WILLIAMS UBALDO HUAMANI QUISPE

ESSAYS ON CARTELS STABILITY UNDER ANTITRUST POLICIES

Thesis submitted to the Applied Economics Graduate Program of the Universidade Federal de Viçosa in partial fulfillment of the requirements for the degree of *Doctor Scientiae*

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VIÇOSA - MINAS GERAIS
2022

Ficha catalográfica elaborada pela Biblioteca Central da Universidade Federal de Viçosa - Campus Viçosa

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H874e
2022

Huamani Quispe, Williams Ubaldo, 1989-
Essays on cartels stability under antitrust policies / Williams Ubaldo Huamani Quispe. – Viçosa, MG, 2022.
1 tese eletrônica (59 f.): il.

Texto em inglês.
Orientador: Marcelo José Braga.
Tese (doutorado) - Universidade Federal de Viçosa, Departamento de Economia Rural, 2022.
Referências bibliográficas: f. 57-59.
DOI: https://doi.org/10.47328/ufvbbt.2022.271
Modo de acesso: World Wide Web.

1. Organização industrial (Teoria econômica). 2. Cartéis. 3. Direito antitruste. I. Braga, Marcelo José, 1969-. II. Universidade Federal de Viçosa. Departamento de Economia Rural. Programa de Pós-Graduação em Economia Aplicada. III. Título.

CDD 22. ed. 338.7

Bibliotecário(a) responsável: Alice Regina Pinto CRB6 2523

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APPROVED: March 28, 2022.

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To my parents.

Acknowledgements

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

Abstract

HUAMANI, Quispe Williams Ubaldo, D.Sc. Universidade Federal de Viçosa, March 2022. **Essays on cartels stability under antitrust policies**. Adviser: Marcelo José Braga. Co-adviser: Lucas Campio Pinha.

This thesis is composed of three essays about cartels' stability under antitrust policies. The first essay aims to create a new approach to analyze optimal antitrust policies to combat cartels in a more general context. Using the repeated games theory we build a scenario where a cartel — any collusive strategy can be considered — operates under an economy with antitrust policies. We show that antitrust policies — antitrust enforcement without leniency program, ex-ante leniency program, and ex-post leniency program — negatively affect cartels' stability. However, there are no optimal antitrust enforcement parameters without a leniency program to destabilize cartels. On the other hand, we find that total immunity of the whistle-blower cartel member is more effective than partial immunity in destabilizing cartels. The second essay aims to answer the question: is antitrust enforcement more effective against cartels when products are horizontally more homogeneous or differentiated? Based on Cournot's (Bertrand 's) horizontally differentiated duopoly model, we conclude that antitrust enforcement is more effective on cartel stability when products are highly differentiated. However, it should be noted that antitrust enforcement negatively affects cartel stability — regardless of the degree of differentiation. The third essay aims to answer: is antitrust enforcement more effective against cartels when products are vertically more homogeneous or differentiated? Based on Bertrand's vertically differentiated duopoly model, we show that antitrust enforcement uniformly and negatively affects cartel stability, for every product quality differentiation degree — antitrust enforcement does not strongly affect the stability of a cartel that has a specific product quality differentiation degree.

Keywords: Market Structure. Collusion Economics. Antitrust Policies.

Resumo

WILLIAMS, Ubaldo Huamani Quispe, D.Sc. Universidade Federal de Viçosa, março de 2022. **Essays on cartels stability under antitrust policies**. Orientador: Marcelo José Braga. Coorientador: Lucas Campio Pinha.

Esta tese é composta por três ensaios sobre a estabilidade dos cartéis diante das políticas antitruste. O primeiro ensaio visa criar uma nova abordagem para analisar políticas antitruste ótimas para combater cartéis em um contexto mais geral. Usando a teoria dos jogos repetidos, construímos um cenário onde um cartel – qualquer estratégia de conluio pode ser considerada – opera sob uma economia com políticas antitruste. Mostra-se que as políticas antitruste — antitrust enforcement sem programa de leniência, programa de leniência ex-ante, e programa de leniência ex-post — afetam negativamente a estabilidade dos cartéis. No entanto, não há parâmetros ótimos de antitrust enforcement sem um programa de leniência para desestabilizar os cartéis. Por outro lado, verificamos que a imunidade total do denunciante membro do cartel é mais eficaz do que a imunidade parcial na desestabilização de cartéis. O segundo ensaio visa responder à pergunta: o antitrust enforcement é mais eficaz contra cartéis quando os produtos são horizontalmente mais homogêneos ou diferenciados? Com base no modelo de duopólio horizontalmente diferenciado de Cournot (Bertrand), concluímos que o antitrust enforcement é mais eficaz desestabilizando carteis quando os produtos são altamente diferenciados. No entanto, deve-se notar que o antitrust enforcement afeta negativamente a estabilidade do cartel — independentemente do grau de diferenciação. O terceiro ensaio visa responder: o antitrust enforcement é mais eficaz contra cartéis quando os produtos são verticalmente mais homogêneos ou diferenciados? Com base no modelo de duopólio verticalmente diferenciado de Bertrand, mostra-se que o antitrust enforcement afeta de maneira uniforme e negativa a estabilidade do cartel, para cada grau de diferenciação da qualidade do produto – o antitrust enforcement não afeta fortemente a estabilidade de um cartel que possui um grau específico de diferenciação de qualidade do produto.

Palavras-chave: Estrutura de Mercados. Economia do Conluio. Políticas Antitruste.

Contents

Introduction

Competition subjects firms to continuous pressure to offer the best variety of products at the best prices, to benefit consumers. Sometimes firms try to ruin the competition. For the proper functioning of competitiveness in an economy, antitrust authorities constantly monitor two groups of anti-competitive practices: mergers and acquisitions, and dominant positions and collusion. On the one hand, mergers and acquisitions are a mechanism for the union of firms to obtain greater market concentration (they can disturb the market price). However, there is also the possibility that efficiency can be improved (by lowering the price). The task of the antitrust authority is to analyze the consequences of the approval (or approval with restrictions) or not of the merger. On other hand, monopolies and coordinated oligopolies (cartels) are clear examples of dominant positions, whose purpose is to create entry barriers for new firms, application of predatory prices (creation of artificial prices to suppress the entry of a new rival firm) to obtain supra-competitive profits (profits above competitive profit). The task of the antitrust authority in this item is to regulate monopolies and punish cartel members (collusive agreements).

The United States is one of the first countries to enact a law to control and punish anti-competitive practices — inserted in 1890 as the Sherman Act. A priori, the objective was to control and sanction monopolies and coordinated oligopolies (currently we call them cartels). A posteriori, the antitrust law in the United States improved by inserting leniency programs to combat cartels in 1993. Thus, seeing significant results, developed and developing countries began to implement their laws, but based on the Sherman Act. Consequently, in economics, an interesting topic arises to study: the effects of leniency policies on cartels stability.

