is related to the policies adopted in education and training of the worker. We can be seen from Figure 1 that among the Four Asian Tigers, South Korea stands out with the highest percentages of government spending on education since 1970 and the year 1982 with more than 6% of Gross Domestic Product (GDP) invested in education. Despite the series of government spending on education not be complete for Singapore and Hong Kong, we can see how they present average investment about 3% of GDP in these countries. Already in Taiwan, government spending on education was throughout the period 1981 to 2014 and close to 3% of GDP, with the years 2001, 2002 and 2003 with little more than 3%.

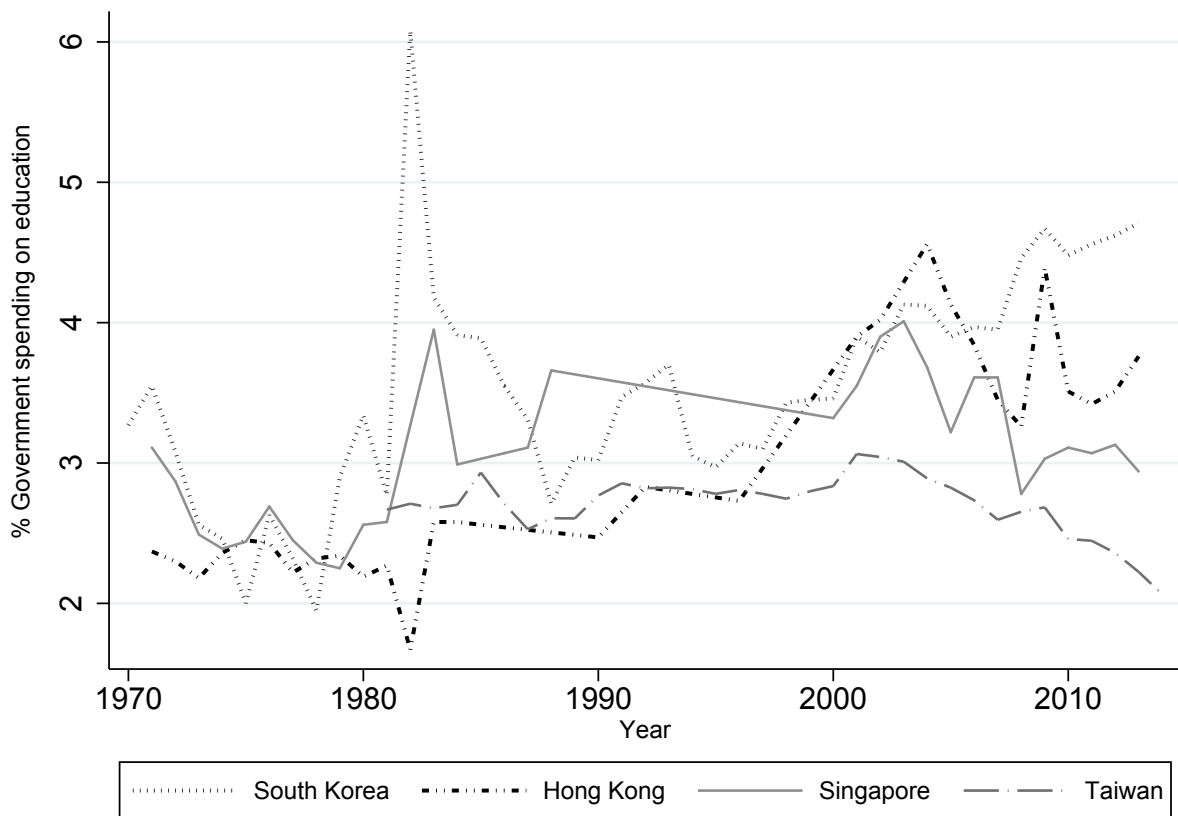


Figure 1 – Government spending on education (% of GDP), 1970 to 2014

Source: Authors' calculations based on data from World Bank (2015) and National Statistics of Republic of China.

The Human Capital Index per capita, collected in the Penn World Table 8.0, was calculated taking into account the years of schooling and mortality rates (BARRO; LEE, 2013), and the returns to education (PSACHAROPOULOS, 1994). This indicator can be considered as a proxy for human capital and through the Figure 2 we noted the growth of the Four Asian Tigers. Over the years, the strategies of East Asian governments were being structured to raise the standard of knowledge toward the creation of high technological products. This strategy has been accompanied by increases in investment levels they were entering those countries and the growing demand for manufactured products made with skilled labor increased.

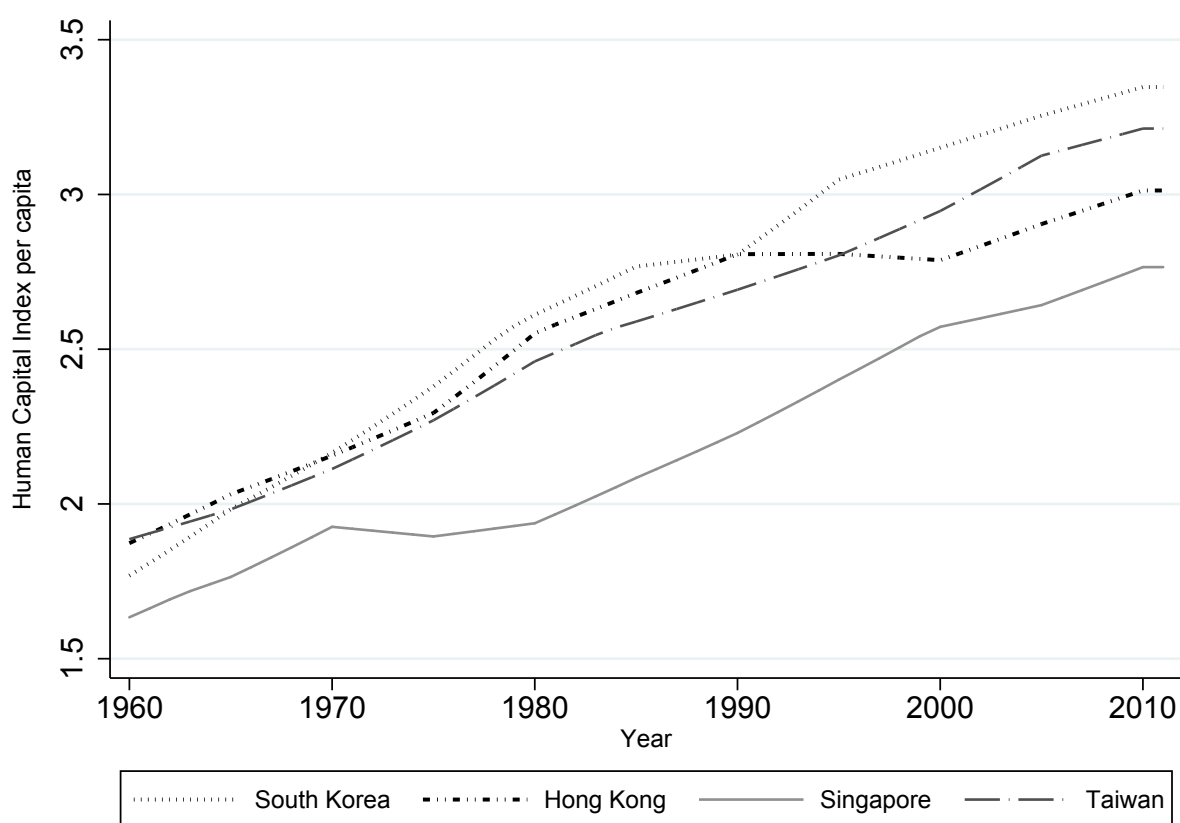


Figure 2 – Human Capital Index per capita, 1960 to 2011

Source: Authors' calculations based on data from Penn World Table 8.0

Note: Based on years of schooling and mortality rates (BARRO; LEE, 2013) and returns to education (PSACHAROPOULOS, 1994).

The average rate of growth of the human capital index of the four countries studied was approximately 1% per year<sup>1</sup>. South Korea had the highest average annual growth rate, with 1.22%, while Hong Kong increased human capital, on average, 0.90% per year. Singapore and Taiwan have very close increases, 1.03% and 1.08% per year, respectively.

During the Cold War, the United States (USA) was fundamental to finance the development process and further strengthen the technological progress of the Four Asian Tigers. The USA fears that Asia was dominated by communism meant that American investments were concentrated in the region to try to strengthen capitalism (CASTELLS, 1992). In addition to Foreign Direct Investment (FDI) and providing credit lines, it was the United States' interest to use the Asian production platforms manufactured with efficient labor and low-paid.

Lines of credit and foreign investment enable access to imported technology and the financing of public projects to improve the infrastructure of the country. The priority

<sup>1</sup> The calculation of average annual growth rates was performed using a regression of the logarithm of the explained variable, and the tendency as an explanatory variable. That is,  $\ln Y = \alpha + \beta t + \varepsilon$ , where the value of the angular coefficient ( $\beta$ ) is the growth rate of the variable  $Y$  for each period.

in these countries was to form a national strong and focused on the production of products with high technological level industry.

With the increase of imports of technology and the expansion of industries, there was the accumulation of capital by these countries, which we can be seen in Figure 3, which it shows the stock of capital per capita based on Purchasing Power Parity (PPP)<sup>2</sup>. Most notably, there is Singapore with about 185 billion dollars in capital stock per capita in 2011, then with about 147 billion dollars, is Hong Kong, with approximately 116 billion dollars is South Korea followed Taiwan with just over 83 billion dollars in 2011.

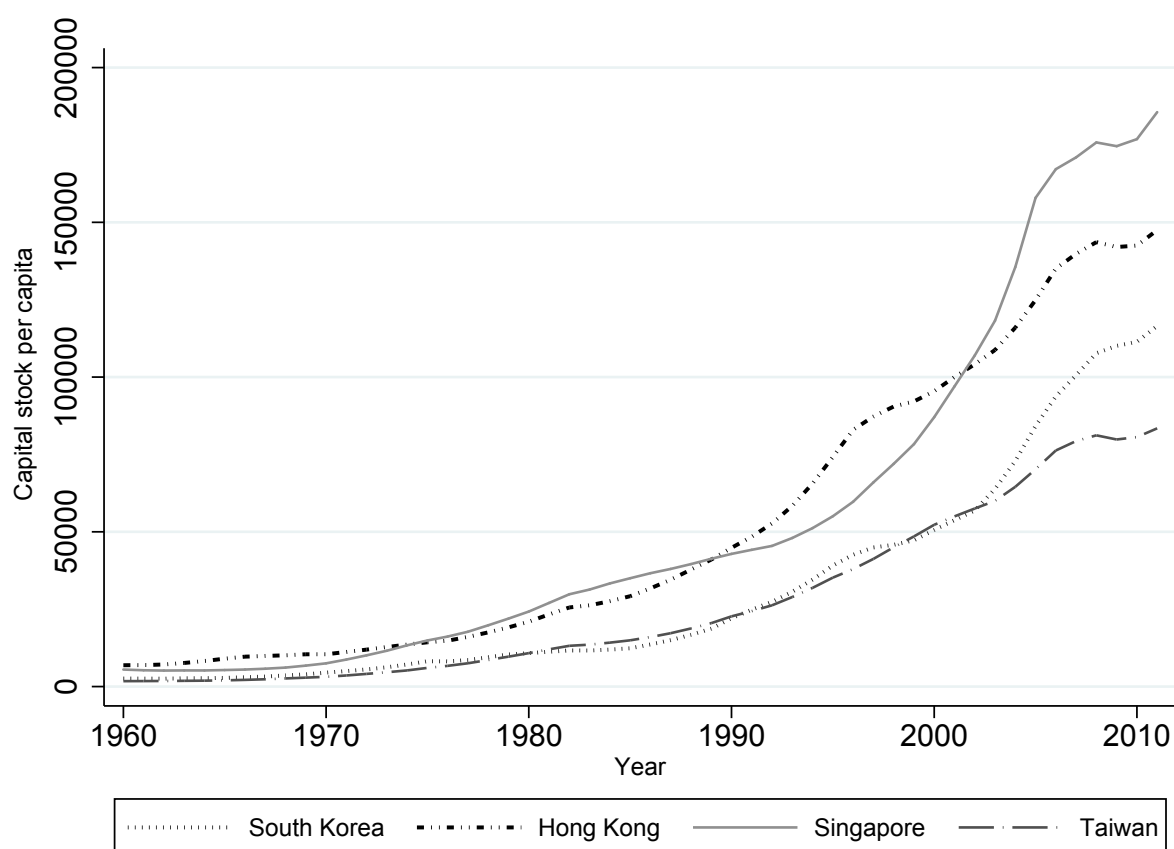


Figure 3 – Current capital stock per capita based on Purchasing Power Parity (PPP) (in millions of dollars 2005), 1960 to 2011

Source: Authors' calculations based on data from the Penn World Table 8.0.

According to [Hong & Yeung \(2015\)](#), while Hong Kong and Singapore have developed financial centers characteristics, South Korea and Taiwan are the main information technology manufacturers and electronic products in the world. Figure 3 shows that the stock of capital per capita of countries representing financial centers developed is higher than those technology vendors' countries. However, without considering the population of

<sup>2</sup> The Purchasing Power Parity (PPP) measures how a particular currency can buy internationally, since goods and services has different prices from one country to another. In other words, relates the purchasing power of a particular person with the local cost of living. Therefore, the PPP takes into account both income differences as differences in the cost of living.

these countries, the capital stock of the producing economies of technology such as South Korea and Taiwan are around 6 trillion and 2 trillion dollars in 2011, respectively, while Hong Kong and Singapore revolve around 1 trillion dollars.

In South Korea and Taiwan, the domestic industry was the key piece of gear approaching the technological frontier process with large South Korean conglomerates and small and medium enterprises in Taiwan. In Singapore, the technology transfer was more connected with the installation of transnational in the country. In Hong Kong, the industrialization process was less intense and technological innovation capacity was more limited, since the deepening of industrialization not proved fundamental in the dynamics of modernization (CUNHA, 2012).

The average rate of growth stock of physical capital per capita in South Korea, Singapore and Taiwan, ranged around 8% per year, while the growth of Hong Kong was 6.75% per year. By splitting the total period of analysis (1960-2011) in two, 1960-1985 (first period) and 1986-2011 (second period), we have that the only countries that have increased the average annual growth rate were South Korea and Hong Kong, as can be confirmed by Figure 3. While Taiwan and Singapore have reduced their average rates of growth capital stock per capita of 9.93% and 8.96% per year in the first period to 7% and 7.56% per year in the second period, respectively. South Korea raised its average growth in the first period (7.64% annually) for the second period (8.62% annually), while increasing the capital stock per capita of Hong Kong from the first to the second period was 5.89% to 6.29%.

According to the World Bank (1993), the main growth factor of these economies is related to high rates of investment, state spending on education, exports of manufactured products of high technology and efficiency of services.

From 1980 to 2014, countries of the Four Asian Tigers had high average Gross Domestic Product growth (GDP) per capita per year. The country with the highest average growth rate per year was South Korea, with 7.97%. Taiwan obtained the second highest average growth rate of 7.37% per year. Despite having the highest GDP per capita values of the review period, Singapore and Hong Kong had the lowest average rates of GDP growth per capita per year, 6.38% and 5.72%, respectively, but this growth is still higher than the average annual growth of most countries of the world.

Kim & Nelson (2005) points out that these four countries have experienced a late industrialization process and transformed from agrarian economies with low standard of technological development in countries considered as growth and prosperity reference. This growth can be seen by GDP per capita analyzed by the purchasing power parity between the period 1980-2014, in Figure 4. The rise of these countries is closely linked to successful policies of investment in education, industry and stability economic attracting financial capital available for further investment.

Singapore has the highest per capita income among the Tigers, due to high levels of investment by transnational companies operating in the country (investment in physical capital) and in the laboratories of Research and Development (R&D) (investment in knowledge). The Singapore government has made heavy investments in technology infrastructure, while established an appropriate environment for foreign investment in high technology sectors. It has adopted policies aimed to establish adequate structure for training, development of new technologies, provide services to companies and transfer technology to other productive sectors of the country (HODGSON, 2006).

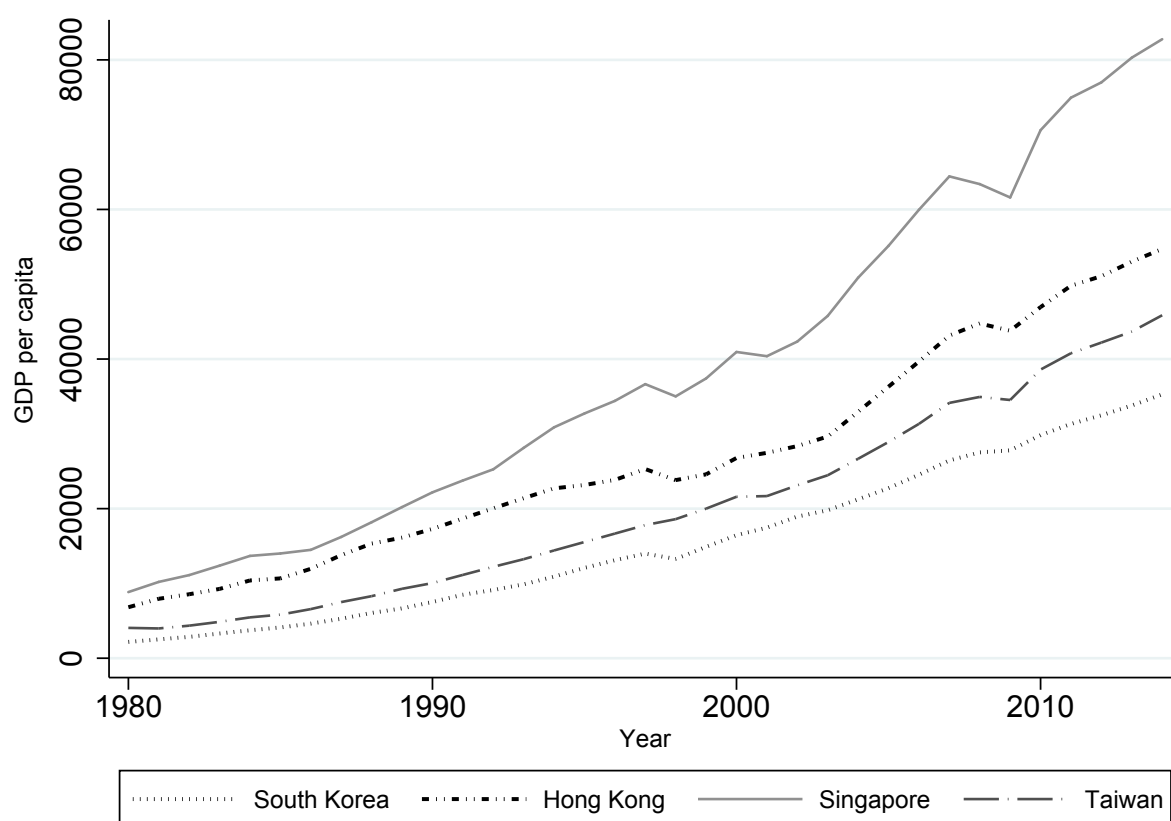


Figure 4 – Gross Domestic Product (GDP) per capita based on Purchasing Power Parity (PPP) in millions of dollars, 1980-2014

Source: Authors' calculations based on data from the International Monetary Fund (2015).

The Hong Kong government played a decisive role in creating the conditions for economic growth and competitiveness, although it was a way to smoother and indirect intervention. Though it had the second highest per capita income among the Tigers, Hong Kong was the country that least developed technological capabilities. As previously mentioned, the country specializes in the service sector and it is considered one of the leading financial centers of Asia.

According to Castells (1992), most of Taiwan's development was made possible by a flexible combination of decentralized networks of local family origin companies

acting as subcontractors for foreign manufacturers located in this country and suppliers of international commercial networks, usually connected by intermediate. From 1960 there was an economic restructuring to transform the country into an export platform for high technology products and through strategic partnerships in the production process, Taiwanese companies had contact with the North American industrial sector, and this has leveraged growth country due to high investments.

According to Cunha (2012), the massive investment in education is a key factor for South Korea to become the great technological leader in almost every innovation indicator. In just a few decades the country of agrarian based economy became equipped with high technological standard and moved quickly from imitation to innovation.

The period of growth and development of the Four Asian Tigers led to increases in productivity levels due to the efficiency, competitiveness and the accumulation of physical capital in these countries. Veloso, Ferreira & Pessoa (2004) show that in the period from 1966 to 1990, while the growth rate of Total Factor Productivity (TFP) was around -0.17% worldwide, the Four Asian Tigers this indicator reached to about 2.09% per year.

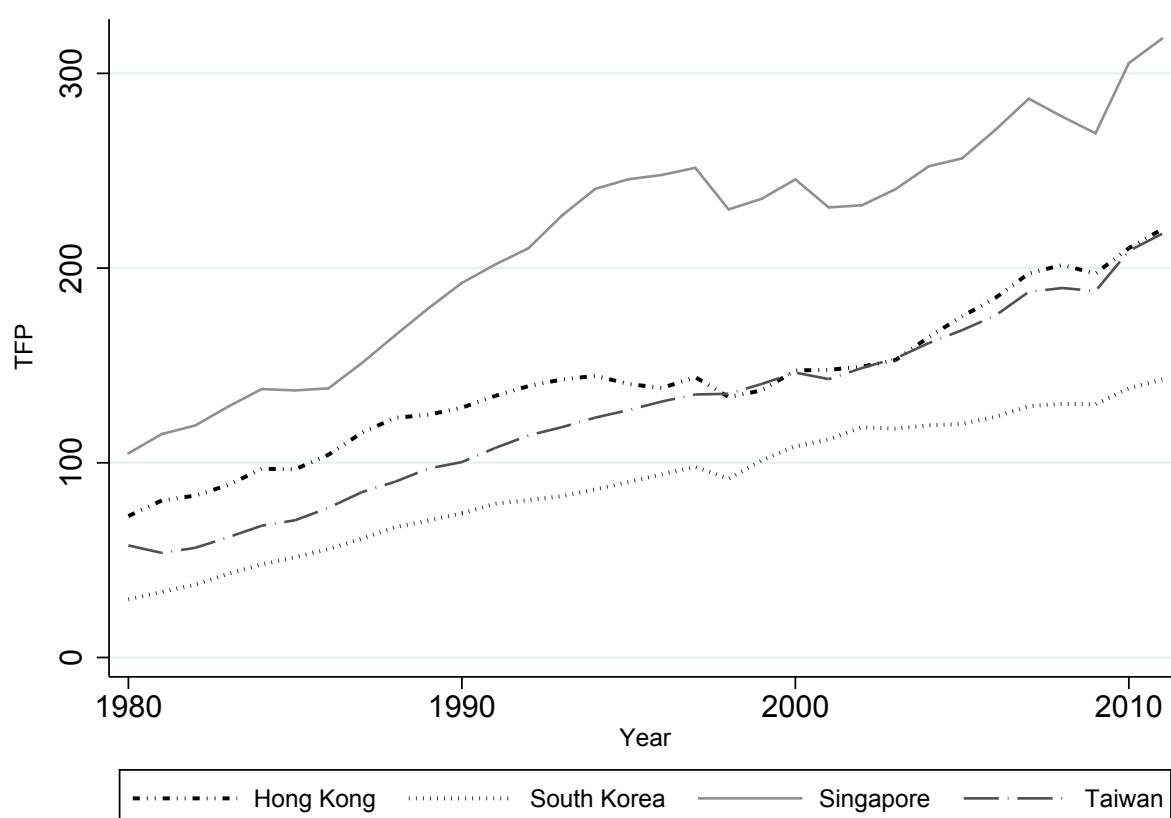


Figure 5 – Total Factor Productivity (TFP) the current level based on Purchasing Power Parity (PPP), 1980 to 2011

Source: Authors' calculations based on data from the Penn World Table 8.0.

We use the data from the Penn World Table 8.0 and the methodology of calculation

made by [Veloso, Ferreira & Pessoa \(2004\)](#)<sup>3</sup>, we can calculate the TFP for the Four Asian Tigers during the period from 1980 to 2011. Through Figure 5, we note the upward trend TFP in the countries concerned.

For the period in Figure 5 (1980-2011), the average growth rate of annual productivity of these countries is in the range of 2.9% to 4.45%. South Korea, in addition to presenting the highest average annual growth rates of human capital index and GDP per capita also has the highest average annual productivity growth (4.45%). Then Taiwan has as average productivity growth 4.25% per year. Already, Singapore and Hong Kong have average rates of TFP growth of 3.10% and 2.92% per year, respectively.

When calculating the partial correlation between GDP per capita and TFP for the period between 1980 and 2011, we can realize that this was 0.92. On the other hand, the partial correlation between GDP per capita and physical capital per capita was 0.90, while the partial correlation between GDP per capita and human capital per capita was only 0.32. Despite the low partial correlation between human capital per capita and GDP per capita, the human capital was important for there to be increases in the hand of the qualifying labor and that countries could grow from the targeted efficiency for the manufacture of high technology products. The large share of the capital stock to raise the growth of countries is given by the entry of new machinery to increase productivity and makes these countries produce with lower costs and increased competitiveness.

Table 1 – Partial correlation between GDP per capita, physical capital per capita, human capital per capita and productivity of the Four Asian Tigers, 1980-2011

	Physical Capital per capita (k)	Human Capital per capita (h)	Total Factor Productivity (TFP)
GDP per capita (y)	0.90	0.32	0.92

Source: Authors' calculations based on data from the Penn World Table 8.0.

Growth and economic development of Asian has been at the center of the main discussions on the strategies and policies to be adopted. The experience of East Asia is a successful example of action and coordinated public policies, aimed at generating skills, technological capabilities and bringing the countries of the world technological frontier.

<sup>3</sup> The authors start from a production function with physical capital, human capital and total factor productivity. For they check the level of productivity of the countries, the authors arrive at the following equation  $A_{it} = \frac{y_{it}}{k_{it}^\alpha H_{it}^{1-\alpha}}$ , where  $A_{it}$  is the TFP country  $i$  at time  $t$ ;  $y_{it}$  is the per capita GDP of country  $i$  at time  $t$ ;  $k_{it}$  e  $H_{it}$  are, respectively, the stock of capital per capita of country  $i$  at time  $t$  and the human capital stock of the country  $i$  at time  $t$ ; capital participation parameter  $\alpha = 0.4$  was calibrated following [Gollin \(2002\)](#) and [Veloso, Ferreira & Pessoa \(2004\)](#). The variable  $H_{it}$ , was collected from the Penn World Table 8.0 and was based on years of schooling [Barro & Lee \(2013\)](#) and returns to education [Psacharopoulos \(1994\)](#), and is called the human capital index per capita.

## 4 THEORETICAL MODEL

As a way we answer the objectives of this research, we have developed a theoretical model able to measure the level of investment and human capital externality in a market economy with perfect competition. In this theoretical model are inserted three generations of agents (children, adults and old agents) and a representative firm. Adults face the trade-off between current consumption (invest in your child's education) or investment in future capital. Children who do not study need to work and they become unskilled workers in the future. The old agents are the only holders of capital that economy and rent this capital for the firm.

We inserted a Social Planner to internalize the externalities caused and compare with the inefficiency of Pareto caused in a market economy. The Social Planner encourages investment in education and raises the future productivity of labor. The productivity of physical capital increases and raises economic growth.

### 4.1 Firm's Problem

Without loss of generality, we assume that there is on the market a representative firm that behaves competitively and choose a sequence of inputs  $(K_t, L_{u_t}, L_{s_t})$  that maximizes the total profit  $(\pi_t)$ . Since each period inputs are rented (the firm does not make the capital accumulation decision) the firm will have a static problem. The firm's production function is represented by a production function with Constant Elasticity of Substitution (CES).

$$Y_t = A(\Omega)L_{u_t}^\nu[\gamma K_t^\rho + (1 - \gamma)L_{s_t}^\rho]^\frac{\alpha}{\rho}$$

where  $A(\Omega)$  is the total factor productivity, with  $\Omega$  aggregate state variables  $\frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}}$ ; where  $\frac{L_{s_t}}{L_{u_t}}$  measures the qualification of labor or skilled labor relative to unskilled labor;  $L_{u_t}$  and  $L_{s_t}$  are unskilled and skilled labor, respectively;  $K_t$  is the capital factor;  $0 < \alpha < 1$  is the degree of homogeneity;  $\rho \leq 1$  determines the elasticity of substitution between capital and skilled labor and the type of production function; and,  $0 \leq \gamma \leq 1$  is the participation of capital and skilled labor in the product.

Thus, if  $\alpha + \nu > 1$  we have increasing return to scale; if  $\alpha + \nu < 1$  we have decreasing returns to scale; and, if  $\alpha + \nu = 1$  we have constant return to scale<sup>1</sup>. If  $\alpha \rightarrow 1$  and  $\rho \rightarrow 1$  the CES converges to linear function and the elasticity between capital

<sup>1</sup> A constant-returns production function at the aggregate level can reflect learning by doing and spillovers of knowledge. According Barro & Sala-i-Martin (1999) this kind of technology may support endogenous growth, but the outcomes tend not to be Pareto optimal because the spillovers constitute a form of externality. Hence, these models may have implications for desirable government policy.

and skilled labor is infinite ( $S_{K,L_s} = \infty$ ); if  $\alpha \rightarrow 1$  and  $\rho \rightarrow -\infty$  the CES function converges to Leontief production function and the elasticity is zero ( $S_{K,L_s} = 0$ ); if  $\alpha \rightarrow \mathbb{R}$  and  $\rho \rightarrow 0$  the CES function converges to Cobb-Douglas function and the elasticity of substitution is unitary ( $S_{K,L_s} = 1$ ). When the elasticity of substitution is greater than 1 will be substitutability between factors of production ( $S_{K,L_s} > 1$ ), and when less than 1 will be complementary ( $S_{K,L_s} < 1$ ). Thus, the closer to zero is the elasticity of substitution, the greater the complementarity of the factors, and the closer to the infinite, the greater the substitutability. It will be assumed constant returns to scale, therefore, the production function becomes:

$$Y_t = A(\Omega)L_{u_t}^{(1-\alpha)}[\gamma K_t^\rho + (1-\gamma)L_{s_t}^\rho]^{\frac{\alpha}{\rho}} \quad (4.1)$$

Note that representative firm do not take into account the increase in total productivity  $\left(A\left(\frac{L_{s_t}}{L_{u_t}}\right)\right)$  by increasing demand for skilled labor (externality). Note the consistent link between different types of workers (skilled and unskilled) with the assumption of complementarity between physical and human capital. Note that there is a function of the Cobb-Douglas between unskilled and skilled labor and a CES function determining the degree of complementarity/substitutability between capital and skilled labor<sup>2</sup>.

Thus, the problem of the representative firm is to maximize the profit:

$$\max_{\{L_{u_t}, L_{s_t}, K_t\}} \pi_t = Y_t - w_t^u L_{u_t} - w_t^s L_{s_t} - r_t K_t \quad (4.2)$$

where  $w_t^u$  and  $w_t^s$  are the wage of workers with the unskilled and skilled labor, respectively, and  $r_t$  is the interest rate. The term  $r_t K_t$  is that firms pay rent to capital belonging to families looking for an investment.

First order conditions of the problem of the Firm:

$$w_t^u(\Omega) = A\left(\frac{L_{s_t}}{L_{u_t}}\right)(1-\alpha)\left[\gamma\left(\frac{K_t}{L_{u_t}}\right)^\rho + (1-\gamma)\left(\frac{L_{s_t}}{L_{u_t}}\right)^\rho\right]^{\frac{\alpha}{\rho}} \quad (4.3)$$

$$w_t^s(\Omega) = A\left(\frac{L_{s_t}}{L_{u_t}}\right)\alpha\left[\gamma\left(\frac{K_t}{L_{u_t}}\right)^\rho + (1-\gamma)\left(\frac{L_{s_t}}{L_{u_t}}\right)^\rho\right]^{\frac{\alpha-\rho}{\rho}}(1-\gamma)\left(\frac{L_{s_t}}{L_{u_t}}\right)^{\rho-1} \quad (4.4)$$

$$r_t(\Omega) = A\left(\frac{L_{s_t}}{L_{u_t}}\right)\alpha\left[\gamma\left(\frac{K_t}{L_{u_t}}\right)^\rho + (1-\gamma)\left(\frac{L_{s_t}}{L_{u_t}}\right)^\rho\right]^{\frac{\alpha-\rho}{\rho}}\gamma\left(\frac{K_t}{L_{u_t}}\right)^{\rho-1} \quad (4.5)$$

<sup>2</sup> The neoclassical aggregate production function suggests that if skilled and unskilled workers are imperfect substitutes, and the key feature of the technology is the complementarity between capital stock and skill labor. It means that the elasticity of substitution between capital stock and unskilled labor is higher than that between capital stock and skilled labor. A key implication of capital-skill complementarity is that growth in the stock of equipment increases the marginal product of skilled labor, but decreases the marginal product of unskilled labor ((KRUSELL et al., 2000); (MORETTI, 2004); (ACEMOGLU; AUTOR, 2012)).

## 4.2 Preferences

There are three generations in each period: children, adult and old person. It is assumed that just adult agents make the economic decisions. The final good produced by firm is a composite good used for consumption and investment in physical capital. The adult chooses consumption level,  $c$ , future capital for retirement purposes,  $k_{t+1}$ , and if he will educate his children or not,  $e \in \{0, 1\}$ .

If the children go to school, they will become skilled workers next period ( $h' = s$ ). If parents decide to not educate their children, children will work as unskilled labor force with productivity  $l$ . Children who work will become unskilled workers next period ( $h' = u$ ).

## 4.3 Individual State Variables

1. Old Agents: Capital level –  $k$ .
2. Adult Agents: (1) Idiosyncratic labor productivity:  $z$ , which follows a Markov chain<sup>3</sup> with transition matrix  $P$  and; (2) Level of human capital:  $h \in \{s, u\}$ .

Aggregate State Variables

- $\frac{K_t}{L_{u_t}}$  and  $\frac{L_{s_t}}{L_{u_t}}$ . Where,  $\frac{L_{s_t}}{L_{u_t}}$  measure the qualification of labor or skilled labor relative to unskilled labor.

Households Problem:

1. Old Agent: When old, agents just consume ( $c^o$ ) the lifetime savings in form of capital:

$$\max_{\{c^o\}} u(c^o) \tag{4.6}$$

Subject to:

$$c^o \leq (1 + r - \delta)k$$

Note that capital is reversible, the owners of total physical capital in the economy is the old agents. The solution for old agent problem is trivial and equals to:  
 $c^o = (1 + r - \delta)k$ .

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<sup>3</sup> A Markov chain is a random process that undergoes transitions from one state to another on a state space. It must possess a property that is usually characterized as "memoryless": the probability distribution of the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes.

2. Adult Problem: The Bellman equation associated with the adult agent problem with  $z$  level of idiosyncratic labor productivity,  $h$  level of human capital, where the individual state variables are:

$$\{z \in \{z_1, z_2\}; h \in \{s, u\}\} \quad (4.7)$$

$$\begin{aligned} & V_z^h \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) = \\ & = \max_{(c, k_{t+1}, e)} \left\{ u(c) + \beta E \left[ \theta V_{z'}^{h'} \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) + (1 - \theta) u \left( 1 + r \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) - \delta \right) k_{t+1} \right] \right\} \end{aligned} \quad (4.8)$$

Subject to:

- (i)  $c + k_{t+1} + \varphi e \leq w^h \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) z + (1 - e) l w^u \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right)$ ;
- (ii)  $c \geq 0, k_{t+1} \geq 0, e \in \{0, 1\}$ .

Note that if adult parent decides to educate their children, then  $V_{z'}^{h'} \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) = V_{z'}^s \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right)$ , otherwise,  $V_{z'}^{h'} \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) = V_{z'}^u \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right)$ , therefore, Bellman equation can also be written as:

$$\begin{aligned} V_z^h \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) & = \max_{(c_z^h, k_{t+1}^h, e_z^h)} u(c) + \beta E \left[ \theta \left( e V_{z'}^s \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) + (1 - e) V_{z'}^u \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) \right) \right] \\ & \quad + \beta E \left[ (1 - \theta) u \left( 1 + r \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) - \delta \right) k_{t+1} \right] \end{aligned} \quad (4.9)$$

where  $\beta > 0$  is the subjective discount factor and  $\theta \in [0, 1]$  is the altruism parameter that governs the utility that adult agent receives from children future utility versus his own future utility when old. Hence, higher  $\theta$ , higher the incentives that parents have to invest in their children's human capital and lower the incentives to invest in physical capital for their own future consumption.