Nowadays, the literature on cartels' stability under antitrust policies has gained considerable importance due to the search for optimal antitrust policies to combat cartels (Block et al., 1981; Souam, 2001; Motta and Polo, 2003; Harrington Jr, 2008; Chen and Rey, 2013; Houba et al., 2015; Pinha and Braga, 2021). Cartels occur when a group of firms in a certain industry agrees on their behavior to obtain supra-competitive profits. Anti-competitive activities such as cartels are considered a type of organized crime that damages the economic order because it mainly harms the well-being of consumers with high prices, restricted production capacity, and others (Spagnolo, 2008; Harrington Jr, 2017). Antitrust authorities, to combat anti-competitive activities such as cartels, make use of leniency policies. The leniency policies (leniency program) guarantee partial or total immunity from the fine to the cartel whistle-blower firm in exchange for denouncing their illicit activity and providing relevant information on the behavior of their former partners until the end of the investigation process. Thus, the antitrust authority can fulfill its task, successfully prosecuting and sanctioning the case. The leniency program is applied by the antitrust authority in two stages, ex-ante (ex-post) before (after) having initiated its investigation. Analyzing the cartels stability in these scenarios — different antitrust policy regimes — has led to studying the effects of antitrust policies on cartels' stability.

Early cartel stability studies sought to analyze the factors intrinsic to the structure that destabilize cartels. For example, the difference in production costs (Rothschild, 1999; Vasconcelos, 2005; Collie, 2006; Miklos-Thal, 2011; Ciarreta and Gutiérrez-Hita, 2012) volatility in the demand system (Staiger and Wolak, 1992; Lambertini, 1996), product differentiation degree Ross (1992); Majerus (1988); Häckner (1994); Andaluz (2010), and among others. Here, we mainly studied the trade-off between the product differentiation[1] degree (horizontal and vertical) and cartel stability. Nowadays, an external factor has been inserted — antitrust policies (antitrust enforcement without leniency program, ex-ante leniency program, and ex-post leniency program)— in the dynamics of cartel stability. Therefore, this is how a new topic is born: cartels' stability under antitrust policies.

This thesis is made up of three essays on cartels' stability under antitrust policies. Although the works are presented independently, we consider that these three topics are related. Essentially, these three essays try to understand that cartel's stability is affected by internal factors (product differentiation) and external factors (antitrust policies) that destabilize it.

The main objective of the first essay is to create a general approach — analysis that is independent of the collusive strategy adopted by the cartel — to analyze the cartels' stability under different antitrust policies regimes — antitrust enforcement without a leniency program, ex-ante leniency program, and ex-post leniency program—, and find optimal leniency policies to destabilize cartels. Using repeated game theory we construct a scenario of a cartel operating under an economy with antitrust policies. The inclusion of antitrust policy parameters defines the scenarios: cartel stability under antitrust enforcement without leniency program, ex-ante leniency program, and ex-post leniency program. To contribute to the literature, our approach allows diagnosing optimal antitrust policies for all types of cartels. We also highlight that our approach allows us to analyze the cartels' stability under internal factors (product differentiation degree, the difference in production costs, and among others) and external factors (antitrust policies) simultaneously. This topic is important because there are factors internal to the market structure that can mitigate the effects of antitrust policies on cartels' stability.

The second essay aims to analyze the cartel's stability — with a collusive quantity-fixing and price-fixing strategy — under antitrust enforcement at different levels of horizontal product differentiation. Essentially, we want to answer: is antitrust enforcement more effective against cartels when products are horizontally more homogeneous or differentiated? The literature from its beginnings has tried to understand the relationship between the horizontal product differentiation degree and cartel stability because product differentiation plays an important role in the cartel's stability, and is

[1]Product differentiation is a strategy adopted by companies to attract more consumers and thus gain more market share. This differentiation can be divided into two. First, horizontally differentiated products: it is intrinsically linked to the consumer and the degree of substitution of the consumer, essentially it is a matter of taste. Second, vertically differentiated products (or products differentiated by quality): there is a consensus among consumers, that they know how to differentiate which product is better than another.

also a good indicator for cartels in different industries. To contribute to the literature, we include antitrust enforcement in this context.

The third essay aims to examine the cartel's stability — with a collusive price-fixing strategy— under antitrust enforcement at different levels of product quality differentiation. Essentially, we want to answer: is antitrust enforcement more effective against cartels when products are vertically more homogeneous or differentiated? The literature has not found a clear trade-off between the product quality differentiation degree and cartel stability, due to the complexity of the parameters. However, it is known that the behavior is not the same as in the case of horizontally differentiated products. To contribute to the literature, we find a new trade-off and include antitrust enforcement in this context.

The assumptions of the theoretical model from the first essay serve as the basis for the second and third essays. Since our approach enjoys versatility, we insert to cartels stability, factors intrinsic to the market structure (product differentiation degree) in the second and third essays. Thus, we analyze the effects of antitrust enforcement on cartels' stability at different levels of product differentiation. Economies are made up of different industries and with different levels of product differentiation. Therefore, it is timely to understand how cartels in different industries are affected by antitrust enforcement. This analysis is essential for antitrust policymakers and antitrust authority activity.

The essays in this thesis are connected and present a comprehensive version of cartel stability under antitrust policies and cartel stability under antitrust enforcement at different levels of product differentiation. The first essay presents a general approach to cartels' stability under different antitrust policy regimes. While the second and third essays present the effects of antitrust enforcement on cartels' stability at different levels of product differentiation, horizontal and vertical, respectively. The development of the thesis contributes to the advancement of the literature on the economics of collusion, providing a basis for the study of cartel stability under internal (degree of product differentiation, the elasticity of demand, the difference in production costs, among others) and external (antitrust policies) factors simultaneously. In practice, it can help guide antitrust policymakers and the antitrust authorities' activities, indicating higher fine for firms that participate in more stable cartels because they enjoy excessive cooperative profit relative to competitive profit.

Chapter 1

Effect of Leniency Program on Cartels Stability: A General Approach

Abstract: This paper examines cartels stability under different antitrust policy regimes — antitrust enforcement without leniency program, ex-ante leniency program, and ex-post leniency program. Our approach presents results in a simple and generalized way — the results are independent of the collusive strategy adopted by the cartels. We show that antitrust policies negatively affect the cartel's stability. However, there are no optimal antitrust enforcement parameters to destabilize cartels. On the other hand, we show that the total immunity regarding the fine for the cartel whistle-blower member, when applied to the leniency program (ex-ante or ex-post leniency), guarantees greater instability in the cartels. Our approach is versatile, for future research, in our models, we can insert factors intrinsic to the market structure to diagnose which mitigate the effects of antitrust policies.