Constraint (i) is the budget constraint for adult agent with  $z$  level of idiosyncratic labor productivity level,  $h$  level of human capital with  $h \in \{s, u\}$ . Define  $\varphi$  as the education cost per children. Total expenditures on consumption,  $c$ , investment in future capital for retirement,  $k_{t+1}$  and total education cost of children,  $\varphi e$ , if agent decides to educate his children,  $e = 1$ , equals total income in the period: wage income of the agent with skill level  $h$  and  $z$  idiosyncratic labor productivity, if agent decides to not educate his children,  $e = 0$ , then children will work and receive:  $l w^u \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right)$ . Note that the choice set for education is a non-convex set. Finally, the instantaneous utility function is assumed strictly increasing and strictly concave in consumption.

The necessary and sufficient conditions for adult agents are (from First Order Conditions (FOC)), it is important to note that you cannot derive a FOC for education, since the choice for this variable is non-convex):

$$u'(c) = (1 - \theta)\beta E \left[ u' \left( \left( 1 + r \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) - \delta \right) k_{t+1} \right) \left( 1 + r \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) - \delta \right) \right] \quad (4.10)$$

Equation above can also be written as:

$$u'(c) = (1 - \theta)\beta E \left[ u'(c^o) \left( 1 + r \left( \frac{K_{t+1}}{L_u}, \frac{L_{s_{t+1}}}{L_u} \right) - \delta \right) \right] \quad (4.11)$$

where  $c^o$  is future consumption when old, equation above is a version of the renowned Euler equation. It shows how this agent trades present and future consumption. The difference between equation above and the ordinary Euler equation is the  $(1 - \theta)$  term. This term comes from the fact that there is an additional trade-off to invest in physical capital in this model: investing in children human capital. It is straightforward to show that this additional trade-off decreases the level of investment in physical capital for households in comparison with a model without human capital.

Moreover:

if  $e = 0$

$$c = w^h \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) z + l w^u \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) - k_{t+1} \quad (4.12)$$

if  $e = 1$

$$c = w^h \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) z - \varphi - k_{t+1} \quad (4.13)$$

Let  $\mu_t$  be the unconditional distribution of  $(h, z)$  pairs, i.e.,  $\mu_{t+1}(h', z') = \sum_z \sum_{\{h: e=g(h, z)\}} \mu_t(h, z) P(z, z')$ . Where  $g(h, z)$  is the optimal education level policy for agent with skill level  $h$  and idiosyncratic productivity  $z$ . We will look for a time-invariant distribution, i.e.,  $\mu_t = \mu_{t+1}, \forall t$ .

## 4.4 Competitive Equilibrium Definition

A stationary recursive competitive equilibrium for this economy consists of value functions  $V_z^h$ , policy functions  $(c_z^h, k_{z_{t+1}}^h, e_z^h)$  for  $z \in \{z_1, z_2\}$  and  $h \in \{s, u\}$ ; consume function for old agents ( $c^o$ ); price functions  $(w^s, w^u, r)$  and a stationary distribution  $\mu = (h, z)$  such that:

- $c^o = (1 + r - \delta)k$ ;
- $V_z^h$  satisfies young household Bellman equation and  $(c_z^h, k_{z_{t+1}}^h, e_z^h)$  are the associated policy functions;
- $w^s, w^u, r$  are given by equations 4.3, 4.4 and 4.5;
- The stationary distribution  $\mu(h, z)$  is induced by the Markov chain for  $z$  and the optimal

policy  $\{e_z^h\}$ ;

e) Market clear:

i.  $L_s = \sum_j z_j \mu(s, z)$

ii.  $L_u = \sum_j z_j \mu(u, z)$

iii.  $K_{t+1} = \sum_{h,z} k_{z_{t+1}}^h \mu(h, z)$

## 4.5 Solving for the stationary competitive equilibrium

Prices are constant (there is no aggregate uncertainty), therefore, Euler equation:

$$u'(c) = (1 - \theta)\beta(1 + r(\Omega_{t+1}) - \delta)[u'(1 + r(\Omega_{t+1}) - \delta)k_{t+1}] \quad (4.14)$$

where

$$c = w^h(\Omega)z + lw^u(\Omega) - k_{0_{t+1}} \text{ if } e = 0,$$

$$c = w^h(\Omega)z - \varphi - k_{1_{t+1}} \text{ if } e = 1.$$

**Proposition 1** *If  $e = 0$  then  $(k_{z_{t+1}}^h)_{e=1} < (k_{z_{t+1}}^h)_{e=0}$ . There is a trade-off between human and physical capital.*

**Proof 1** *By contradiction, let  $e = 0$  and  $k_{1_{t+1}} \geq k_{0_{t+1}}$ ,  $(c)_{e=1} < (c)_{e=0}$ , then  $u'(c_{e=1}) > u'(c_{e=0})$  since  $u(\cdot)$  is strictly concave. Therefore, contradicts the Euler equation:*

$$\begin{aligned} u'[(1 + r(\Omega_{t+1}) - \delta)k_{1_{t+1}}] &< u'[(1 + r(\Omega_{t+1}) - \delta)k_{0_{t+1}}] \\ k_{1_{t+1}} &> k_{0_{t+1}} \end{aligned}$$

**Assumption 1** *The unskilled Agent does not invest in education if the future benefit of educating is less than the utility of the unskilled Agent does not educate their children.*

$$\begin{aligned} \beta E[\theta(V_{z_{t+1}}^s - V_{z_{t+1}}^u)] &< u(w^u(\Omega)z_1 + lw^u(\Omega) - k_{0_{t+1}}) - u(w^u(\Omega)z_1 - \varphi - k_{1_{t+1}}) \\ &+ \beta(1 - \theta)u[(1 + r(\Omega) - \delta)k_{0_{t+1}}] - \beta(1 - \theta)u[(1 + r(\Omega) - \delta)k_{1_{t+1}}] \end{aligned}$$

**Proposition 2** *Under assumption 1, then not all types of unskilled Agents with idiosyncratic low ( $z_1$ ) invested in their children's education.  $e_{z_1}^u = 0$ , if and only if  $V_{z_1}^u(e_{z_1}^u = 1) < V_{z_1}^u(e_{z_1}^u = 0)$ .*

**Proof 2**

$$\begin{aligned} V_{z_1}^u(e_{z_1}^u = 1) &= u(w^u(\Omega)z_1 - \varphi - k_{1_{t+1}}) + \beta E[\theta V_{z_1}^s(\Omega_{t+1})] + \beta(1 - \theta)u[(1 + r(\Omega_{t+1}) - \delta)k_{1_{t+1}}] \\ &< u(w^u(\Omega)z_1 + lw^u(\Omega) - k_{0_{t+1}}) + \beta E[\theta V_{z_1}^u(\Omega_{t+1})] + \beta(1 - \theta)u[(1 + r(\Omega_{t+1}) - \delta)k_{0_{t+1}}] \\ &= V_{z_1}^u(e_{z_1}^u = 0) \end{aligned}$$

The utility of the agent with the unskilled labor ( $u$ ), low productivity ( $z_1$ ) and investing in education ( $e = 1$ ), will be less than the utility of the agent with unskilled labor ( $u$ ) and that does not invest in education ( $e = 0$ ). The Adult with unskilled labor and low productivity, who invests in the education of children, will replace consumption and investment in physical capital to pay the cost of education of children, in addition, the children will not receive ( $lw^u$ ) paid for the lower productivity of the unskilled labor.

The costs of investing in education are related to the cost of education and the loss of the children would receive if they work with unskilled labor, even if their productivity is less than an Adult worker any.

**Proposition 3** *If  $e_{z_1}^u = 1$ , then  $e_z^h = 1 \forall z, \forall h$*

The utility function of Adult agent with unskilled labor and low productivity is higher than if he does not invest in education (Assumption 1), but if the worst Adult agent decides to invest in human capital, then all the other agents will invest in education too. However, if parents decide to invest in children's education, the future benefit of children educated will be greater than the benefit if the Adult agent decides not to invest in education ( $\beta E[V_{z_1}^s(\Omega_{t+1})] > \beta E[V_{z_1}^u(\Omega_{t+1})]$ ).

$$\text{If } e_{z_1}^u = 1 \{e_{z_2}^s = 1; e_{z_1}^s = 1; e_{z_2}^u = 1\}$$

**Proof 3** (3.1) *The agent's utility with skilled labor, higher productivity and investing in education is higher than the agent's utility with unskilled labor, lower productivity and investing in education.*

$$\begin{aligned} & V_{z_2}^s(e_{z_2}^s = 1) > V_{z_1}^u(e_{z_1}^u = 1) \\ & u(w^s(\Omega)z_2 - \varphi - k_{1,t+1}) - u(w^u(\Omega)z_1 - \varphi - k_{1,t+1}) + \beta E\theta[V_z^s(\Omega_{t+1}) - V_z^s(\Omega_{t+1})] \\ & + \beta(1 - \theta)[u(1 + r(\Omega_{t+1}) - \delta)k_{1,t+1} - u(1 + r(\Omega_{t+1}) - \delta)k_{1,t+1}] > 0 \\ & w^s(\Omega)z_2 > w^u(\Omega)z_1 \end{aligned}$$

However, in cases (3.1) and (3.3), the future physical capital are different given that individuals have different idiosyncratic productivity ( $z_1 < z_2$ ). The probabilities of the future benefits of the agents ( $V_z^s(\Omega_{t+1})$ ) will be different, and follow a Markov chain in which:

$$\text{Prob}(z_2'|z_1) > \text{Prob}(z_1'|z_2)$$

(3.2) *The agent's utility with skilled labor, lower productivity and investing in education is higher than the agent's utility with unskilled labor, lower productivity and*

investing in education.

$$\begin{aligned}
& V_{z_1}^s(e_{z_1}^s = 1) > V_{z_1}^u(e_{z_1}^u = 1) \\
& u(w^s(\Omega)z_1 - \varphi - k_{1_{t+1}}) - u(w^u(\Omega)z_1 - \varphi - k_{1_{t+1}}) + \beta E\theta[V_{z_1}^s(\Omega_{t+1}) - V_{z_1}^s(\Omega_{t+1})] \\
& + \beta(1 - \theta)[u(1 + r(\Omega_{t+1}) - \delta)k_{1_{t+1}} - u(1 + r(\Omega_{t+1}) - \delta)k_{1_{t+1}}] > 0 \\
& w^s(\Omega)z_1 > w^u(\Omega)z_1
\end{aligned}$$

In this case, the agents have the same idiosyncratic productivity ( $z_1$ ) and the probabilities of future benefits for agents ( $V_{z_1}^s(\Omega_{t+1})$ ) will be the same, follow a Markov chain.

(3.3) The agent's utility with unskilled labor, higher productivity and investing in education is higher than the utility of the agent with unskilled labor, lower productivity and investing in education.

$$\begin{aligned}
& V_{z_2}^u(e_{z_2}^s = 1) > V_{z_1}^u(e_{z_1}^u = 1) \\
& u(w^u(\Omega)z_2 - \varphi - k_{1_{t+1}}) - u(w^u(\Omega)z_1 - \varphi - k_{1_{t+1}}) + \beta E\theta[V_z^s(\Omega_{t+1}) - V_z^s(\Omega_{t+1})] \\
& + \beta(1 - \theta)[u(1 + r(\Omega_{t+1}) - \delta)k_{1_{t+1}} - u(1 + r(\Omega_{t+1}) - \delta)k_{1_{t+1}}] > 0 \\
& w^u(\Omega)z_2 > w^u(\Omega)z_1
\end{aligned}$$

**Proposition 4** In the Competitive Equilibrium  $L_{ut} > 0 \iff e_{z_1}^u = 0$ .

**Proof 4** Under Assumption 1 and Proposition 2, the unskilled worker with low productivity does not invest in education because their future benefit is greater than the value of investing in education. Therefore, the unskilled labor is no negative for all types of qualification and idiosyncratic productivity, and the production function will be different to zero ( $L_{ut} > 0$ ).

## 4.6 Social Planner Problem

Let the benevolent social planner maximizes a social welfare function and internalizes the external effects. The social planner must allocate the society's resources  $\left(\frac{K_t}{L_u}, \frac{L_{st}}{L_u}\right)$  between consumption of the three generations alive the time  $t$  and the capital stock in period  $t + 1$ .

- i. Each generation has measure one;
- ii. No productivity;
- iii. For given  $\Psi \in [0, 1]$  and  $\Gamma \in [0, 1]$ .

where,  $\Gamma$  is the weight that the social planner gives to intergenerational (child, adult and

old person) and  $\Psi$  is the weight that the social planner gives to same generation, between unskilled and skilled adult.

$$\max_{\{c_t^u, c_t^s, c_t^o, L_{s_{t+1}}, L_{u_{t+1}}^A, K_{t+1}^0\}} \sum_{t=0}^{\infty} \beta^t \{ \Gamma [\Psi L_{u_t}^A u(c_t^u) + (1 - \Psi) L_{s_t} u(c_t^s)] + (1 - \Gamma) u(c_t^o) \} \quad (4.15)$$

Subject to:

$$L_{u_t}^A c_t^u + L_{s_t} c_t^s + c_t^o + K_{t+1}^0 + \varphi L_{s_{t+1}} = A \left( \frac{L_{s_t}}{L_{u_t}} \right) L_{u_t}^{(1-\alpha)} [\gamma K_t^\rho + (1 - \gamma) L_{s_t}^\rho]^{\frac{\alpha}{\rho}} + (1 - \delta) K_t \quad (4.16)$$

$$L_{u_t} = L_{u_t}^A + (1 - L_{s_{t+1}}) l \quad (4.17)$$

$$K_0 > 0; L_{u_0}, L_{s_0} \in [0, 1] \quad (4.18)$$

Since  $L_{s_{t+1}} = 1 - L_{u_{t+1}}^A$  and  $L_{s_0} = 1 - L_{u_0}^A$  then we can write the production above

$$L_{u_t} = (1 - L_{s_{t+1}}) l + (1 - L_{s_t}) \quad (4.19)$$

Note that, the social planner choice  $L_{s_{t+1}}$  (future skilled labor). Therefore, the children that studying are  $L_{s_{t+1}}$  and  $(1 - L_{s_{t+1}})$  are the children who don't studying (child labor).

$$\max_{\{c_t^u, c_t^s, c_t^o, L_{s_{t+1}}, K_{t+1}^0\}} \sum_{t=0}^{\infty} \beta^t \{ \Gamma [\Psi (1 - L_{s_t}) u(c_t^u) + (1 - \Psi) L_{s_t} u(c_t^s)] + (1 - \Gamma) u(c_t^o) \} \quad (4.20)$$

Subject to:

$$(1 - L_{s_t}) c_t^u + L_{s_t} c_t^s + c_t^o + K_{t+1}^0 + \varphi L_{s_{t+1}} = A \left( \frac{L_{s_t}}{(1 - L_{s_{t+1}}) l + (1 - L_{s_t})} \right) \quad (4.21)$$

$$[(1 - L_{s_{t+1}}) l + (1 - L_{s_t})]^{(1-\alpha)} [\gamma K_t^\rho + (1 - \gamma) L_{s_t}^\rho]^{\frac{\alpha}{\rho}} + (1 - \delta) K_t$$

$$c_t^u, c_t^s, c_t^o \geq 0, K_{t+1}^0 \geq 0, L_{s_{t+1}} \in [0, 1] \quad (4.22)$$

$$K_0 > 0; L_{s_0} \in [0, 1] \quad (4.23)$$

First order conditions of the social planner problem

$$\begin{aligned} \frac{\partial L}{\partial L_{s_{t+1}}} &= -\beta^{t+1} \Gamma \Psi u(c_{t+1}^u) + \beta^{t+1} \Gamma (1 - \Psi) u(c_{t+1}^s) \\ &+ \lambda_{t+1} \left[ \frac{\partial A_{t+1}}{\partial L_{s_{t+1}}} L_{u_{t+1}}^{1-\alpha} [\gamma K_{t+1}^\rho + (1 - \gamma) L_{s_{t+1}}^\rho]^{\frac{\alpha}{\rho}} + A_{t+1} (1 - \alpha) L_{u_{t+1}}^{-\alpha} \frac{\partial L_{u_{t+1}}}{\partial L_{s_{t+1}}} [\gamma K_{t+1}^\rho + (1 - \gamma) L_{s_{t+1}}^\rho]^{\frac{\alpha}{\rho}} \right] \\ &+ \lambda_{t+1} \left[ A_{t+1} L_{u_{t+1}}^{1-\alpha} \left( \frac{\alpha}{\rho} \right) [\gamma K_{t+1}^\rho + (1 - \gamma) L_{s_{t+1}}^\rho]^{\frac{\alpha-1}{\rho}} \rho (1 - \gamma) L_{s_{t+1}}^{\rho-1} \right] - \lambda_{t+1} [-c_{t+1}^u + c_{t+1}^s] - \lambda_t \varphi = 0 \end{aligned} \quad (4.24a)$$

$$\begin{aligned}
& \beta^{t+1}\Gamma(1-\Psi)u(c_{t+1}^s) + \lambda_{t+1}c_{t+1}^u + \lambda_{t+1}\left[\frac{\partial A_{t+1}}{\partial L_{s_{t+1}}}L_{u_{t+1}}^{1-\alpha}[\gamma K_{t+1}^\rho + (1-\gamma)L_{s_{t+1}}^\rho]^\frac{\alpha}{\rho}\right] \\
+ \lambda_{t+1}\left[A_{t+1}L_{u_{t+1}}^{1-\alpha}\left(\frac{\alpha}{\rho}\right)[\gamma K_{t+1}^\rho + (1-\gamma)L_{s_{t+1}}^\rho]^\frac{\alpha-1}{\rho}\rho(1-\gamma)L_{s_{t+1}}^{\rho-1}\right] &= \beta^{t+1}\Gamma\Psi u(c_{t+1}^u) + \lambda_{t+1}c_{t+1}^s \\
& + \lambda_{t+1}\left[-A_{t+1}(1-\alpha)L_{u_{t+1}}^{-\alpha}\frac{\partial L_{u_{t+1}}}{\partial L_{s_{t+1}}}[\gamma K_{t+1}^\rho + (1-\gamma)L_{s_{t+1}}^\rho]^\frac{\alpha}{\rho}\right] + \lambda_t\varphi
\end{aligned} \tag{4.24b}$$

$$\frac{\partial L}{\partial K_{t+1}} = -\lambda_t + \lambda_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + (1-\delta)\right] = 0 \tag{4.25a}$$

$$\lambda_t = \lambda_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + (1-\delta)\right] \tag{4.25b}$$

$$\frac{\partial L}{\partial c_t^u} = \beta^t\Gamma\Psi(1-L_{s_t})u'(c_t^u) - \lambda_t(1-L_{s_t}) = 0 \tag{4.26a}$$

$$\lambda_t = \beta^t\Gamma\Psi u'(c_t^u) \tag{4.26b}$$

$$\frac{\partial L}{\partial c_t^s} = \beta^t\Gamma(1-\Psi)L_{s_t}u'(c_t^s) - \lambda_t L_{s_t} = 0 \tag{4.27a}$$

$$\lambda_t = \beta^t\Gamma(1-\Psi)u'(c_t^s) \tag{4.27b}$$

$$\frac{\partial L}{\partial c_t^o} = \beta^t(1-\Gamma)u'(c_t^o) - \lambda_t = 0 \tag{4.28a}$$

$$\lambda_t = \beta^t(1-\Gamma)u'(c_t^o) \tag{4.28b}$$

$$\frac{\partial L}{\partial \lambda_t} = (1-L_{s_t})c_t^u + L_{s_t}c_t^s + c_t^o + K_{t+1} + \varphi L_{s_{t+1}} = Y_{t+1} + (1-\delta)K_t \tag{4.29}$$

Then, using equations 4.26b, 4.27b and 4.28b we have to:

$$\beta^t\Gamma\Psi u'(c_t^u) = \beta^t\Gamma(1-\Psi)u'(c_t^s) = \beta^t(1-\Gamma)u'(c_t^o) \tag{4.30a}$$

$$\Gamma\Psi u'(c_t^u) = \Gamma(1-\Psi)u'(c_t^s) = (1-\Gamma)u'(c_t^o) \tag{4.30b}$$

The relation of the agents' marginal utility of consumption is given only by the weights of the social planner.

Then, without loss of generality we can assume that  $\Psi = \frac{1}{2}$ . So,  $c_t^u = c_t^s, \forall t$ .

$$\lambda_{t+1} \left[ \frac{\partial A_{t+1}}{\partial L_{s_{t+1}}} \frac{Y_{t+1}}{A_{t+1}} + \frac{\partial Y_{t+1}}{\partial L_{s_{t+1}}} \right] = \lambda_{t+1} \left[ -\frac{\partial Y_{t+1}}{\partial L_{u_{t+1}}} \frac{\partial L_{u_{t+1}}}{\partial L_{s_{t+1}}} \right] + \frac{\lambda_t}{\lambda_{t+1}} \varphi \quad (4.31a)$$

$$\frac{\partial A_{t+1}}{\partial L_{s_{t+1}}} \frac{Y_{t+1}}{A_{t+1}} + \frac{\partial Y_{t+1}}{\partial L_{s_{t+1}}} = \frac{\partial Y_{t+1}}{\partial L_{u_{t+1}}} \left( -\frac{\partial L_{u_{t+1}}}{\partial L_{s_{t+1}}} \right) + \varphi \left( \frac{\partial Y_{t+1}}{\partial K_{t+1}} + 1 - \delta \right) \quad (4.31b)$$

If the term  $\left( \frac{\partial A_{t+1}}{\partial L_{s_{t+1}}} \right)$  is positive, then there is externality. Otherwise, it will be zero. With externality, equation 4.31b of the social planner optimal is larger on the left side, as well, to the social planner maintain equality, the marginal productivity of skilled labor is expected to fall and/or the marginal productivity of the unskilled labor should increase. In other words, the social planner in Pareto optimal, choose a higher level of human capital than in competitive equilibrium, to internalize the beneficial effects of human capital externality.

As the number of skilled workers increases, the marginal productivity of skilled labor tends to reduce. Thus, the marginal productivity of unskilled labor increases because the increasing number of skilled workers in regarding the number of unskilled workers.

The sum of the right side of the social planner equation represents the substitutability between capital and unskilled labor. In other words,  $L_s$ ,  $L_u$  and  $K$  are such that the left side of the social planner equation is greater than the right side (case of competitive equilibrium). Therefore, to reach the Pareto optimum, the social planner should decrease the left and/or increase the right side of the equation 4.31b.

To decrease the left side of the equation, the social planner must reduce the marginal productivity of skilled labor, that is, increase the number of skilled workers in regarding the number of unskilled workers. Similarly, to increase the right side, the social planner should increase marginal productivity of the unskilled labor, in other words, to increase skilled labor in relation to the unskilled labor.

Note that increasing skilled labor relative to unskilled labor, social planner also increases the marginal productivity of capital because skilled labor is complementary to capital and substitute the unskilled labor. Physical capital and human capital are complementary in the model, and then the increase in skilled individuals raises the stock of physical capital, which leads to economic growth.

According [Barbosa Filho & Pessoa \(2010\)](#) and [Acemoglu, Gallego & Robinson \(2014\)](#) the human capital causes economic growth because that rich countries have educated workers (human capital), greater levels of productivity, tools and factories (physical capital), and more machines. The little existing human capital in poor countries is a constraint that hinders the use of investment in physical capital, and limited economic growth.

Thus, the return of skilled workers reduces when compared to return given to unskilled worker to the extent that the difference between these two returns is minimized by the social planner. The social planner minimizes the gap between private and social return on investment in human capital.

## 5 METHODOLOGY

We will use the simulation to measure the human capital externality and the welfare level of the agents from the theoretical model created. The model will be applied to the region called the Four Asian Tigers. The main equations of the model to be calibrated and simulated using MATLAB software are:

$$w_t^u(\Omega) = A \left( \frac{L_{st}}{L_{ut}} \right) (1 - \alpha) \left[ \gamma \left( \frac{K_t}{L_{ut}} \right)^\rho + (1 - \gamma) \left( \frac{L_{st}}{L_{ut}} \right)^\rho \right]^{\frac{\alpha}{\rho}} \quad (5.1)$$

$$w_t^s(\Omega) = A \left( \frac{L_{st}}{L_{ut}} \right) \alpha \left[ \gamma \left( \frac{K_t}{L_{ut}} \right)^\rho + (1 - \gamma) \left( \frac{L_{st}}{L_{ut}} \right)^\rho \right]^{\frac{\alpha - \rho}{\rho}} (1 - \gamma) \left( \frac{L_{st}}{L_{ut}} \right)^{\rho - 1} \quad (5.2)$$

$$r_t(\Omega) = A \left( \frac{L_{st}}{L_{ut}} \right) \alpha \left[ \gamma \left( \frac{K_t}{L_{ut}} \right)^\rho + (1 - \gamma) \left( \frac{L_{st}}{L_{ut}} \right)^\rho \right]^{\frac{\alpha - \rho}{\rho}} \gamma \left( \frac{K_t}{L_{ut}} \right)^{\rho - 1} \quad (5.3)$$

where  $w_t^u$  and  $w_t^s$  are the wage of workers with the unskilled and skilled labor, respectively, and  $r_t$  is the interest rate;  $A \left( \frac{L_{st}}{L_{ut}} \right)$  is the total productivity (externality);  $L_{ut}$  and  $L_{st}$  are unskilled and skilled labor, respectively;  $K_t$  is the capital factor;  $\alpha > 0$  is the degree of homogeneity;  $\rho \leq 1$  determines the elasticity of substitution between capital and skilled labor and the type of production function; and,  $0 \leq \gamma \leq 1$  is the participation of capital and skilled labor in the product.

In the competitive equilibrium, firms maximize the profit subject to restrictions. The first order conditions above will be calibrated and simulated to make it possible to measure the prices in a market economy.

The problem of the old agent is:

$$c^o = (1 + r - \delta)k$$

where  $c^o$  is consume of old agent;  $\delta$  is the depreciation;  $k$  is the capital level per capita. Note that capital is reversible, the owners of total physical capital in the economy is the old agents.

Prices are constant, in other words, there is no aggregate uncertainty. Therefore, the adult problem is:

$$u'(c) = (1 - \theta)\beta(1 + r(\Omega_{t+1}) - \delta)[u'(1 + r(\Omega_{t+1}) - \delta)k_{t+1}] \quad (5.4)$$

where

$$\begin{aligned} c &= w^h(\Omega)z + lw^u(\Omega) - k_{0t+1} \text{ if } e = 0, \\ c &= w^h(\Omega)z - \varphi - k_{1t+1} \text{ if } e = 1. \end{aligned}$$

The equation above is a version of the renowned Euler equation. It shows how this agent trades present and future consumption. The term  $(1 - \theta)$  comes from the fact that there is an additional trade-off to invest in physical capital in this model: investing in children human capital.

Let  $\mu_t$  be the unconditional distribution of  $(h, z)$  pairs, i.e.,  $\mu_{t+1}(h', z') = \sum_z \sum_{\{h: e=g(h, z)\}} \mu_t(h, z) P(z, z')$ . Where  $g(h, z)$  is the optimal education level policy for agent with skill level  $h$  and idiosyncratic productivity  $z$ . We will look for a time-invariant distribution, i.e.,  $\mu_t = \mu_{t+1}, \forall t$ .

We inserted a benevolent government to represent the social planner in the simulated model. The benevolent government taxes the consumption of all agents (skilled and unskilled) and he subsidizes a percentage of the cost of education. The benevolent government internalizes the Pareto inefficiency that economy. Thus, the consumption of the agents who do not invest in education ( $e = 0$ ) will be represented by:

$$\begin{aligned} c_{e=0}^s &= w^s(\Omega)z + lw^u(\Omega) - k_{t+1} - lump \\ c_{e=0}^u &= w^u(\Omega)z + lw^u(\Omega) - k_{t+1} - lump \end{aligned}$$

The government taxes the consumption of adult agents with a lump-sum tax and returns this revenue as a subsidy to the cost of education. Thus, the consumption of the agents who invest in education ( $e = 1$ ) will be:

$$\begin{aligned} c_{e=1}^s &= w^s(\Omega)z - \varphi(1 - \tau) - k_{t+1} - lump \\ c_{e=1}^u &= w^u(\Omega)z - \varphi(1 - \tau) - k_{t+1} - lump \end{aligned}$$

The consumption of the skilled and unskilled agent who invests in education will be increased by the percentage of the subsidy ( $\tau$ ) of the cost of education and reduced from the tax ( $lump$ ). The lump-sum tax ( $lump$ ) is equal to  $\frac{\phi\tau}{2}$  and it returns in form of subsidy ( $\tau$ ) just to agent who decided to invest in child education.

In a market economy there is underinvestment in education that reduces labor productivity and impacts on long-run growth. With fewer skilled workers, there will be a direct impact on productivity of physical capital that limits the development of the economy. In presence of the benevolent government, as well as in presence of the Social Planner in the theoretical model, the investments in education will be encouraged and the increasing number of skilled agents will increase the productivity of capital. This increase will generate greater long-run growth.

## 5.1 Simulation and Calibration forms

The simulation is a method of numerical analysis of a dynamic model that allows the development experiences and to quantify them. The simulation uses the mathematical techniques used in computers, which allow imitate the operation of almost any type of

operation or real world process.

According to Hansen & Heckman (1996), the simulation model implies a modeling of a process or system, so that the model imitates the responses of the real system a sequence of events that occur over time. Thus, we can develop analysis from changing some system parameters, checking to what extent and in what sense this system can be affected.

Shannon (1998) points out that there are two reasons justifying the use of simulation models in economic scientific papers. Initially, there is the fact that the simulation method enables the observation of the intertemporal behavior of a theoretical and analytical representation of a model in which we can establish complex relationships among many variables and observe emerging dynamic phenomena of these interactions, which would be difficult, achieved through purely theoretical analysis. Secondly, this method makes possible the dynamic analysis without the need for extensive simplifications that may disfigure the object under examination.

An advantage of using mathematical and computational models is its economicity. The several alternatives of configuration of a system need to be studied several times, constructed, decomposed and reconstructed to a real system, and it makes the other alternatives very costly (PASSOS, 2008).

In addition, real-world systems tend to be more complex than desired and, above all, do not have a predictable behavior. Simplifications about these systems aiming analytical studies can lead to poor solutions and even unreliable. While analytical models require a very large number of simplifications to make them mathematically tractable, simulation models have no such restrictions. Moreover, in the analytic models, analyzes include only on a limited number of performance measures. On the other hand, the information generated by simulation models allows the analysis of almost any conceivable measure. A simulation study usually shows as possibly a system operates, as opposed to the way everyone thinks it operates (Freitas Filho, 2001).

Kyndland & Prescott (1991) argue that it is the model calibration process that defines how to process the data so that the models imitate the reality as much as possible within a limited number of dimensions. Some economic issues have known the answers and model study needs to be able to reproduce them, so it can be considered reliable and adequate to meet the unknown issues.

In general, in the calibration procedure, first determine if ranges of values that each parameter can have, in order, whenever possible, relying on estimated empirically or counted data. In this step, also feeds the software with historical series of the main variables of the model. Then, the data processing system is done in obtaining the values of all parameters. Soon after, must be observed trajectories produced by the model with

the variables and values entered. If these paths represent approximations of the actual trajectories of the historical series used, indicates that the set of calibrated values is appropriate to describe the studied phenomenon.

## 5.2 Database and value of the parameters

In the calibration process, we used database regarding the countries of the Four Asian Tigers (Hong Kong, Taiwan, South Korea and Singapore).

According to [Gomes, Bugarin & Ellery-Jr \(2005\)](#), [Gollin \(2002\)](#) and [Parente & Prescott \(2000\)](#), the international share of capital in income is about 30 percent. [Barro & Sala-i-Martin \(1999\)](#) calculated the share of the Four Asian Tigers for the period between 1966-1990, Hong Kong has  $\alpha$  equal to 0.37, Singapore with  $\alpha$  equal to 0.49, South Korea and Taiwan have  $\alpha$ , respectively, equal to 0.30 and 0.26. Therefore, the values calculated by [Barro & Sala-i-Martin \(1999\)](#) will be used to calibrate the parameter  $\alpha$  (share of capital and skilled labor in the product).

We define that the child productivity labor,  $l$ , is equal to 0.5. The parameters,  $\rho$ , which is the curvature that governs the elasticity of substitution between capital and skilled labor, and  $\gamma$ , which is the weight of capital given by the CES production function, have been defined following the calibration [Krusell et al. \(2000\)](#) and [Alexopoulos \(2012\)](#), and they are presented in Table 2.

As a positive real interest rate is a necessary condition to be impatience by agents and considering the Asian economies have been studied, they have average interest rates of 6.05 percent (Hong Kong), 4.27 percent (Singapore), 3.97 percent (South Korea) and 2.80 percent (Taiwan<sup>1</sup>), between 2000 and 2014, collected from the [World Bank \(2015\)](#).

In [De La Croix & Doepke \(2003\)](#), the altruism factor,  $\theta$ , is equal to 27.1 percent and represents the weight of the parent invests in the child's education. In [Kang & Sawada \(2003\)](#), the altruism parameter in South Korea is represented by private transfers intra altruist household and it is equal to 0.2170. [Horioka \(2010\)](#) used the proportion of incomes that respondents are willing to donate to their (grown) children in urban Chinese areas. Its value is equal to 12.24 percent, and we could use for Hong Kong. Then, we choose the value of altruism factor parameter,  $\theta$ , following these authors.

Regarding the depreciation rate,  $\delta$ , [Bouzahzah, De La Croix & Docquier \(2002\)](#) show that in industrialized economies this rate varies around 0.4 percent per month, which would 4.8 percent per year. Already [Otsu \(2008\)](#) used a 4.7 percent depreciation rate for South Korea. [Lu \(2012\)](#) used 4.93 percent to South Korea, 7.60 percent to Hong Kong,

<sup>1</sup> The real interest rate series of Taiwan was collected from the Annual Report 2015 website Central Bank of Republic of China (Taiwan).

5.87 percent to Singapore and 6.10 percent to Taiwan. Therefore, in this study, we adopt the depreciation rate values in Lu (2012) for the four countries.

The model will be calibrating under the assumption that each one period (or generation) has a length of 25 years. Therefore, the model will be calibrating for 75 years. Then, the intertemporal discount rate, in the Euler Equation, will be calculated as  $\beta = \frac{1}{(1-\theta)(1+r-\delta)^{25}}$ .  $\beta$  is the intertemporal discount rate and it measures the impatience of the representative agent. It will be 1.6839 to Hong Kong, 2.0530 to Singapore, 1.6254 to South Korea and Taiwan to 3.1740. We emphasize that this rate relates the present and future consumption decisions, so that reductions in  $\beta$  increase the weight of future consumption in the utility of economic agents.

The parameter of relative risk aversion,  $\sigma$ , indicates the sensitivity of the agent's consumption patterns in response to changes in the interest rates. A risk averse agent (with a high coefficient of relative risk aversion or low temporal substitution elasticity of consumption) slightly changes its consumption patterns in response to changes in interest rate (ISSLER; PIQUEIRA, 2000) (KRUSELL et al., 2000). In Cho (2010), the risk aversion parameter is equal to 1.5 for South Korea. Yoshino, Kaji & Asonuma (2016) used a risk aversion parameter equal to 2.0 for China and Singapore. Walsh (2003) and Nuutilainen & Pääkkönen (2013) also used the value 2.0 for China. Therefore, we will use these values for South Korea, Hong Kong and Singapore. Taiwan has a low interest rate and this may mean that agents are not risk averse. For this reason, we can use a lower risk aversion coefficient (1.5) as in South Korea.

The  $z$  parameter is technological innovation (productivity shock) set from the autoregressive first order process, such that  $\log z_t = \rho_z \log z_{t-1} + \varepsilon_t$ , where  $\log z_t$  is the Solow residual and  $0 < \rho_z < 1$ , where  $\rho_z$  is the parameter of persistent shock and it is assumed that  $\varepsilon_t$  is serially uncorrelated with zero mean. Conceptually, the technological shock is obtained by estimating the Solow residual ( $z$ ) or total factor productivity (TFP), from the production function. In other words, it corresponds to the aggregated product part that is not explained neither by capital nor the labor. The parameters corresponding persistence productivity,  $\rho_z$ , and the variance of productivity shock,  $\sigma_z$ , were obtained from the estimation of the Solow residual.

Kim & Chang (2008) and Lee & Moon (2013) used the persistence parameter,  $\rho_z$ , equal to 0.8 to South Korea. Nuutilainen & Pääkkönen (2013) and Miao & Peng (2011) used equal to 0.9 to China, and Hwang & Ho (2012) used  $\rho_z$  equal to 0.9 to Taiwan. Moreover, Miao & Peng (2011) used the productivity shock,  $\sigma_z$ , equal to 0.2 for China, while Lee & Moon (2013) used the same value to South Korea.