Keywords: Market Structure · Antitrust Enforcement · Leniency Programs · Cartels Stability.

JEL Classification: L13 · L41 · D43 · C73

1.1 Introduction

Nowadays, the literature on cartels' stability under antitrust policies has gained considerable importance due to the search for optimal antitrust policies to combat cartels. Cartels occur when a group of firms in a certain industry agrees on their behavior to obtain supra-competitive profits. Anti-competitive activities such as cartels are considered a type of organized crime that damages the economic order because it mainly harms the well-being of consumers with high prices, restricted production capacity, and others (Spagnolo, 2008; Harrington Jr, 2017). Antitrust authorities, to combat anti-competitive activities such as cartels, make use of leniency program. The leniency program guarantee partial or total immunity from the fine to the cartel whistle-blower firm in exchange of denouncing their illicit activity and providing relevant information on the behavior of their former partners until the end of the investigation process. Thus, the antitrust authority can fulfill its task, successfully prosecuting and sanctioning the case. The leniency program is applied by the antitrust authority in two stages, ex-ante (ex-post) before (after) having initiated its investigation. Analyzing the cartels stability in these scenarios — the presence of the authority with antitrust policies — has led to studying the effects of antitrust policies on cartels' stability.

The literature on cartels stability under different leniency policy regimes has been continuously explored for cases of cartels with collusive price-fixing strategies (Block et al., 1981; Souam, 2001; Motta and Polo, 2003; Harrington Jr, 2008; Houba et al., 2015; Pinha and Braga, 2021). Our paper proposes to create a general approach — simple and generalized — to study cartels' stability under antitrust policies —antitrust enforcement without leniency program, ex-ante leniency program, and ex-post leniency program—, and thus find optimal antitrust policies to combat cartels. Having this different approach promises future work on cartel stability under internal factors (product differentiation degree, volatility of the demand system, and among others) and external factors (presence of the antitrust authority with antitrust policies) at the same time. On the other hand, the results can guide the activity of the antitrust authorities, indicating the design of antitrust policies regarding the fight against cartels.

To unwrap the dynamics of cartel stability under different leniency policy regimes we use the sub-game perfect Nash equilibrium of the modified trigger strategy — trigger strategy is a sub-game of the infinitely repeated game induced by the cartel dilemma. The scenario is an economy that operates a cartel (with any collusive strategy) made up of N symmetrical firms under antitrust law and an antitrust authority that applies leniency program. In this sense, we understand: (i) cartel stability under antitrust enforcement without leniency program; (ii) cartel stability under leniency programs (ex-ante leniency program and ex-post leniency program).

The effects of antitrust enforcement without leniency program on cartels' stability are continually discussed. The works of Souam (2001), and Motta and Polo (2003) converge on some points in the case of cartels with a collusive price-fixing strategy: antitrust enforcement negatively affects cartels stability — antitrust enforcement causes the collusive price to decrease (Houba et al., 2015). Extending to a more general result — an analysis that is independent of the cartel's collusive strategy. Our paper also shows that antitrust enforcement negatively affects cartel stability. However, there are no antitrust enforcement parameters to destabilize cartels (Souam, 2001).

On the other hand, the effects of leniency programs on cartels' stability are also strongly discussed. The results of Houba et al. (2015) converge to the point for the case of price-fixing cartels: (i) leniency programs cause the collusive price to decrease, making consumers' welfare better; (ii) to further destabilize cartels, the antitrust authority must guarantee total immunity from the fine to cartel whistle-blower member (Block et al., 1981; Motta and Polo, 2003; Harrington Jr, 2008; Houba et al., 2015; Pinha and Braga, 2021). However, poorly designed leniency program could lead to efficiency in fighting cartels — leniency programs would be taken advantage of by cartel members due to high cooperative profitability and non-serious fines (Chen and Rey, 2013). Extending to more general results — regardless of the collusive strategy adopted by the cartel. Through our approach, we show that indeed the leniency program negatively affects (destabilizes cartels) on cartel's stability, and item (ii) is valid. Based on our generalized results, we guarantee that total immunity from fines for cartel members (cartel with any type of collusive strategy) that applies to the leniency program (ex-ante or ex-post) will guarantee greater efficiency in combating cartels.

The remaining of the paper is organized as follows. Section 1.2 describes the capture of profits that form cartel dilemma, and cartels stability. Section 1.3 describes cartels stability under antitrust enforcement and some important results. Section 1.4 describes cartel's stability under leniency program and conditions to generate greater instability in the cartels. Finally, Section 1.5 concludes the paper and makes recommendations for the implementation of leniency program.

1.2 Competition Models

The models developed by Cournot (1838) and Bertrand (1883) characterize competition in imperfect markets, where the strategic interaction of firms is by quantity to be produced and by price to be charged respectively. Moving to a collusive (or cartel) environment, collusive agreements commonly have quantity-fixing and price-fixing as strategies. This section describes the profit capture that constitutes the cartel's dilemma.

1.2.1 Quantity-Fixing

Let us consider a market with either homogeneous or differentiated products where an oligopoly of $N = \{1, 2, \cdots, n\}$ firms serves a inverse demand system.

$$P_i(q_1, q_2, \cdots, q_n), \quad \text{for all} \quad i \in \{1, 2, \cdots, n\} \tag{1.1}$$

The strategic interaction of the N firms via Cournot consists in that: i) each firm $i \in N$ choosing $q_i \in [0, \infty)$; ii) each firm $i \in N$ is endowed with a payoff function (profit function) π_i — operating at constant marginal cost $c > 0$ — defined as:

$$\pi_i(q_i, q_{-i}) = (P_i(q_i, q_{-i}) - c_i)q_i, \quad \text{for all} \quad i \in N. \tag{1.2}$$

The behavior of firms in the market leads to different profitability. In a non-cooperative (competitive) environment, firms have as a solution the individual maximization of their profits — Nash-Cournot equilibrium $q^N = (q_i^N, q_{-i}^N)$ — and this can be expressed concerning their best response functions:

$$q_i \in \arg\max_{q_i}\{\pi_i(q_i, q_{-i}) = (P_i(q_i, q_{-i}) - c_i)q_i\}, \quad \text{for all} \quad i \in N. \tag{1.3}$$

Then, evaluating the Nash-Cournot equilibrium $q^N = (q_i^N, q_{-i}^N)$ in its profit functions, for every firm $i \in N$, we obtain its non-cooperative profit π_i^N. On the other hand, in a cooperative environment, the firms maximize the joint profit of the industry and the solution is a maximization process — the optimal point q^C of the sum of profit functions — and this can be expressed as the first-order conditions.