According to Mcewan (1999), families with children in public schools of developing countries face direct and indirect costs. Direct costs include uniforms, school supplies, books, transportation, contribution to parent groups, and even tuition. Indirect costs include the

foregone income of the child's work in the labor market, the foregone contribution of the child to home or farm production, and the value of parents' time contributed to school activities. In [UNESCO \(2016\)](#), the education expenditure as % of GDP is 4.62 percent for South Korea, 3.26 percent for Hong Kong and 2.91 percent for Singapore. In [Sorensen \(1994\)](#), the cost of education in Taiwan and South Korea are equal to 3.6 percent and 4.5 percent of Gross National Product (GNP), respectively. [Heckman \(2005\)](#) used 3.3% of China's GDP, 3% of Singapore's GNP and 3.7% of South Korea's GNP.

$\eta$  represents the percentage of wealth consumed, then we will use the average of household final consumption expenditure as % of GDP, from 2000 to 2014, in the [World Bank \(2015\)](#). The values of  $\eta$  are 0.5238, 0.6026, and 0.3941 to South Korea, Hong Kong and Singapore, respectively. Moreover,  $(1-\eta)$  represents the bequest left for the next generations. Taiwan's percentage of wealth consumed values was not available in [World Bank \(2015\)](#), so we used the average of percentage of wealth consumed of Hong Kong, South Korea and Singapore.

$\frac{L_s}{L_u}$  and  $\frac{K}{L_u}$  measure the ratio of skilled labor to unskilled labor and the ratio of capital stock to unskilled labor, respectively. To represent the skilled labor, we use the number of workers with full tertiary education (university education) and to quantify the unskilled labor we use the number of workers with primary and secondary education, all data are available at the [World Bank \(2015\)](#). The capital stock variable, used for the calculation of  $\frac{K}{L_u}$ , was extracted from Penn World Table 9.0. We use the average of the period between 2000 and 2011 to represent these ratios. We found 0.4997 and 0.2530 for South Korea, 0.3107 and 0.3205 for Hong Kong, 0.2980 and 0.3568 for Singapore, and 0.3609 and 0.1858 for Taiwan, for  $\frac{L_s}{L_u}$  and  $\frac{K}{L_u}$  respectively.

Taiwan's skilled and unskilled labor data were not available at [World Bank \(2015\)](#). Then, we use the average proportion of skilled and unskilled labor in South Korea, Hong Kong, and Singapore over the total population of those countries. After this calculation, we found the percentage of workers with primary, secondary and tertiary education in the entire population of these countries. Then we use these average percentages to find the number of skilled and unskilled workers in the total population of Taiwan.

Table 2 shows the parameters used to calibrate the model developed in the previous section as well as its values.

Table 2 – Values of parameters for the calibration of the model

		South Korea	Hong Kong	Singapore	Taiwan
$\alpha$	Share of capital and skilled labor in the product	0.30	0.37	0.49	0.26
$\beta$	Intertemporal discount rate	1.6254	1.6839	2.0530	3.1740
$\delta$	Physical capital depreciation rate	0.0493	0.0760	0.0587	0.0610
$r$	Real interest rate	0.0397	0.0605	0.0427	0.0280
$l$	Productivity of unskilled child labor	0.5	0.5	0.5	0.5
$\eta$	Percentage of wealth consumed	0.5238	0.6026	0.3941	0.5068
$\rho_z$	Persistence parameter of productivity shock	0.8	0.9	0.9	0.9
$\sigma_z$	Conditional variance of productivity shock	0.2	0.2	0.2	0.2
$\gamma$	Weight of capital given by the CES production function	0.5	0.5	0.5	0.5
$\rho$	Curvature governs the elasticity of substitution	-0.495	-0.495	-0.495	-0.495
$\varphi$	Cost of education	0.0462	0.0326	0.0291	0.036
$\theta$	Altruism adult agent educate your child	0.2170	0.1224	0.2710	0.2710
$\sigma$	Coefficient of risk aversion of the function utility of consumption	1.5	2.0	2.0	1.5
$\frac{L_s}{L_u}$	Skilled to unskilled labor ratio	0.4997	0.3107	0.2980	0.3609
$\frac{K}{L_u}$	Capital stock to unskilled labor ratio	0.2530	0.3205	0.3568	0.1858

Source: Authors' calculations.

## 6 SIMULATION AND RESULTS

This section presents the results of the model after calibration. First, we present the results of the competitive equilibrium model. Next, we will discuss the results of a model with government taxing adults agents and gives subsidies as a way to encourage investment in education. In this sense, we will analyze which optimal subsidy will give the highest level of utility of the agents and elevation of welfare.

We have the results for the Four Asian Tigers only in the competitive equilibrium model. The model with subsidies and government intervention does not present results for Taiwan. The lack of results for Taiwan can be justified because the model did not reach convergence and there were no computational results when we modified the levels of subsidies. Therefore, the subsection 6.2 is limited to analyzing the results for the remaining three Asian Tigers (South Korea, Hong Kong and Singapore).

### 6.1 The Competitive Equilibrium Results

This subsection shows the results for the Four Asian Tigers in the competitive equilibrium model. The results are analyzed as the human capital externality reduces. The results, separated for each country, are also available in the Tables 8, 9, 10 and 11 in appendix B.

The human capital externality reduces when the externality goes from concave (weight of externality is less than 1.0) to convex (weight of externality is greater than 1.0). Therefore, when we increase the weight of the externality, we can verify that there is reduction of the human capital externality. There will be an increase in social benefit over personal benefit when the externality reduces. However, no agent will be paying for this increase in social benefit. The human capital externality was about 72% for Taiwan (concave, 0.2) to about 8% (convex, 2) for all countries analyzed in the competitive equilibrium model <sup>1</sup>.

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<sup>1</sup> We do not have the results with weight of externality is greater than 1.5 for South Korea. In this case, the model became explosive and the results tended to infinity.

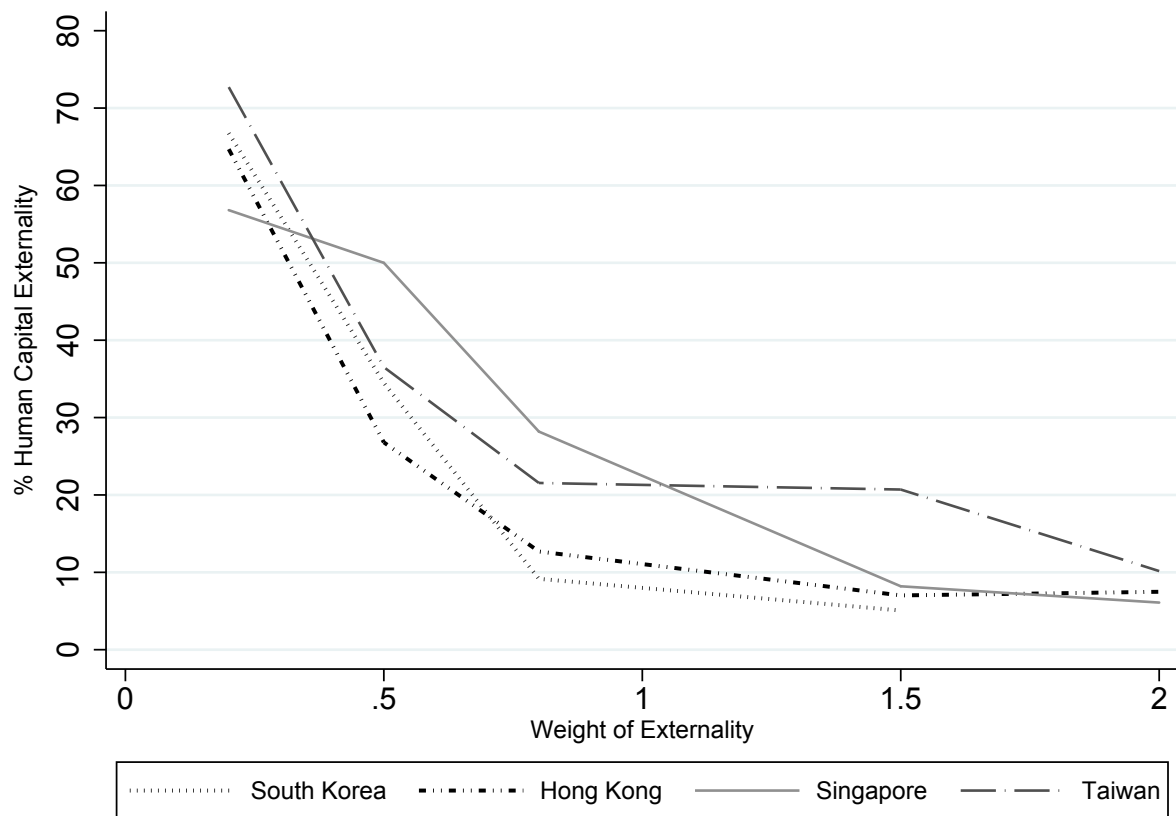


Figure 6 – Percentage of Human Capital Externality with weight of externality  
Source: Calibration's results.

When the externality is reduced, in other words, when we increased the weight of the externality, countries tend to invest more in capital stock ( $K$ ). The ratio of capital stock per product ( $\frac{K}{Y}$ ) increases and the capital stock is close to 4 times greater than product in Singapore, while the average real value of the  $\frac{K}{Y}$  ratio is 2.35 times. Hong Kong and Taiwan are 1.88 and 1.34 times of the  $\frac{K}{Y}$  ratio, respectively, and their average real values are 2.58 and 1.96 times <sup>2</sup>.

<sup>2</sup> These real values can be seen in Table 7 in the appendix A.

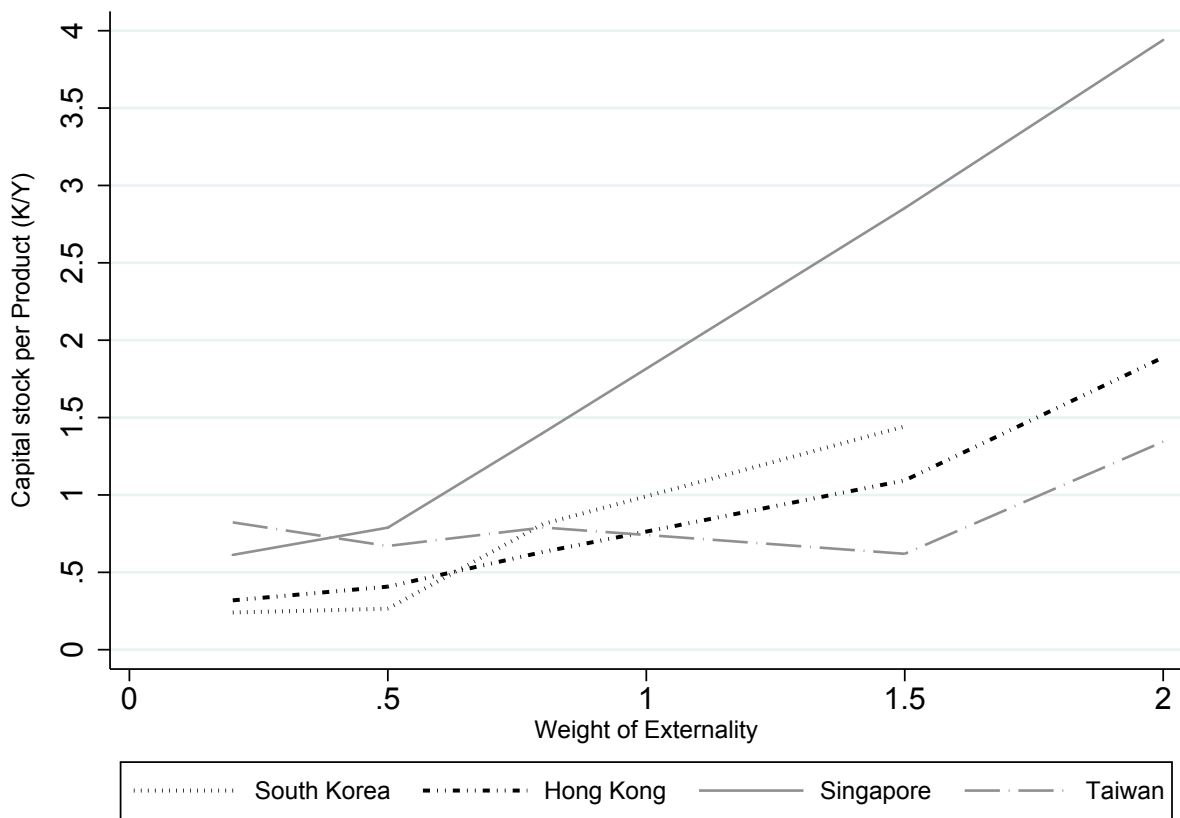


Figure 7 – Capital stock per Product ratio, with weight of externality  
Source: Calibration's results.

The increase in investment in physical capital per product can also be confirmed by Figure 8 of Gini Asset. As reduces the Gini Asset, reduces the risk of investment assets and increase the investment in physical capital. We can realize that while South Korea, Hong Kong and Singapore have fall risk investment trend, Taiwan has a tendency to increase the risk of investment in assets. Taiwan has a risk aversion parameter equal to 1.5 and interest rate equal to 2.8%. These values indicate that Taiwan is not risk averse and can invest in physical capital and human capital even when there is an increased level of risk.

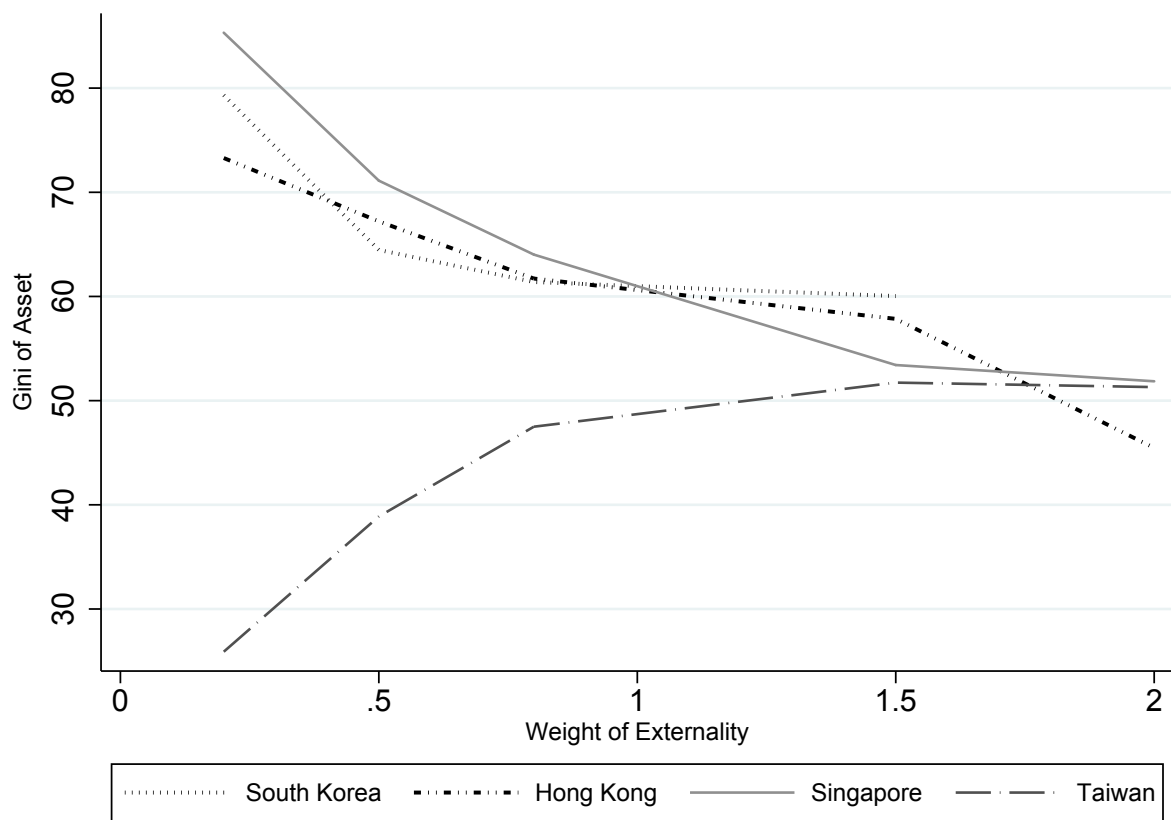


Figure 8 – Gini Asset with weight of externality

Source: Calibration's results.

Capital stock ( $K$ ) and skilled labor ( $L_s$ ) are complementary in the model. With the increase in investment in  $K$  there is also an increase in the percentage of skilled workers while the human capital externality is reduced. The percentage of skilled workers approaches average real values of 49% for South Korea, 31% for Hong Kong, 29.8% and 36% for Singapore and Taiwan, respectively, when the human capital externality reduces<sup>3</sup>.

<sup>3</sup> See Table 4 for real values of the percentage of skilled agents in the appendix A.

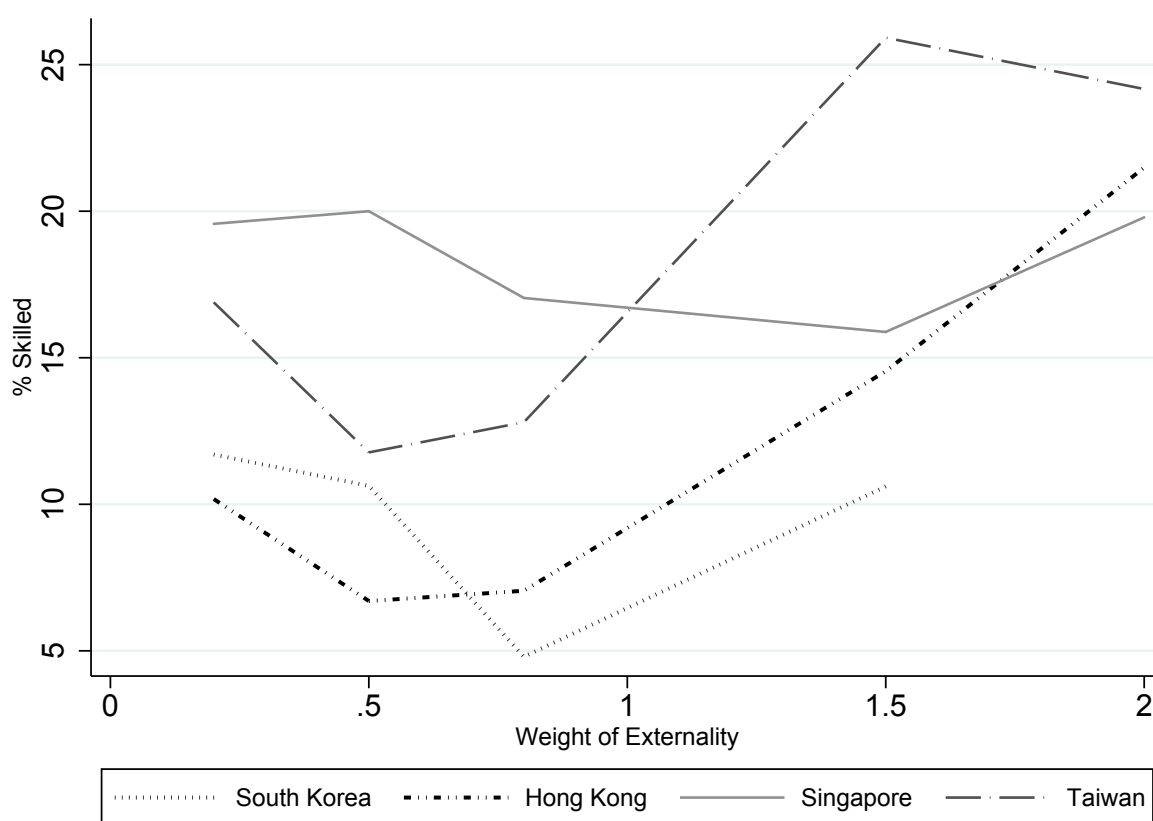


Figure 9 – Percentage of skilled labor with weight of externality  
Source: Calibration's results.

The upward trend in the number of skilled workers with the reduction of the externality has a direct impact on the skill premium. In Figure 10 we can see that the increase in the number of skilled agents in relation to the number of unskilled agents, reduces the skill premium of those skilled agents. When the weight of externality is equal to 0.8 (concave) the skill premium is 3.6 times in South Korea, 2.8 times in Hong Kong, 2.24 and 1.07 times in Singapore and Taiwan, respectively. Already, when the weight of externality is equal to 2 (convex) the skill premium is 0.7 times in Hong Kong, 1.48 times in Singapore and 0.37 times in Taiwan.

It means that the wage of skilled agents compared to wage of unskilled agents tends to reduce while the human capital externality reduces in the competitive equilibrium model.

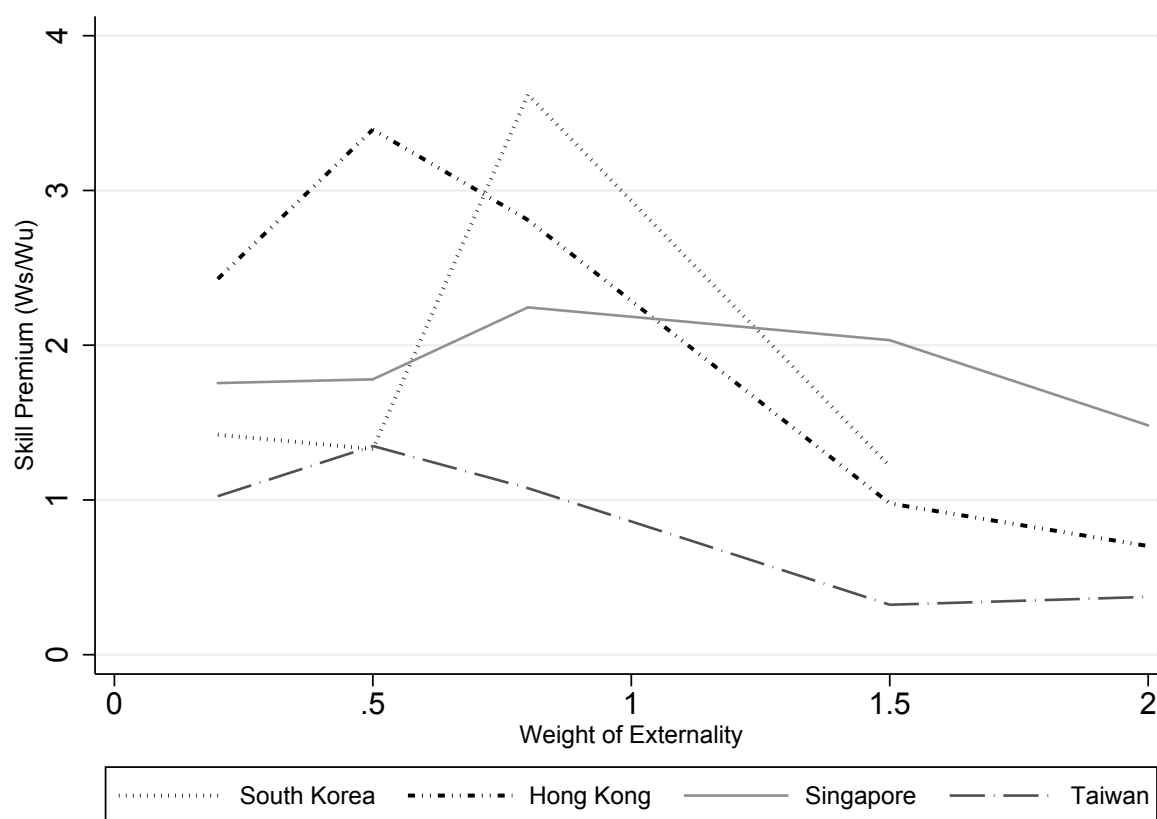


Figure 10 – Skill Premium with weight of externality

Source: Calibration's results.

As the weight of externality increases, in other words, when the externality reduces, there is a tendency to reduce the skill premium in the competitive equilibrium model. This reduction of the skill premium given by the increase in the number of skilled agents tends to make the economy more equal in the distribution of income. This relationship can be seen in Figure 11 of the Gini Index.

The Gini Index is between 0 and 100, and the closer to 0 greater the country's income equality. The countries analyzed present the respective real values for the Gini Index: South Korea 30.2 in 2013; Hong Kong, 53.7 in 2011; Singapore, 46.4 in 2014; and Taiwan, 33.6 in 2014. We can see these values in Table 6 in the appendix A.

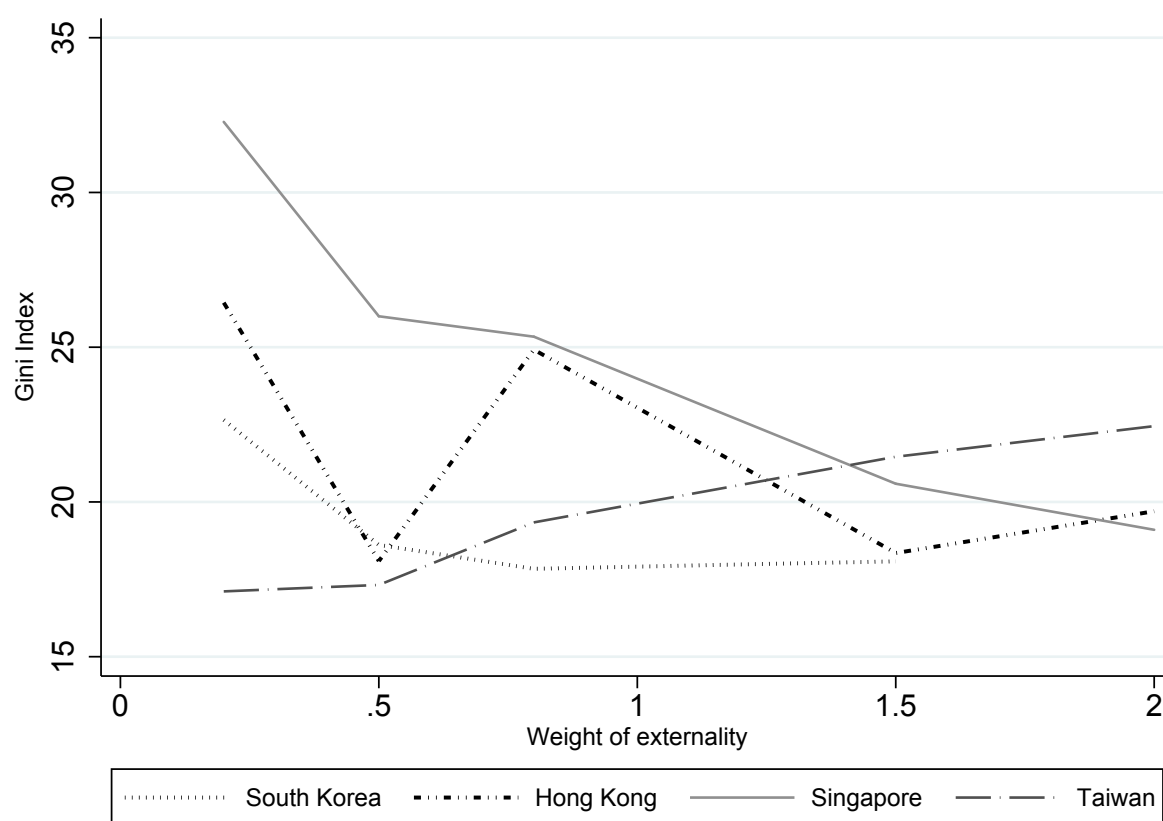


Figure 11 – Gini Index with weight of externality

Source: Calibration's results.

Although the Gini index is moving further away from the real Gini values of the countries analyzed, we can intuit that with the fall of the human capital externality, economies tend to become more equality. In this model, only Taiwan increases the level of inequality and concentration of income as well as has a higher level of risk for investment in assets as it reduces the externality and increases the percentage of skilled agents.

## 6.2 The Benevolent Government Results

We insert a government in the competitive equilibrium model that acts benevolently for we analyze the level of utility of skilled and unskilled agents that invest in education. The government taxes the consumption of adult agents with a lump-sum tax (non-distorting tax) and returns this revenue as a subsidy to the cost of education.

The consumption of the skilled and unskilled agent who invests in education will be increased by the percentage of the subsidy ( $\tau$ ) of the cost of education and reduced from the tax (*lump*).

$$c_{e=1}^s = w^s \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) z - \varphi(1 - \tau) - k_{t+1} - lump$$

$$c_{e=1}^u = w^u \left( \frac{K_t}{L_{u_t}}, \frac{L_{s_t}}{L_{u_t}} \right) z - \varphi(1 - \tau) - k_{t+1} - lump$$

The tax affects the consumption of all adult agents (skilled and unskilled) equality and returns to agents who decide to invest in education in form of subsidy. Then, we can analyze the level of utility of the agents as the subsidy to education increases. Therefore, we can analyze the evolution of utility when modifying subsidies, to encourage investment in human capital and, consequently, in physical capital.

When there is no subsidy to education (subsidy equal 0%), nothing is taxed and the economy acts in a competitive equilibrium without government intervention. At this point, the utility of agents tends to be lower than when small percentages of taxation, and hence subsidies.

As we said before, we will only present the results for South Korea, Hong Kong and Singapore in this subsection. Unfortunately, we do not have results for Taiwan because the model with government did not reach convergence and there were no computational results when we modified the levels of subsidies <sup>4</sup>.

Figure 12 shows the utility of skilled agents who invest in your child's education and receive subsidy values from 0% to 100%. The highest levels of utility for South Korea are in the education subsidies offered by the government are 20%, 40% and 60%. The skilled agents of Hong Kong have the highest level of utility when this subsidy is 5%. Singapore has the highest level of utility when the government contributes with 10% of the costs of education. From these values, the utility tends to be less, since the taxes will be increasing more and more.

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<sup>4</sup> There is sensitivity in the parameters calibration model since  $\alpha$  may be underestimated.  $\alpha$  parameter was based on a Cobb-Douglas function in the paper of Barro & Sala-i-Martin (1999), while the production function of this research is a Constant Elasticity of Substitution (CES) and Cobb-Douglas function.

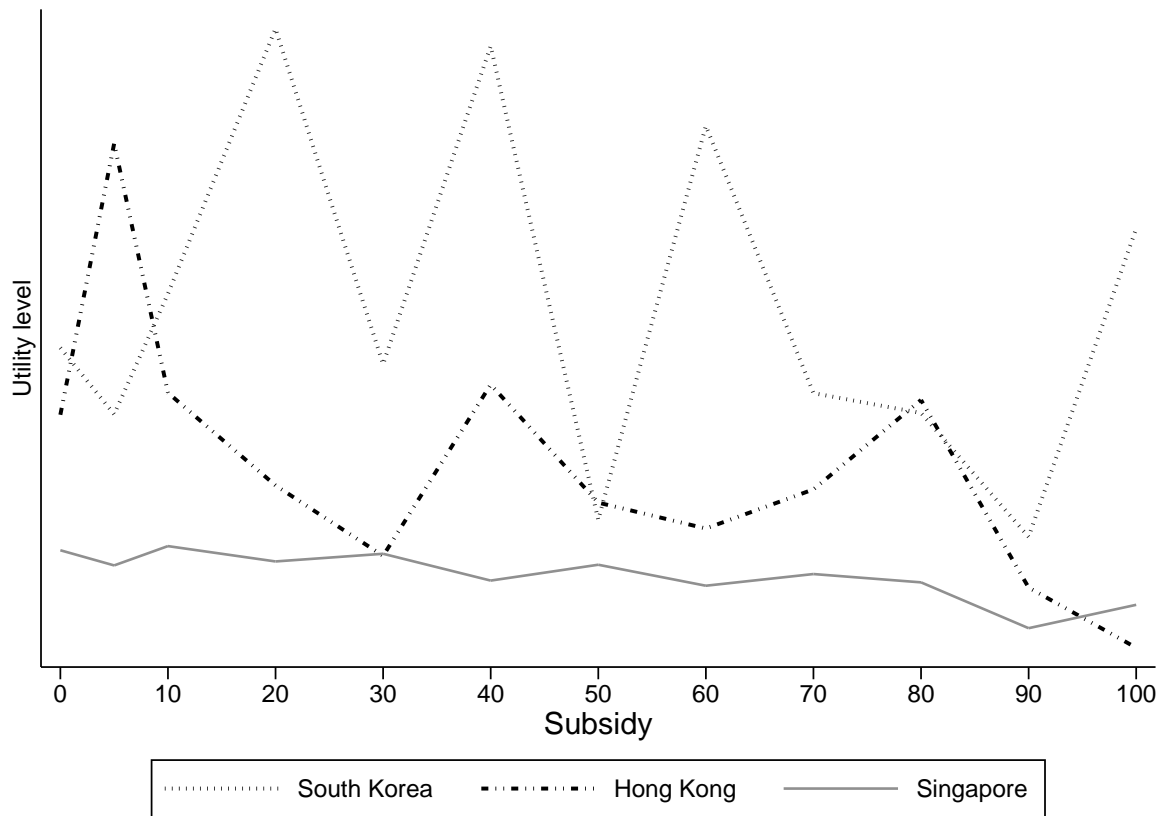


Figure 12 – Utility level of the skilled adult for percentage of subsidy  
Source: Calibration's results.

Through the figures, we can assimilate the trajectory of utilities with Laffer curves as agents are taxed. In this sense, we can check the possible optimal subsidies and taxes to reduce the human capital externality and increase investment in education and physical capital stock.

The Laffer curve is a theoretical representation of the relationship between the tax value collected and the different rates. It seems obvious that a subsidy equal 0% will not generate great utility of the agents, since agents will pay all costs of education and not receiving any value as a subsidy (competitive equilibrium case). However, in Laffer's hypothesis, a 100% tax rate, in which the government taxes and fully subsidizes education costs, will not generate the highest level of utility.

This fact can be explained as the government encourages investment in education beyond the optimal, the number of skilled agents will rise in relation to the number of unskilled workers, and this will affect the skill premium value. The reduction in the skill premium will eventually discourage agents to invest in education even if this cost of education is fully subsidized by the government. Another factor is that for the government to subsidize 100% of the costs of education, the agents will be highly taxed in their consumption functions. If both subsidy rates (0% to 100%) cause falls utility of the agents,

then, there should be rates between 0% and 100% that reaches the maximum level of utility.

In this sense, Figure 13 shows the values of subsidies of 0% to 10% on the costs of education for skilled agents who decide to invest in education. Skilled agents of South Korea have a highest level of utility when the education subsidy is 8%. Hong Kong has some points where the subsidy increases the utility of the skilled agent when the subsidies are 7.5%, 2% and 0.5%, respectively. Singapore has highest utility when the subsidy is 8%.



Figure 13 – Utility level of the skilled adult for percentage of subsidy  
Source: Calibration's results.

For Singapore, we can see that there is not a very discrepant level of utility. This fact can be explained since the percentage of skilled agents of this economy is higher (about 20% in competitive equilibrium <sup>5</sup>). This makes the agents do not have a very significant increase in utility given a subsidy to invest more in education. In addition, the value of the education cost calibrated for Singapore is lowest among the countries analyzed (2.91%), and this encourages agents to invest more in quality of their child labor. Therefore, comparing with the other countries, Singapore's utility curve is the lower.

Figure 14 shows the utility levels of unskilled workers who decide to invest in their child's education. South Korea has the highest levels of utility when subsidies are 20% and

<sup>5</sup> See this value in Table 9 in appendix.

80%. Unskilled agents in Hong Kong and Singapore have highest levels of utility when education subsidies are 5% and 10%, respectively.