$$\frac{\partial \pi(q)}{\partial q_i} = (P_i(q_i, q_{-i}) - c) + \frac{\partial P_i(q_i, q_{-i})}{\partial q_i} q_i + \frac{\partial \sum\limits_{i \in N-i}^{N-i} (P_i(q_i, q_{-i}) - c) q_{-i}}{\partial q_i} = 0. \tag{1.4}$$

Then, evaluating the optimal point q^C in the profit functions, for each firm $i \in N$ we obtain its cooperative profit π_i^C. Finally, in a deviation environment — one firm deviates from the cooperative agreement and the others maintain the cooperative action. We assume that firm $i \in N$ deviates from the agreement since the firms $j \in N - \{i\}$ cooperate. The solution for firm $i \in N$ is individual maximization given that the others cooperate — optimal point (q_i^D, q_{-i}^C) of the profit function $\pi(q_i, q_{-i}^C)$ —, and this can be expressed as the following first-order condition.

$$\frac{\partial \pi_i(p_i, p_{-i}^C)}{\partial p_i} = D_i(p_i, p_{-i}^C) + (p_i - c_i)\frac{\partial D_i(p_i, p_{-i}^C)}{\partial p_i} = 0. \tag{1.5}$$

Therefore, evaluating the optimal point (q_i^D, q_{-i}^C) in the profit functions, we obtain diversion profit π_i^D for firm $i \in N$ and damage profit π_{jD}, for each firm $j \in N - \{i\}$.

The profits captured in this section constitute the cartel dilemma[1] — all firms have cooperate (C) and non-cooperate (NC) actions. The cartel dilemma induces an infinitely repeated game, in particular, the sub-game called grim-trigger strategy.

1.2.2 Price-Fixing

Let us consider a market structure with either homogeneous or differentiated products where an oligopoly of $N = \{1, 2, \cdots, n\}$ firms serves a demand system.

$$D_i(p_1, p_2, \cdots, p_n), \quad \text{for all } \; i \in \{1, 2, \cdots, n\} \tag{1.6}$$

The strategic interaction of the N firms via Bertrand consists in that: i) each firm $i \in N$ chooses the price to charge $p_i \in [0, \infty)$; ii) each firm $i \in N$ is endowed with a payoff function π_i (profit function) — operating at a constant marginal cost $c > 0$ — defined as:

$$\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i}), \quad \text{for all } \; i \in N. \tag{1.7}$$

The profitability of firms depends on their behavior in the market. In a competitive environment (non-cooperative), firms individually maximize their profits. The solution is the Nash-Bertrand equilibrium $p^N = (p_i^N, p_{-i}^N)$ and this can be expressed as its best response functions:

$$p_i \in \arg\max_{p_i}\{\pi_i(p_i, p_{-i}) = (p_i - c_i)D_i(p_i, p_{-i})\}, \quad \text{for all } \; i \in N. \tag{1.8}$$

Next, evaluating the Nash-Bertrand equilibrium $p^N = (p_i^N, p_{-i}^N)$ in the profit functions, for each firm $i \in N$ we obtain its non-cooperative profit π_i^N. On the other hand, in a cooperative environment, the firms maximize the joint profit of the industry. The solution

[1]The bimatricial representation of the cartel dilemma for two symmetrical firms is:

$$\begin{pmatrix} \pi^C, \pi^C & \pi_D, \pi^D \\ \pi^D, \pi_D & \pi^N, \pi^N \end{pmatrix}$$

where the first row (column) are the payoffs of the cooperate action of firm 1 (firm 2) and the second row (column) are the payoffs of the non-cooperate action of firm 1 (firm 2).

is the optimal point p^C of profit functions sum and this can be expressed as the first-order conditions:

$$\frac{\partial \pi(p)}{\partial p_i} = D_i(p_i, p_{-i}) + (p_i - c_i)\frac{\partial D_i(p_i, p_{-i})}{\partial p_i} + \frac{\partial \sum_{i \in N-i}^{N-i} (p_{-i} - c_{-i}) D_{-i}(p_i, p_{-i})}{\partial p_i} = 0. \quad (1.9)$$

Then, evaluating the optimal point p^C in the profit functions, for each firm π_i^N we obtain its profit cooperative π_i^C. Finally, in a cheating environment, one firm maximizes its profit while the other firms cooperate. Suppose firm $i \in N$ deviates and firms $N - \{i\}$ cooperate. The solution is the optimal point (p_i^D, p_{-i}^C) of function $\pi_i(p_i, p_{-i}^C)$ and this can be expressed as the first-order conditions:

$$\frac{\partial \pi_i(p_i, p_{-i}^C)}{\partial p_i} = D_i(p_i, p_{-i}^C) + (p_i - c_i)\frac{\partial D_i(p_i, p_{-i}^C)}{\partial p_i} = 0. \quad (1.10)$$

Therefore, evaluating the optimal point (p_i^D, p_{-i}^C) in the profit functions of the firms, we obtain deviation profit π_i^D for firm $i \in N$ and damage profit π_{-iD} for the firms $N - \{i\}$.

Analogously to Subsection 1.2.2, the profits found in this section constitute the cartel's dilemma and are related: profit diversion π_i^D is greater than cooperative profit π_i^C; cooperative profit $\pi_i^C$is greater than competitive profit π_i^N, and competitive profit π_i^N is greater than damage profit π_{iD}^N.

1.2.3 Cartel Stability

The sub-game perfect Nash equilibrium (SPNE) of the grim-trigger strategy[2] (Friedman, 1971) — sub-game of the infinitely repeated game induced by the cartel dilemma — characterizes cartel stability. A cartel made up of $N = \{1, \cdots, i, \cdots, n\}$ firms is said to be stable if for all firm $i \in N$ there exists $\delta \in (0, 1)$, such that $\delta = \max\{\delta_1, \cdots, \delta_i, \cdots, \delta_n\}$ and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \iff \delta \geq \delta_i = \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^N}, \quad (1.11)$$

where V_i^C is the liquid present value of firm $i \in N$, when it maintains the collusive agreement — in each period firm $i \in N$ takes the cooperate action —, V_i^D is the liquid present value of firm $i \in N$, when it deviates from the agreement — in the first period the firm $i \in N$ deviates from the agreement and then decides to compete forever, and $\delta_i \in (0, 1)$ is the discount factor with which firm $i \in N$ maintains cartel stability. The minimum discount factor of firm $i \in N$ — discount factor satisfies the equality of the expression 1.11 — is called the critical discount factor of firm $i \in N$. The set defined as

[2]To analyze cartel stability we choose the sub-game perfect Nash equilibrium (SPNE) of the trigger strategy because we want to observe the effect of the degree of product differentiation. There is an extensive literature that uses the grim-trigger strategy to analyze cartel stability affected by factors intrinsic to the market structure. Essentially, the critical discount factor measures the degree of cartel stability. See Ross (1992); Majerus (1988) and among others.

$\{\delta \in (0,1) : \delta_i \leq \delta < 1\}$ is called the stability interval of firm $i \in N$ — the cartel member firm that has a smaller stability interval is more likely to destabilize the cartel, i.e., the firms operate at a higher discount factor. Note that the right-hand side of the inequality 1.11 can only be perturbed by factors intrinsic to the market structure. The set defined as $\{\delta \in (0,1) : \max\{\delta_1, \cdots, \delta_i, \cdots, \delta_n\} \leq \delta < 1\}$ is called cartel stability interval. The cartel tends to be less stable if the cartel stability interval tends to decrease.

To analyze the cartel's stability made up of N symmetrical firms, it is enough to analyze the behavior of only one cartel member firm. A cartel made up of N symmetric firms is said to be stable if there is a discount factor $\delta \in (0,1)$, such that it satisfies the incentive compatibility constraint.

$$\delta \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^N}. \tag{1.12}$$

Below we exemplify cartels stability — cartels made up of symmetrical duopoly with a collusive strategy of quantity-fixing and price-fixing — in a market with horizontally differentiated products.

Example 1.1. Consider the strategic interaction via Cournot of symmetric firms, 1 and 2, serving a demand system (inverse), $p_1 = 100 - (q_1 + \theta q_2)$, $p_2 = 100 - (q_2 + \theta q_1)$, with a horizontal product differentiation degree $\theta = 0.5$, at a constant marginal cost equal to $c = 0$. Then, using the mechanism of Subsection 1.2.1 we obtain the profits[3]: competitive profit $\pi^N = 1600.0$; cooperative profit $\pi^C = 1666.7$; diversion profit $\pi^D = 1736.1$, and damage profit $\pi_D = 1527.8$. Then, substituting these values in the inequality 1.11 we obtain the characterization of cartel stability made up of firms 1 and 2. The cartel made up of firms 1 and 2 is said to be stable it satisfies incentive compatibility constraints:

$$\delta \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^N} = \frac{1736.1 - 1666.7}{1736.1 - 1600.0} = 0.50992.$$

The cartel's stability interval is $[0.50992, 1)$. In other words, the cartel is stable for all discount factors $\delta \in [0.50992, 1)$.

Example 1.2. Consider the strategic interaction via Bertrand of symmetric firms, 1 and 2, serving a demand system, $q_1 = \dfrac{100(1-\theta) - p_1 + \theta p_2}{1-\theta^2}$, $q_2 = \dfrac{100(1-\theta) - p_2 + \theta p_1}{1-\theta^2}$, with a horizontal product differentiation degree $\theta = 0.5$, at a constant marginal cost equal to $c = 0$. Then using the mechanism of Subsection 1.2.2 we obtain the profits: competitive profit $\pi^N = 1481.5$; cooperative profit $\pi^C = 1666.7$; diversion profit $\pi^D = 1875.0$, and damage profit $\pi_D = 1250.0$. Then, substituting these values in the inequality 1.11 we obtain the characterization of cartel stability made up of firms 1 and 2. The cartel made of firms 1 and 2 is said to be stable if satisfies incentive compatibility constraints:

$$\delta \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^N} = \frac{1875.0 - 1666.7}{1875.0 - 1481.5} = 0.52935.$$

[3] The bimatricial representation of the cartel's dilemma is

$$\begin{pmatrix} 1666.7, 1666.7 & 1527.8, 1736.1 \\ 1736.1, 1527.8 & 1600.0, 1600.0 \end{pmatrix}$$

where the first row (column) are the payoffs of the cooperate action of firm 1 (firm 2) and the second row (column) are the payoffs of the non-cooperate action of firm 1 (firm 2).

The cartel's stability interval is $[0.52935, 1)$. In other words, the cartel is stable for all discount factors $\delta \in [0.52935, 1)$.

Looking at Examples 1.1 and 1.2 we diagnose that the quantity-fixing cartel is more stable than the price-fixing cartel — the stability interval of the quantity-fixing cartel is greater than the stability interval of the price-fixing cartel, i.e., $[0.52935, 1) \subset [0.50992, 1)$.

1.3 Antitrust Enforcement without Leniency Program

This section describes cartel stability under antitrust enforcement without leniency program — represented by the probability of cartel detection and rate of fine imposed on cartel members—, characterized by the sub-game perfect Nash equilibrium (SPNE) of the modified grim-trigger strategy[4].

The elements that characterize cartel stability are described below. Denote as V_i^C the expected present value of firm $i \in N$, when it maintains the collusive agreement — in each period firm $i \in N$ maintains the cooperative action knowing that the cartel can be discovered with a certain probability and applied a certain rate of fine — , and define as the following recursive dynamics:

$$V_i^C = \pi_i^C + \beta \left[-f\pi_i^C + \bar{\delta}_i \frac{\pi_i^N}{1-\bar{\delta}_i} \right] + (1-\beta)\bar{\delta}_i V_i^C, \tag{1.13}$$

where $\beta \in (0,1)$ is the probability of cartel detection, $f \in (0,1)$ is the fine rate applied to cartel members, $\bar{\delta}_i \in (0,1)$ is the discount factor with which firm $i \in N$ maintains the cartel stability. On the other hand, denote as V_i^D the liquid present value of firm $i \in N$, when it deviates from the collusive agreement — firm $i \in N$ deviates in the first period and then competes forever — and define as:

$$V_i^D = \pi_i^D + \frac{\bar{\delta}_i \pi_i^N}{1-\bar{\delta}_i} = \frac{(1-\delta)\pi_i^D + \bar{\delta}_i \pi_i^N}{1-\bar{\delta}_i}. \tag{1.14}$$

Since $1 - \bar{\delta}_i > 0$, and $1 - \beta(1-\bar{\delta}_i) > 0$, the sub-game perfect Nash equilibrium (SPNE) of the modified grim-trigger strategy characterizes cartel stability under antitrust enforcement. A cartel made up of N firms is said to be stable under antitrust enforcement without leniency program if for all firm $i \in N$ there exist $\bar{\delta} \in (0,1)$ such that $\bar{\delta} = \max\{\bar{\delta}_1, \cdots, \bar{\delta}_i, \cdots, \bar{\delta}_n\}$, and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \Longleftrightarrow \bar{\delta}_i \geq \frac{\pi_i^D - (1-\beta f)\pi_i^C}{(1-\beta)[\pi_i^D - \pi_i^N]}. \tag{1.15}$$

[4] To analyze cartel stability under antitrust enforcement we choose the sub-game perfect Nash equilibrium (SPNE) of the modified grim-trigger strategy because we want to observe the effects of the degree of product differentiation and antitrust enforcement simultaneously. There is extensive literature that uses the modified grim-trigger strategy to analyze what factors external to the market structure destabilize cartels, for example, antitrust enforcement. See Motta and Polo (2003); Harrington Jr (2008), and among others.