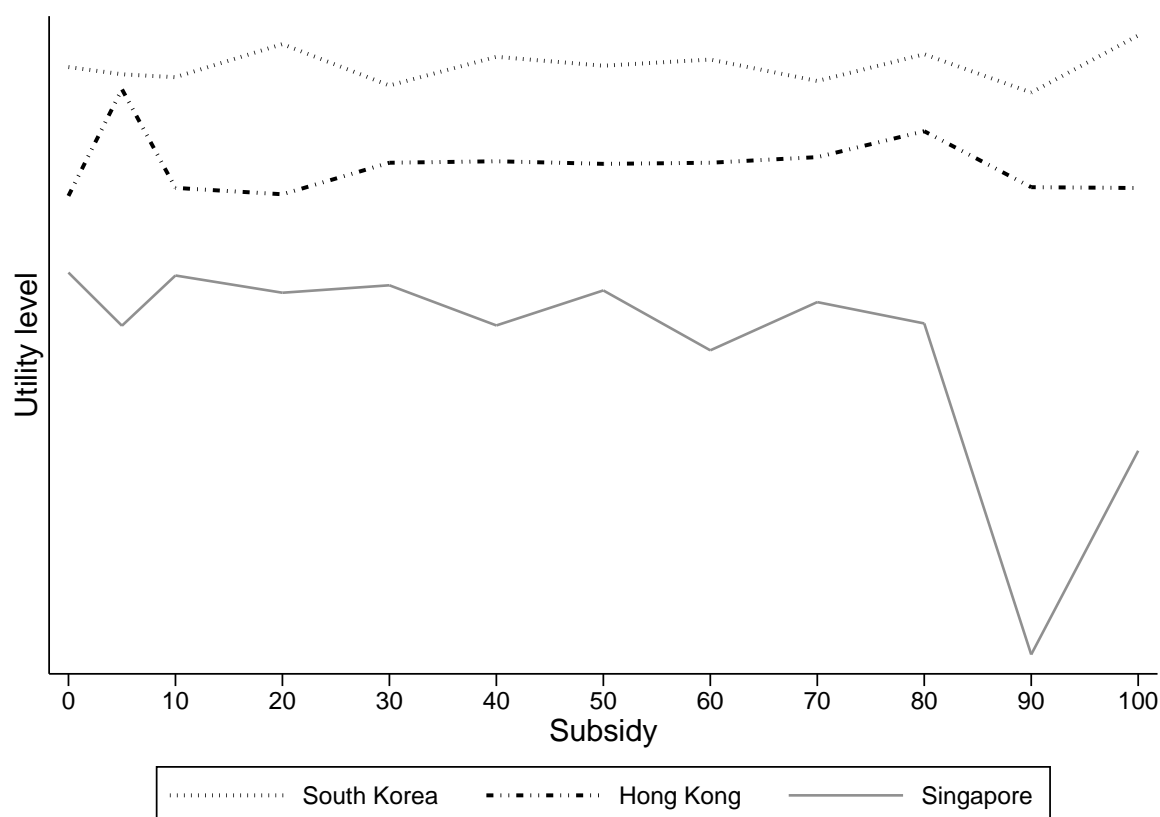


Figure 14 – Utility level of the unskilled adult for percentage of subsidy  
Source: Calibration's results.

When the subsidy is equal zero (competitive equilibrium case), the utility level is very close to the optimal level for the unskilled workers. It means that probably there are public policies, in competitive equilibrium, that encourage unskilled agents to invest more in education in Singapore and South Korea. Therefore, even when there is no subsidy (0%), there is internalisation of externalities in Singapore and South Korea for unskilled workers.

Figure 15 shows the utility levels of the unskilled agents when the education subsidies are 0% to 10%. South Korea has the highest level of utility when the subsidy is 6.5%. The unskilled agents in Hong Kong have greater utility when subsidies are 7.5%, 0.5% and 1.5%, respectively. Already, Singapore has the highest utility when subsidies are 8% and 7%. We can verify that the level of utility of the unskilled agents for South Korea is the highest, followed by Hong Kong and Singapore.

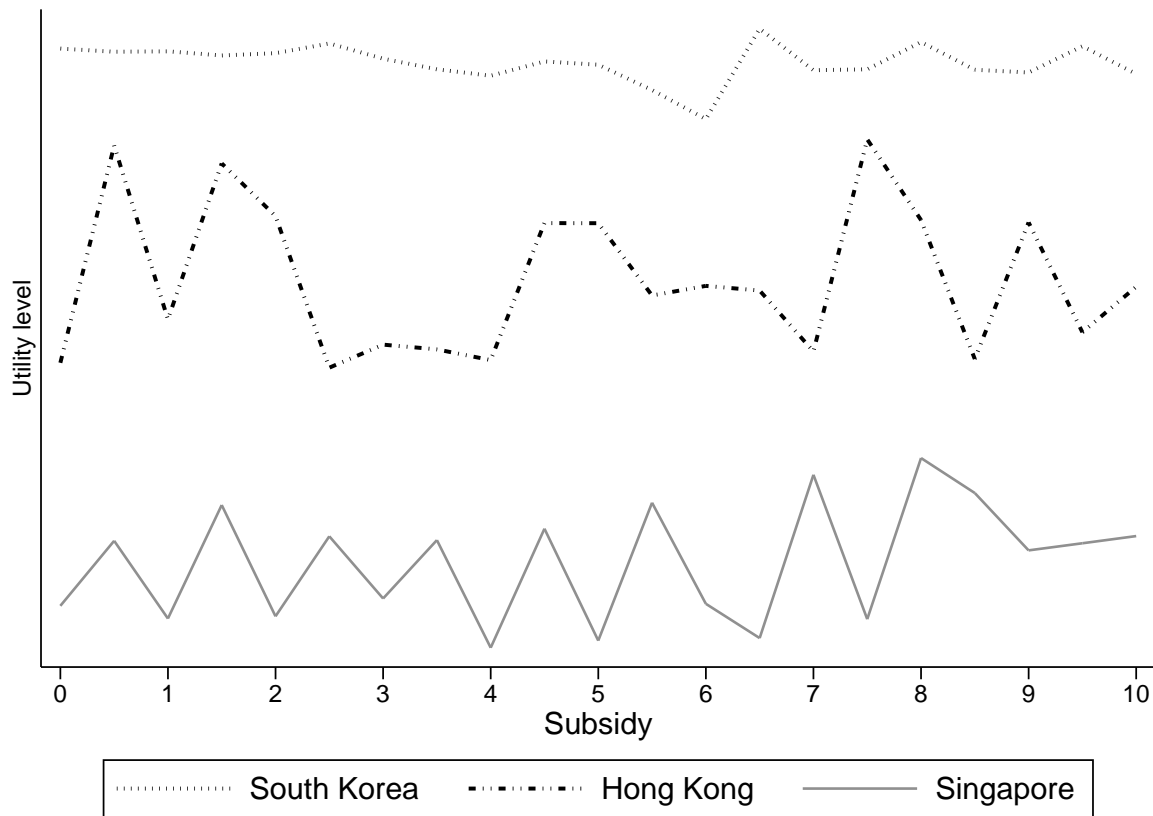


Figure 15 – Utility level of the unskilled adult for percentage of subsidy  
Source: Calibration's results.

As the subsidy increases, the percentage of skilled labor increases as well as reduces the skill premium. This skill premium reduction decreases the incentive to invest in education and the utility of the agents. Given the increasing in the skill premium, the agents have incentives to invest in education, then the percentage of skilled workers increases and reduces the utility of these agents again.

The  $K/Y$  ratio and skill premium are the determinants of non-monotonic utility behavior. As the subsidy increases the agents invest more in education and less in capital stock. However, the interest rate begins to rise due to the complementarity between physical and human capital. Then there will be incentives for agents to increase investment in physical capital and decrease investment in human capital. In Figures 17, 18 and 19 in appendix C, there appears to be a negative correlation between the  $K/Y$  ratio and percentage of skilled workers, or, a positive correlation between  $K/Y$  ratio and premium skill.

An important point is that in the competitive equilibrium, model without government intervention, the reduction of the externality will cause an increase in the percentage of skilled workers and consequently reduction of the skill premium. Already, in the model with government intervention, the points with lower externality show higher levels of skill

premium and greater utility of the agent in investing in education. We can understand that the agent will have more incentive to invest in education when the human capital externality is low.

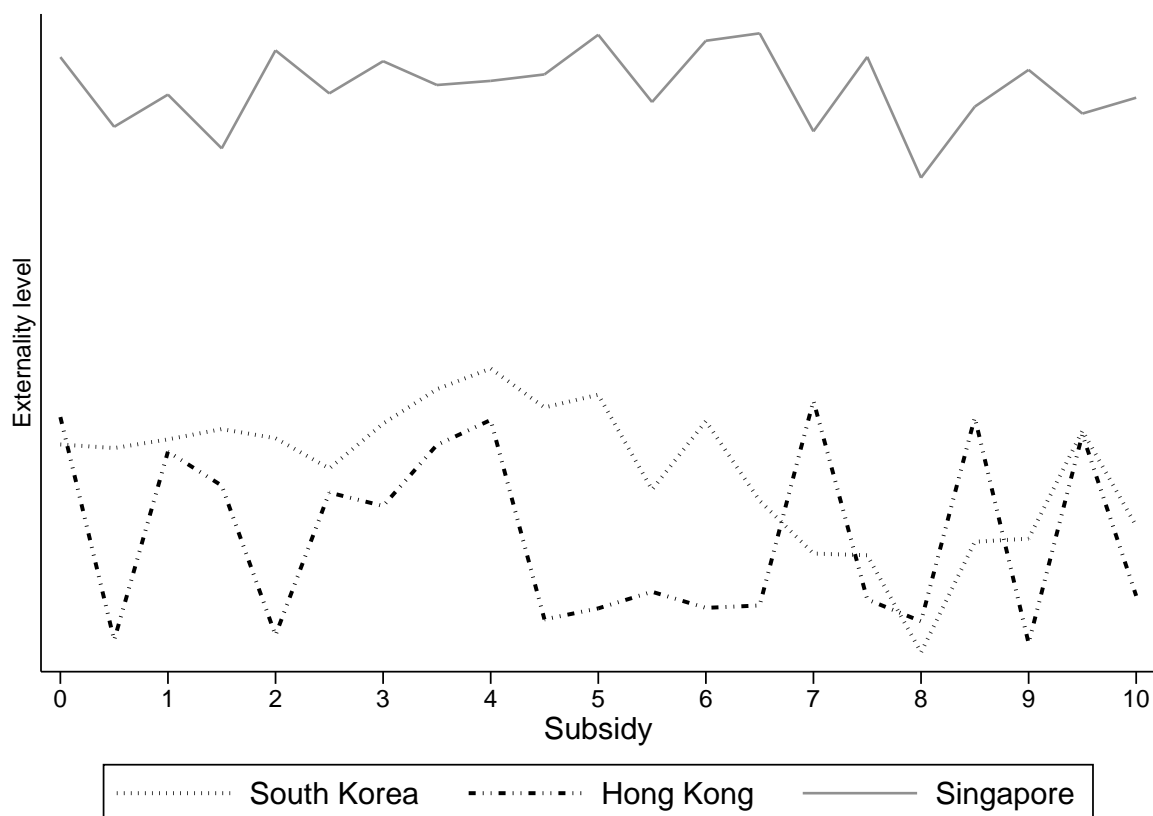


Figure 16 – Externality of the adult agent for percentage of subsidy  
Source: Calibration's results.

In other words, the reduction of the human capital externality will generate greater utility of the agents who invests in education. In the competitive equilibrium model, the reduction of the externality generates reduction of the skill premium given by the increase of the number of skilled workers. In the model with subsidies, the reduction of the externality generates increase of the skill premium and increase of the utility.

Therefore, if government subsidy is well directed, it will tend to increase the utility of the agent who invests in education, increase investment in physical capital, reduce the human capital externality, and improve equality of income distribution. The subsidy has the power to direct investment and impact government spending. In theory, if subsidies are well directed, they can induce investments in strategic sectors and generate increased productivity.

## 7 CONCLUSION

The impact of education on labor income represents a real gain for society, it follows that policies that raise the educational level will have a strong impact on the long-run income, and therefore on growth. In this case, the social return to education will be higher than the private return.

Then, the search problem was to measure the human capital externality and the economic growth by simulation of the theoretical model developed in this research. As general goal to investigate size and sources of externalities in human capital accumulation and Pareto inefficiency generate in the competitive equilibrium. More specifically, we analyzed the decisions of households of investing in human capital and trade-offs faced by these agents; Investigated complementarity between physical and human capital in technology and its implications for human capital investment; Examined the Pareto inefficiency created by externalities in human capital; Compared, in theoretical model, the Pareto inefficiency with “the first best” set of allocations through a Social Planner Problem.

The group of countries called Four Asian Tigers (Hong Kong, South Korea, Singapore and Taiwan) is famous for their fast economic and industrial growth since the 1960s. These countries showed an accelerated process of industrialization based on technological progress from middle of the twentieth century. Thus, the Four Asian Tigers have maintained high growth rates and captured significant shares of the international markets, transforming their economic bases.

During the 1960s, the “Tigers” were relatively poor, having abundance of cheap labor, and took that advantage by uniting it with the educational reform creating a low-cost workforce, but very productive. The “Tigers” invested in the education system at all levels, including the university system, ensuring that every child to attend primary school and high school. In general, these advances in education allowed high levels of literacy and cognitive skills.

In the theoretical model, we inserted three generations of agents (children, adults and old agents) and a representative firm. Adults face the trade-off between current consumption (invest in your child’s education) or investment in future capital. Children who do not study need to work and they become unskilled workers in the future. The old agents are the only holders of capital that economy and rent this capital for the firm.

We inserted a Social Planner to internalize the externalities caused and compare with the inefficiency of Pareto caused in a market economy. The Social Planner encourages investment in education and raises the future productivity of labor. The productivity of

physical capital increases and raises economic growth. We let the benevolent social planner maximize a social welfare function and internalizes the external effects. The social planner must allocate the society's resources between consumption of the three generations alive the time  $t$  and the capital stock in future period.

After the calibration of the model, we showed the results for competitive equilibrium and a benevolent government. In competitive equilibrium, when the externality is reduced, countries tend to invest more in capital stock ( $K$ ). With the increase in investment in  $K$  there is also an increase in the percentage of skilled workers. As the weight of externality increases, in other words, when the human capital externality reduces, there is a tendency to reduce the skill premium in the competitive equilibrium model. This reduction of the skill premium given by the increase in the number of skilled agents tends to make the economy more equal in the distribution of income.

We insert a government in the competitive equilibrium model that acts benevolently for we analyze the level of utility of skilled and unskilled agents who invests in education. The government taxes the consumption of adult agents with a lump-sum tax (non-distorting tax) and returns this revenue as a subsidy to the cost of education. Therefore, we analyzed the evolution of utility when we modify the percentage of subsidies, to encourage investment in human capital and, consequently, in physical capital.

Through the figures, we assimilated the trajectory of utilities with Laffer curves as agents were taxed. In this sense, we verified the possible optimal subsidies and taxes to reduce the human capital externality and increase the skill premium and physical capital stock.

As the subsidy increases, the percentage of skilled workers increases as well as reduces the skill premium. This skill premium reduction decreases the incentive to invest in education and the utility of the agents. Given the increasing in the skill premium, the agents have incentives to invest in education, and then the percentage of skilled workers increases and reduces the utility of these agents again.

The reduction of the externality will generate greater utility of the agents in investing in education. In the competitive equilibrium model, the reduction of the human capital externality generates reduction of the skill premium given by the increase of the number of skilled workers. In the model with subsidies, the reduction of the human capital externality generates increase of the skill premium, increase of the utility and increase of welfare.

The government subsidy has the power to direct investment and impact government spending. If government subsidy is well directed, it will tend to increase the utility of the agent who invests in education, increase investment in physical capital, reduce the human capital externality, and improve equality of income distribution.

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# APPENDIX

## A Real values of the variables

The skill premium is the relation between skilled and unskilled wage. Then, we use the wage per hour in dollars for Hong Kong, South Korea, Singapore and Taiwan in [International Labor Organization \(2016\)](#) to find the skill premium value. We separated the wages for the skilled labor from the sample by the percentage of workers with tertiary education. In other words, we used this percentage to separate the highest wages from the sample. Then, we divide the average of the highest wages by the average of the lowest wages and we found the premium skill.

Table 3 shows the skill premium values for Hong Kong, South Korea, Singapore and Taiwan from 2000 to 2008. Singapore is the country with the highest skill premium values, 2.78 in 2007. However, Taiwan has the highest average Skill premium (2.60). The average skill premium for Hong Kong are 1.94, South Korea is 1.69, Singapore is 2.55, and Taiwan is 2.60.

Table 3 – Skill Premium

	2000	2001	2002	2003	2004	2005	2006	2007	2008
Hong Kong	2.00	1.98	2.07	2.04	1.90	1.96	1.92	1.83	1.76
South Korea	1.72	1.73	1.69	1.68	1.65	1.69	1.68		
Singapore	2.51	2.59	2.51	2.53	2.55	2.55	2.39	2.78	
Taiwan			2.55	2.63	2.64				

Source: International Labor Organization (2016).

Table 4 shows the values of skilled and unskilled labor ratio for the period between 2000 and 2011. South Korea is the country with the highest average value of skilled and unskilled labor ratio (0.4997), followed by Taiwan (0.3610), Hong Kong ( 0.3107), and Singapore (0.2980). The values used to calculate this series are available in [World Bank \(2015\)](#).

Table 4 – Skilled and Unskilled Labor Ratio

	Hong Kong	Singapore	South Korea	Taiwan
2000	0.2703	0.2046	0.3158	0.2603
2001	0.2703	0.2046	0.3316	0.2655
2002	0.2703	0.2270	0.3643	0.2838
2003	0.2703	0.2563	0.4347	0.3142
2004	0.2983	0.2752	0.4535	0.3367
2005	0.3200	0.3035	0.4771	0.3621
2006	0.3374	0.3055	0.5083	0.3760
2007	0.3520	0.3106	0.5385	0.3910
2008	0.3594	0.3482	0.5850	0.4203
2009	0.2890	0.3643	0.6230	0.4089
2010	0.3455	0.3817	0.6620	0.4507
2011	0.3459	0.3947	0.7022	0.4624
Average	0.3107	0.2980	0.4997	0.3610

Source: World Bank (2015).

In Table 5 we use capital stock data available on Penn World Table 9.0, and unskilled labor data available on [World Bank \(2015\)](#) for the period from 2000 to 2011. Singapore is the country with the largest capital stock and unskilled labor ratio (35.68%), while Taiwan has about 18.5%.

Table 5 – Capital Stock and Unskilled Labor Ratio

	Hong Kong	Singapore	South Korea	Taiwan
2000	0.2404	0.1935	0.1351	0.1298
2001	0.2536	0.2190	0.1438	0.1384
2002	0.2643	0.2491	0.1537	0.1469
2003	0.2754	0.2882	0.1823	0.1567
2004	0.2942	0.3353	0.2075	0.1696
2005	0.3184	0.4113	0.2426	0.1889
2006	0.3448	0.4083	0.2752	0.2019
2007	0.3574	0.4146	0.3001	0.2109
2008	0.3697	0.4288	0.3281	0.2192
2009	0.3457	0.4306	0.3415	0.2123
2010	0.4004	0.4413	0.3519	0.2265
2011	0.3824	0.4622	0.3747	0.2291
Average	0.3205	0.3568	0.2530	0.1858

Source: Penn World Table 9.0 and World Bank (2015).

Table 6 shows the Gini Index values for the four countries analyzed. However, in the [World Bank \(2015\)](#) do not have the Gini Index values for every year. Then, we present the available values of Gini Index.

Table 6 – Gini Index

	1996	1998	2000	2007	2010	2011	2013	2014
Hong Kong	43.4			53.3		53.7		
South Korea		31.6			31.0		30.2	
Singapore		42.5				47.3		46.4
Taiwan			32.6					33.6

Source: World Bank (2015) and CIA (2016).

Table 7 shows the values of capital stock and GDP ratio for the period 1960 to 2014. These data are available in Penn World Table 9.0. We can verify that the capital stock and GDP ratio is increasing over the years. Taiwan is the only country that has an average result lower than 2.

Table 7 – Capital Stock and GDP Ratio

Year	Hong Kong	Singapore	South Korea	Taiwan	Year	Hong Kong	Singapore	South Korea	Taiwan
1960	1.7018	1.3566	1.9815	1.1767	1988	1.6793	1.8118	2.1198	1.7631
1961	1.6901	1.2806	1.8468	1.1555	1989	1.8399	1.9165	2.0305	1.7780
1962	1.5950	1.3026	1.7295	1.1149	1990	1.9721	2.0563	1.9332	1.8780
1963	1.5115	1.2575	1.6215	1.0675	1991	2.0962	2.1278	1.9199	1.8845
1964	1.5480	1.1963	1.7467	1.0044	1992	2.2339	2.2564	1.9075	1.8954
1965	1.5088	1.2069	1.7101	0.9749	1993	2.4155	2.4120	1.8511	1.9648
1966	1.6136	1.1963	1.6202	0.9885	1994	2.6553	2.5424	1.8067	2.0273
1967	1.6661	1.2526	1.5385	1.0074	1995	3.0609	2.6884	1.8445	2.1136
1968	1.6619	1.2970	1.4415	1.0406	1996	3.4479	2.7859	1.8898	2.1643
1969	1.5718	1.3456	1.4277	1.0838	1997	3.5160	2.8414	1.9636	2.2317
1970	1.5107	1.5099	1.4063	1.1390	1998	3.9499	3.1200	2.2369	2.3440
1971	1.5249	1.5291	1.4707	1.1599	1999	3.9925	2.9104	2.3362	2.3830
1972	1.4946	1.5846	1.5001	1.1753	2000	3.8725	2.8796	2.4160	2.4263
1973	1.4571	1.5803	1.5469	1.2051	2001	4.0064	2.9361	2.7105	2.5758
1974	1.5503	1.6995	1.6640	1.3653	2002	4.0708	2.9042	2.8608	2.5433
1975	1.6503	1.8320	1.7756	1.4997	2003	4.0998	3.1497	3.0288	2.5456
1976	1.5095	1.6820	1.8038	1.4816	2004	3.9975	3.4352	3.1812	2.5607
1977	1.4912	1.6592	1.8355	1.5090	2005	4.0158	3.8594	3.4978	2.6417
1978	1.5303	1.7368	1.8791	1.5220	2006	4.1248	4.1077	3.5806	2.8281
1979	1.5376	1.8296	1.9040	1.6073	2007	4.1664	4.2408	3.6284	2.9266
1980	1.5697	2.0335	1.9310	1.7100	2008	4.3294	4.4000	3.8940	3.1286
1981	1.5846	2.0190	1.9488	1.8099	2009	4.6214	4.2589	4.2164	3.3583
1982	1.6657	1.9876	2.0587	1.9091	2010	4.5592	3.9452	3.9879	3.2911
1983	1.6212	1.8137	2.0818	1.8260	2011	4.6994	3.7948	4.1648	3.4885
1984	1.5270	1.7660	2.0951	1.8088	2012	4.7413	3.8352	4.2649	3.4814
1985	1.5853	1.7433	2.2464	1.8316	2013	4.8122	3.8892	4.3616	3.5172
1986	1.6033	1.7753	2.3553	1.7844	2014	4.8754	3.9544	4.5162	3.5196
1987	1.6053	1.7893	2.2401	1.7326	Average	2.5807	2.3513	2.3374	1.9628

Source: Penn World Table 9.0.

## B Calibrations results of the competitive equilibrium for each country

When the externality goes concave (0.2, 0.5 and 0.8) to convex (1.5 and 2) (when increasing the weight of the externality), the stock of capital by product ( $\frac{K}{Y}$ ) of Hong Kong rises approaching of the real capital stock per product. The number of skilled workers in Hong Kong increases relative to the number of Skilled workers. This shows the complementarity between  $K$  and  $L_s$  in the model, when the capital stock increases, we also have an increase in the number of skilled workers compared to the number of unskilled workers, approaching the real  $\frac{L_s}{L_u}$  rate. In this sense, with an increase in the number of skilled workers, the skill premium reduces as there are more skilled labor in Hong Kong.

When the number of skilled workers rises and also the ( $\frac{K}{Y}$ ) ratio, reduces the externality of Hong Kong. The Gini index is different from the real Gini index, however, Hong Kong tends to be more egalitarian with more skilled labor.

Table 8 – Hong Kong’s Simulation Results

Concavity/ Convexity	K/Y	Gini Asset	% Skilled	Skill Premium	Externality	Gini Index
0.2	0.3190	0.7329	0.1018	2.4280	0.6469	0.2644
0.5	0.4077	0.6721	0.0670	3.3938	0.2680	0.1809
0.8	0.6317	0.6172	0.0705	2.8096	0.1270	0.2492
1.5	1.0941	0.5785	0.1454	0.9777	0.0702	0.1835
2	1.8890	0.4546	0.2149	0.7022	0.0749	0.1970

Source: Calibration’s results.

As  $K$  and  $L_s$  are complementary, when the ratio of capital stock per product ( $\frac{K}{Y}$ ) increases, there is an increase in skilled labor in relation to unskilled labor. Thus, the skill premium reduces in Singapore. The reduction of the externality, increases the number of skilled workers in Singapore. The reduction of the Gini index by reducing the externality makes the distribution of income more egalitarian. The Gini Asset shows that there is risk reduction in investment in assets in Singapore.

When the externality decreases in Singapore, the number of skilled workers increases, and the capital stock increases relative to the product. Equality measured by the Gini index improves and reduces the risk for investments.

Table 9 – Singapore’s Simulation Results

Concavity/ Convexity	K/Y	Gini Asset	% Skilled	Skill Premium	Externality	Gini Index
0.2	0.6126	0.8532	0.1957	1.7553	0.5681	0.3228
0.5	0.7888	0.7112	0.2000	1.7795	0.5000	0.2600
0.8	1.4005	0.6402	0.1704	2.2445	0.2819	0.2534
1.5	2.8537	0.5342	0.1588	2.0326	0.0820	0.2059
2	3.9398	0.5186	0.1979	1.4818	0.0609	0.1910

Source: Calibration’s results.

We do not have the results with weight of externality is greater than 1.5 for South Korea. In this case, the model became explosive and the results tended to infinity. However, we found that when there is a reduction of the externality, there is an increase in the capital stock per product ratio and a reduction in the risk of investment in assets. We can also verify that the Gini index does not have a lot of variation, but shows a downward trend.

Table 10 – South Korea’s Simulation Results

Concavity/ Convexity	K/Y	Gini Asset	% Skilled	Skill Premium	Externality	Gini Index
0.2	0.2407	0.7932	0.1170	1.4212	0.6675	0.2265
0.5	0.2652	0.6447	0.1063	1.3292	0.3448	0.1861
0.8	0.8117	0.6139	0.0480	3.6191	0.0916	0.1784
1.5	1.4429	0.5965	0.1061	1.2185	0.0506	0.1881
2	-	-	-	-	-	-

Source: Calibration’s results.

Taiwan is the only country surveyed that increases the risk of asset investment and increases the income inequality while reducing the externality of human capital. The ratio ( $\frac{K}{Y}$ ) has no great variation when the externality goes from concave to convex. In addition, the percentage of skilled agents increases and the skill premium reduces.

Table 11 – Taiwan’s Simulation Results

Concavity/ Convexity	K/Y	Gini Asset	% Skilled	Skill Premium	Externality	Gini Index
0.2	0.8227	0.2590	0.1689	1.0243	0.7271	0.1711
0.5	0.6704	0.3888	0.1177	1.3487	0.3652	0.1732
0.8	0.7898	0.4750	0.1280	1.0764	0.2155	0.1934
1.5	0.6197	0.5173	0.2592	0.3228	0.2070	0.2146
2	1.3447	0.5130	0.2417	0.3737	0.1016	0.2245

Source: Calibration’s results.

## C Calibrations results of the social planner for each country

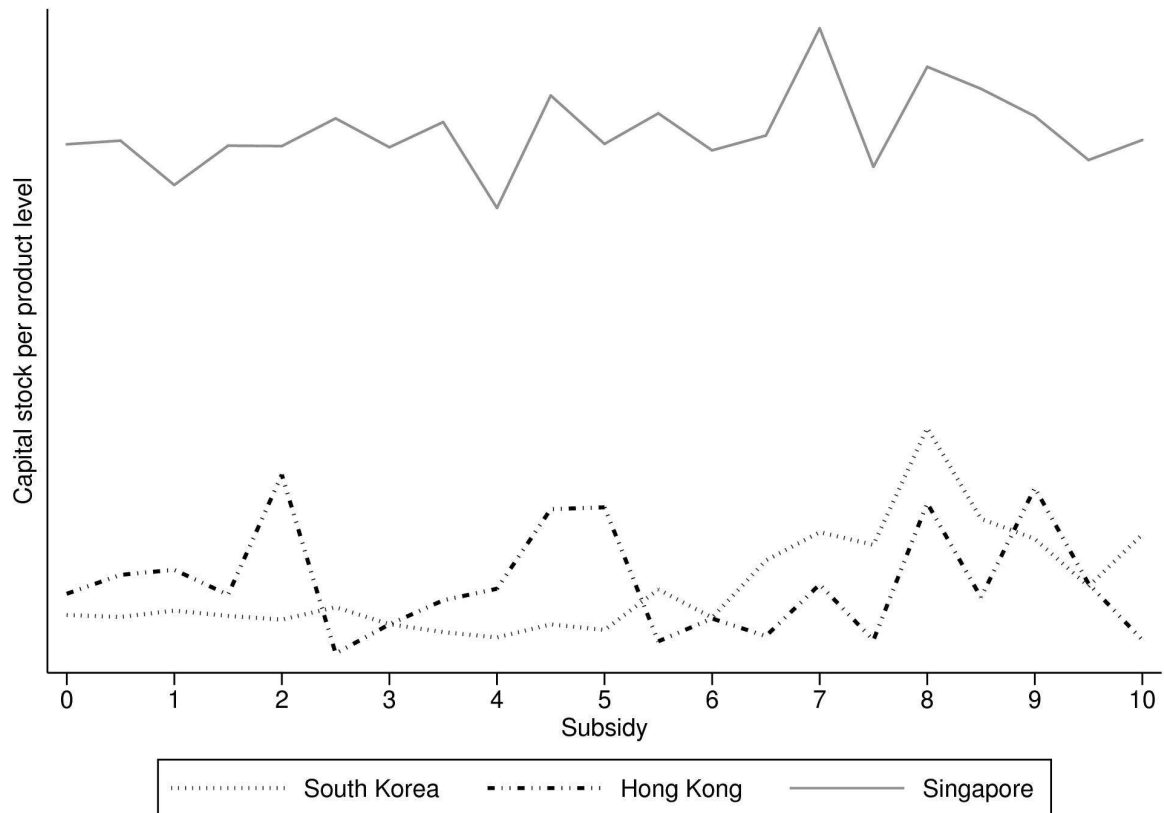


Figure 17 – Capital stock per product level for percentage of subsidy  
Source: Calibration's results.

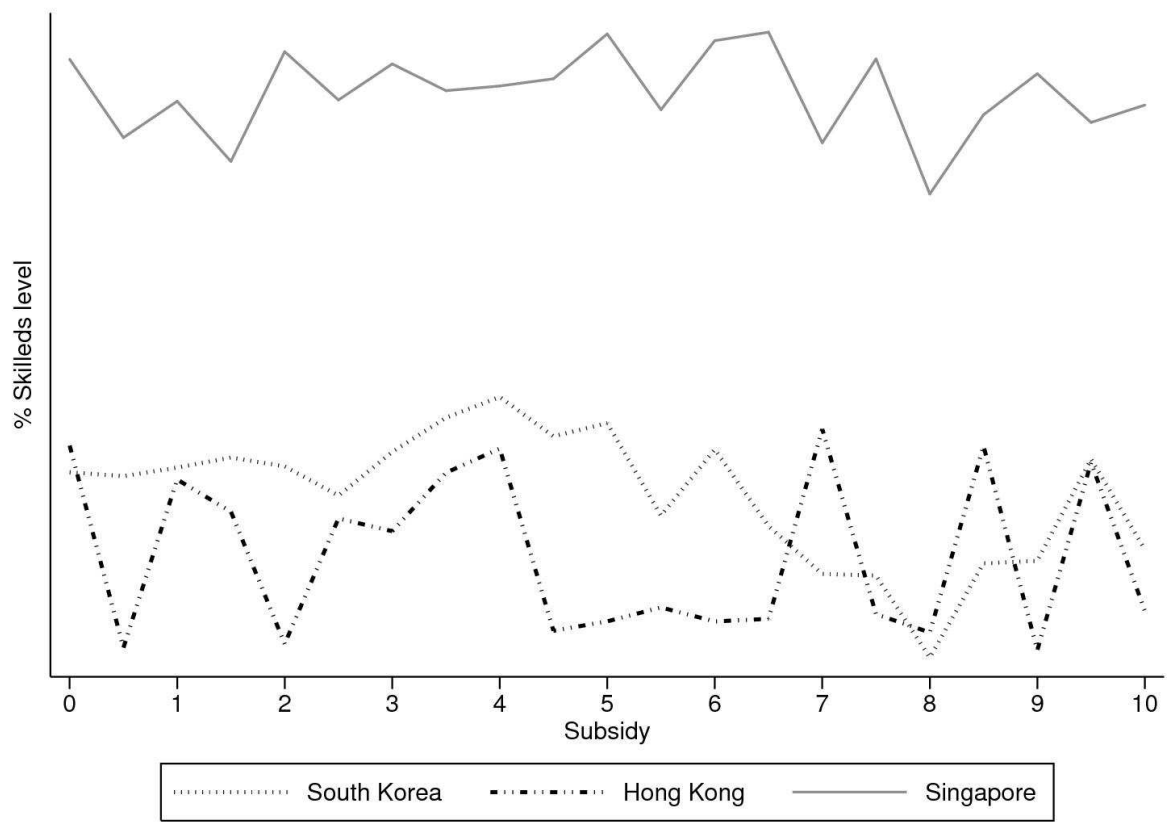


Figure 18 – Perceptual of skilled agents level for percentage of subsidy  
 Source: Calibration's results.

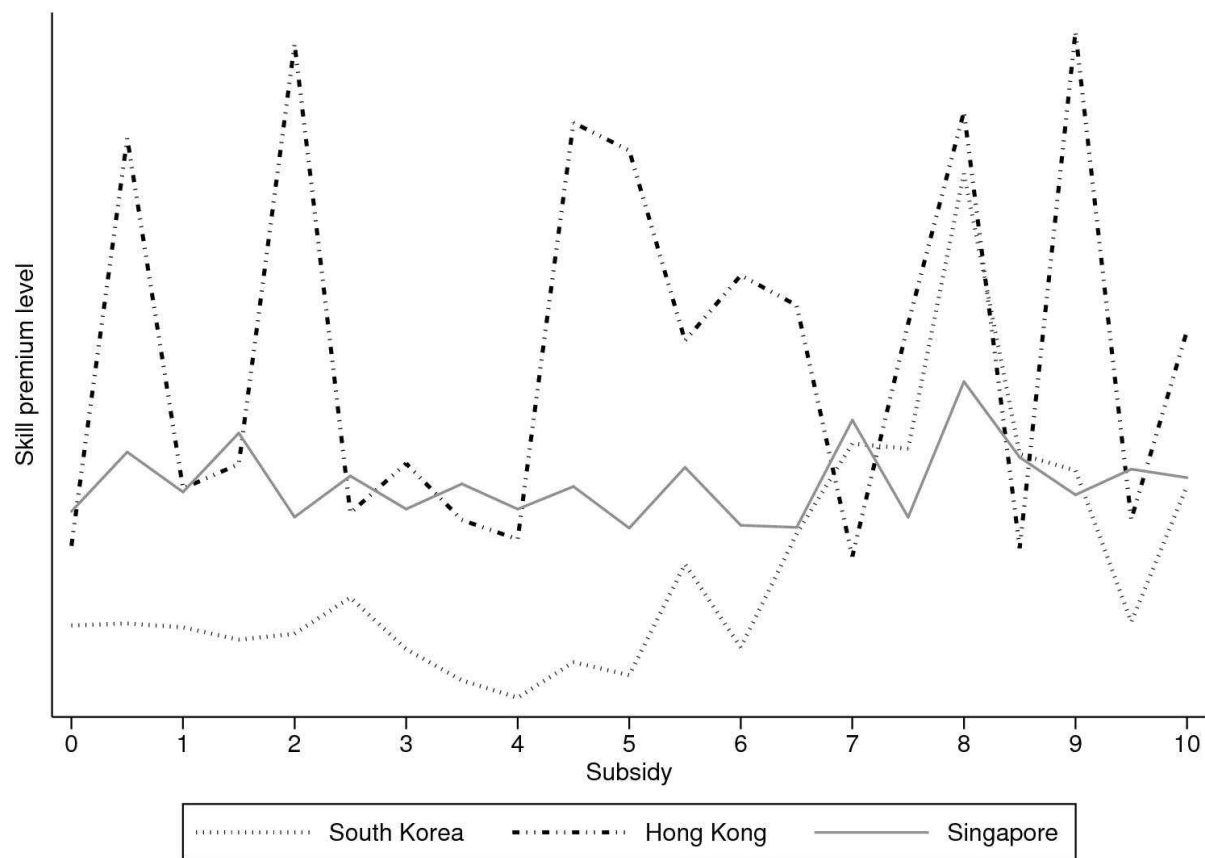


Figure 19 – Skill Premium level for percentage of subsidy  
 Source: Calibration's results.