The minimum discount factor $\bar{\delta}_i$ (discount factor that satisfies equation 1.15) with which firm $i \in N$ maintains cartel stability is called the critical discount factor under antitrust enforcement of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \bar{\delta}_i \leq \delta < 1\}$ is called the stability interval under antitrust enforcement of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \max\{\bar{\delta}_1, \cdots, \bar{\delta}_i, \cdots, \bar{\delta}_n\} \leq \delta < 1\}$ is called the cartel stability interval under antitrust enforcement. If the cartel stability interval tends to decrease, then we say that cartel stability is negatively affected.

Analogously to Subsection 1.2.1, to analyze the cartel's stability made up of symmetric firms, it is enough to analyze the behavior of only one cartel member firm. A cartel made up of N symmetric firms is said to be stable under antitrust enforcement if there is a discount factor $\bar{\delta} \in (0,1)$, such that it satisfies the incentive compatibility constraint:

$$\bar{\delta} \geq \frac{\pi^D - (1-\beta f)\pi^C}{(1-\beta)[\pi^D - \pi^N]}.$$

To measure the impact of antitrust enforcement on cartel stability, we compare the discount factors in both scenarios— under free antitrust enforcement and antitrust enforcement. Doing a simple exercise, observe that if the cartel detection probability is $\beta = 0$, then the critical discount factors in both scenarios are equal $\delta = \bar{\delta}$[5]. Thus, there would be no impact on cartels stability. The definition to follow characterizes the impact of antitrust enforcement on cartels' stability.

Definition 1.3. Consider a cartel made up of N symmetrical firms. Antitrust enforcement negatively affects cartel stability if the discount factor under antitrust enforcement is greater than the critical discount factor, i.e., $\delta < \bar{\delta}$.

The following result essentially shows that antitrust enforcement negatively affects the cartel's stability — it destabilizes cartels regardless of the collusive strategy adopted by the cartel.

Proposition 1.4. *Consider a cartel made up of N symmetrical firms. Antitrust enforcement negatively affects cartel stability, i.e., the critical discount factor under antitrust enforcement is greater than the critical discount factor.*

Proof. To show this conjecture it is sufficient to verify that $\delta < \bar{\delta}$. Since $\pi^D - \pi^C < \pi^D - (1-\beta f)\pi^C$, for all $\beta \in (0, \bar{\beta}] \subset [0,1)$, $\beta \in (0, \bar{f}] \subset [0,1)$; and $(1-\beta)(\pi^D - \pi^N) < \pi^D - \pi^N$, for all $\beta \in (0, \bar{\beta}] \subset [0,1)$. Implies that $\frac{\pi^D - \pi^C}{\pi^D - \pi^N} < \frac{\pi^D - (1-\beta f)\pi^C}{(1-\beta)(\pi^D - \pi^N)}$. Therefore it is fulfilled that $\delta < \bar{\delta}$. □

Proposition 1.4 shows in a general way the effect of antitrust enforcement on cartels stability — such an effect is negative for the cartels' stability, regardless of the collusive cartel strategy. Then, for there to be an impact, it is sufficient that the antitrust law (antitrust enforcement) exists and is in force in the economy where the cartels operate. Therefore, a minimal probability of detection causes instability in the cartels. Doing a simple exercise: if the antitrust law (antitrust enforcement) is not applied, then the

[5]See Bruttel (2009).

probability of cartel detection is $\beta = 0$, which means that the critical discount factors are equal $\bar{\delta} = \delta$. Therefore, there is no instability in the cartels. To illustrate Proposition 1.4 follow the example.

Example 1.5. Consider Example 1.1, and antitrust enforcement parameters $\bar{f} = \min\{f : 0.01 \leq f \leq 0.20\}$ and $\bar{\beta} = 0.15$. We obtain critical discount factor $\delta = 0.50992$ and critical discount factor under antitrust enforcement $\bar{\delta} = 0.68094$. As critical discount factors have the relationship $\delta = 0.50992 < 0.68094 = \bar{\delta}$. Then, antitrust enforcement negatively affects cartel stability.

Taking the critical discount factor under antitrust enforcement as dependent on the probability of cartel detection and fine rate for cartel members. The following proposition diagnoses that there are no optimal antitrust enforcement parameters to destabilize cartels.

Proposition 1.6. *Consider a cartel made up of N symmetrical firms. There are no optimal antitrust enforcement parameters to destabilize cartels.*

Proof. Since $\bar{\delta} : (0, \bar{\beta}] \times (0, \bar{f}] \longrightarrow (0, 1)$ is differentiable, to prove is sufficient verify that

$$H\bar{\delta}(\beta, f) = \begin{bmatrix} \dfrac{2(\pi^D - (1-f)\pi^C)}{(1-\beta)^3(\pi^D - \pi^N)} & \dfrac{\pi^C}{(1-\beta)^2(\pi^D - \pi^N)} \\ \dfrac{\pi^C}{(1-\beta)^2(\pi^D - \pi^N)} & 0 \end{bmatrix}$$

The determinant of the hessian matrix is $|H\bar{\delta}(\beta, f)| = -\left(\dfrac{\pi^C}{(1-\beta)^2(\pi^D - \pi^N)}\right)^2 < 0$, this implies that for all $(\beta, f) \in (0, \bar{\beta}] \times (0, \bar{f}]$ is saddle point. □

Souam (2001) analyzes cartel stability of price-fixing under antitrust enforcement and shows that there are no better or worse antitrust enforcement parameters to destabilize cartels. Proposition 1.4 clearly shows us that antitrust enforcement destabilizes cartels. However, Proposition 1.6 shows us that there are no optimal antitrust enforcement parameters to destabilize cartels.

1.4 Leniency Program

This section describes cartels' stability under the presence of an antitrust authority with two regimes of leniency program. First, it describes cartels' stability under the ex-ante leniency program. Second, it describes cartels' stability under the ex-post leniency program.

1.4.1 Ex-ante Leniency Program

The ex-ante leniency program is a tool that helps the antitrust authority diagnose cartels through a cartel reporting member — a cartel reporting member has to report on the cartel before the antitrust authority initiates an investigation on the cartel. The antitrust authority encourages cartel members to apply to the ex-ante leniency program to provide relevant information about the cartel and partner behavior until the end of the process, and in return guarantees them partial or total immunity from fines.

The elements that characterize cartel stability are described below. Denote as V_i^C the expected present value of firm $i \in N$ when it maintains collusive agreement — in each period the firm $i \in N$ cooperates with the responsibility that the cartel can be detected with certain probability and applied a certain fine rate — define as the following recursive dynamics:

$$V_i^C = \pi_i^C + \beta \left[-f\pi_i^C + \delta \frac{\pi_i^N}{1-\delta} \right] + (1-\beta)\delta V_i^C, \tag{1.16}$$

where $\beta \in (0,1)$ represents the probability of cartel detection, $f \in (0,1)$ is the fine rate applied to cartel member $i \in N$, and δ_i is the critical discount factor with which firm $i \in N$ maintains cartel stability under the ex-ante leniency program. On the other hand, denote as V_i^D the expected present value of firm $i \in N$ when it deviates from the cartel — in each period firm $i \in N$ can deviate from the agreement with a certain probability by prioritizing a ex-ante leniency rate — and define as the following dynamic recursive:

$$V_i^D = \gamma \left[(1-\alpha)\pi_i^C + \delta \frac{\pi_i^N}{1-\delta} \right] + (1-\gamma)\delta V_i^D. \tag{1.17}$$

where γ is the probability that firm iii applies to the ex-ante leniency program, α the immunity rate for those who apply to the ex-ante leniency program. Since $1-\delta > 0$, $1-\beta(1-\delta) > 0$, $1-\delta(1-\gamma) > 0$, the sub-game perfect Nash equilibrium of the modified grim-trigger strategy characterizes cartel stability. A cartel made up of N firms is said to be stable under ex-ante leniency program if for each firm $i \in N$ there exists $\bar{\delta}^{EA} \in (0,1)$ such that $\bar{\delta}^{EA} = \max\{\bar{\delta}_1^{EA}, \cdots, \bar{\delta}_i^{EA}, \cdots, \bar{\delta}_n^{EA}\}$, and satisfies incentive compatibility constraint:

$$V_i^C \geq V_i^D \iff \bar{\delta}_i^{EA} \geq \frac{[(1-\beta f) - \gamma(1-\alpha)]\pi_i^C}{[(1-\beta f) - \gamma(1+(1-\beta f) - \beta(1-\alpha) - \alpha)]\pi_i^C - (\beta-\gamma)\pi_i^N}. \tag{1.18}$$

The minimum discount factor $\bar{\delta}_i^{EA} \in (0,1)$ — discount factor that satisfies the equality of the expression 1.18 — with which firm $i \in N$ maintains cartel stability under ex-ante leniency program is called the critical discount factor under ex-ante leniency of firm $i \in N$. The set of discount factors with which firm $i \in N$ maintains cartel stability under ex-ante program leniency defined as $\{\delta \in (0,1) : \bar{\delta}_i^{EA} \leq \delta < 1\}$ is called the stability interval under ex-ante leniency of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \max\{\bar{\delta}_1^{EA}, \cdots, \bar{\delta}_i^{EA}, \cdots, \bar{\delta}_n^{EA}\} \leq \delta < 1\}$ is called the cartel's stability interval under ex-ante leniency. If the stability interval tends to decrease then the cartel tends to destabilize.

Definition 1.7. Consider a cartel made up of N symmetrical firms. The ex-ante leniency program negatively affects cartel stability if the critical discount factor under ex-ante leniency program is greater than the critical discount factor, i.e., $\delta < \bar{\delta}^{EA}$.

The following proposition shows the importance of implementing an ex-ante leniency program to combat cartels. Essentially, it shows the ex-ante leniency program helps destabilize cartels. Below we make some assumptions to characterize the effect of the ex-ante leniency program on the stability of the cartel formed by N symmetric firms with a collusive strategy of quantity-fixing or price-fixing.

Assumption 1.8. Consider antitrust parameters $x = \beta f - \gamma(1-\alpha)$, $y = [(1-\beta f) - \gamma(1+(1-\beta f) - \beta(1-\alpha) - \alpha)]$, and $z = (\beta - \gamma)$, such that $x > 0$, $y > 0$, and $z > 0$.

The following proposition shows the importance of implementing an ex-ante leniency program to combat cartels. Essentially, it shows that the ex-ante leniency program helps destabilize cartels.

Proposition 1.9. *Consider a cartel made up of N symmetrical firms. The ex-ante leniency program negatively affects cartel stability, i.e., the critical discount factor is less than the critical discount factor under ex-ante leniency program.*

Proof. To show this conjecture it is sufficient to verify that $\delta < \bar{\delta}^{EA}$, where δ is the critical discount factor , and $\bar{\delta}^{EA}$ is the critical discount factor under ex-ante leniency program. Indeed: Since $y\pi^D\pi^C + \pi^C\pi^N \leq \pi^D\pi^C + z\pi^D\pi^N$, for all $y, z \in (0,1)$. This implies that $\frac{\pi^D}{\pi^D - \pi^N} < \frac{\pi^C}{y\pi^C - z\pi^N}$. On the other hand, since $z\pi^C\pi^C + x\pi^D\pi^C < y(\pi^C)^2 + x\pi^C\pi^N$, for all $x, y, z \in (0,1)$, implies that $-\frac{\pi^C}{\pi^D - \pi^N} < -\frac{x\pi^C}{y\pi^C - z\pi^N}$. Then, from the previous inequalities we obtain that:

$$\frac{\pi^D - \pi^C}{\pi^D - \pi^N} \leq \frac{(1-x)\pi^C}{y\pi^C - z\pi^N}.$$

This implies that $\delta < \bar{\delta}^{EA}$. Therefore, the ex-ante leniency program negatively affects cartel stability. □

To illustrate Proposition 1.9 we turn to Example 1.1. Here you can see that the ex-ante leniency program negatively affects cartel stability.

Example 1.10. Consider Example 1.1 and ex-ante leniency program parameters $\bar{f} = \min\{f : 0.01 \leq f \leq 0.20\}$, $\bar{\beta} = 0.30$, $\bar{\alpha} = 0.20$, and $\bar{\gamma} = 0.50$. We obtain the critical discount factor $\delta = 0.50992$ and critical discount factor under ex-ante leniency program $0.68094 = \delta^{EA}$; and we have the following relationship: $\delta = 0.50992 < 0.68094 = \delta^{EA}$. Therefore, the ex-ante leniency program negatively affects cartel stability.

The question of granting total immunity from the fine to the cartel whistle-blower member to generate greater instability in the cartel is something natural. The following proposition makes us see that this effect is valid regardless of the collusive strategy adopted by the cartel.

Proposition 1.11. *Consider a cartel made up of N symmetrical firms. If the antitrust authority grants full immunity from the fine to whistle-blower cartel members (cartel member applying to ex-ante leniency program), then the cartel is strongly destabilized.*

Proof. Consider the following antitrust parameters with total immunity $x^* = \beta f - \gamma(1-\alpha)$, $y^* = [(1-\beta f)-\gamma(1+(1-\beta f)-\beta(1-0)-0)]$, and $z^* = (\beta-\gamma)$, this implies that $x^* < x$, $y < y^*$, and $z = z^*$. To show this proposition, it is enough to verify that $\delta^{EA} < \delta_*^{EA}$, where δ_*^{EA} (δ^{EA}) is the discount factor that sustains the cartel under ex-ante leniency with total (partial) immunity. Since $(1-x)\pi^C < (1-x^*)\pi^C$, and $y^*\pi^C - z^*\pi^N < y\pi^C - z\pi^N$ we have the following implication:

$$\frac{(1-x)\pi^C}{y\pi^C - z\pi^N} < \frac{(1-x^*)\pi^C}{y^*\pi^C - z^*\pi^N}.$$

Therefore, that implies that $\delta^{EA} < \delta_*^{EA}$. This means that total immunity from the fine has a greater impact on the stability of the cartel. □

The analysis of the total immunity for the cartel whistle-blower member is considered by Aubert et al. (2006), Harrington Jr (2008), Chen and Rey (2013), Houba et al. (2015) , and Pinha and Braga (2021), to have a greater efficiency in instability to the cartels (cartels with collusive price-fixing strategy). Our approach shows that total immunity for the cartel whistle-blower member applied by the antitrust authority is effectively more efficient to destabilize cartels. The difference in our approach is that total immunity guarantees greater instability in the cartels — the effect is invariant to the collusive strategy adopted by the cartel.

1.4.2 Ex-post Leniency Program

The ex-ante leniency program is an instrument that helps the antitrust authority to conclude cartel processes investigated through a cartel reporting member - a cartel reporting member has to report on the cartel after the antitrust authority initiates an investigation on the cartel. The antitrust authority encourages cartel members under investigation to apply to the ex-post leniency program to provide relevant information about the cartel and partner behavior to finalize the process, and in return guarantee them partial or total immunity from fines.

The elements that characterize cartel stability are described below. Denote as V_i^C the expected present value of firm $i \in N$ when it maintains the collusive agreement — in each period firm $i \in N$ cooperates with the responsibility that the cartel can be detected with a certain probability and applied a certain fine rate given that the antitrust authority initiates the investigation on the cartel with a certain probability —, and define as the following recursive dynamics:

$$V_i^C = \rho\left\{\pi_i^C + \beta\left[-f\pi_i^C + \delta\frac{\pi_i^N}{1-\delta}\right] + (1-\beta)\delta V_i^C\right\} + (1-\rho)\delta V_i^C, \tag{1.19}$$

where $\rho \in (0,1)$ is the probability with which the antitrust authority begins the investigation on the cartel, $\beta \in (0,1)$ is the probability of cartel detection, $f \in (0,1)$ is the rate of fine applied to cartel member $i \in N$, and $\delta_i \in (0,1)$ is the critical discount factor with which Firm $i \in N$ maintains cartel stability under ex-post leniency program. On the other hand, denote as V^D the expected present value of firm $i \in N$ when it deviates from the cartel — in each period firm $i \in N$ can deviate from the agreement with probability a certain probability by prioritizing an ex-post leniency rate —, and define as the following recursive dynamic:

$$V_i^D = \gamma \left[(1-\alpha)\pi_i^C + \delta \frac{\pi_i^N}{1-\delta} \right] + (1-\gamma)\delta V_i^D, \tag{1.20}$$

where $\gamma \in (0,1)$ is the probability that firm $i \in N$ applies to the ex-post leniency program, $\alpha \in (0,1)$ is the immunity rate for those who apply to the ex-post leniency program. Since $1-\delta > 0$, $1-\delta(1-\beta\rho) > 0$, and $1-\delta(1-\gamma) > 0$; the sub-game perfect Nash equilibrium of the modified grim-trigger strategy characterizes cartel stability. A cartel made of N firms is said to be stable under ex-post leniency program if for each firm $i \in N$ there exists $\bar{\delta}^{EP} \in (0,1)$ such that $\bar{\delta}^{EP} = \max\{\bar{\delta}_1^{EP}, \cdots, \bar{\delta}_i^{EP}, \cdots, \bar{\delta}_n^{EP}\}$, and satisfies the incentive compatibility constraint:

$$V_i^C \geq V_i^D \Longleftrightarrow \bar{\delta}_i^{EP} \geq \frac{[\rho(1-\beta f) - \gamma(1-\alpha)]\pi_i^C}{[\rho(1-\beta f) - \gamma(1+\rho(1-\beta f) - \rho\beta(1-\alpha) - \alpha)]\pi_i^C - (\rho\beta - \gamma)\pi_i^N}. \tag{1.21}$$

The minimum discount factor — discount factor that satisfies the equality of the expression 1.21 — with which firm $i \in N$ maintains cartel stability is called the critical discount factor under ex-post leniency program. The set defined as $\{\delta \in (0,1) : \bar{\delta}_i^{EP} \leq \delta < 1\}$ is called the stability interval of the firm $i \in N$ under ex-post leniency program. The set defined as $\{\delta \in (0,1) : \max\{\bar{\delta}_1^{EP}, \cdots, \bar{\delta}_i^{EP}, \cdots, \bar{\delta}_n^{EP}\} \leq \delta < 1\}$ is called the cartel stability interval under ex-post leniency program. If the cartel stability interval tends to decrease, then the cartel tends to destabilize.

Definition 1.12. Consider a cartel made up of N symmetrical firms. The ex-post leniency program negatively affects cartel stability if the critical discount factor under ex-post leniency program is greater than the critical discount factor, i.e., $\delta < \bar{\delta}^{EP}$.

To facilitate the proofs of the subsequent propositions we define the parameters as follows.

Assumption 1.13. Consider antitrust parameters $\bar{x} = \rho\beta f - \gamma(1-\alpha)$, $\bar{y} = [\rho(1-\beta f) - \gamma(1+\rho(1-\beta f) - \rho\beta(1-\alpha) - \alpha)]$, and $\bar{z} = (\rho\beta - \gamma)$, such that $\bar{x} > 0$, $\bar{y} > 0$, and $\bar{z} > 0$.

The following result is a consequence of Propositions 1.9 — the sense of cartel instability should also prevail when applying for an ex-post leniency program. The following result shows that the ex-post leniency program destabilizes cartels.

Corollary 1.14. *Consider a cartel made up of N symmetrical firms. The ex-post leniency program negatively affects cartel stability, i.e., the critical discount factor under ex-post leniency program is greater than the critical discount factor.*

Proof. To show this conjecture it is sufficient to verify that $\delta < \bar{\delta}^{EP}$, where δ is the critical discount factor , and $\bar{\delta}^{EA}$ is the critical discount factor under ex-ante leniency program. Indeed: Since $y\pi^D\pi^C + \pi^C\pi^N \leq \pi^D\pi^C + z\pi^D\pi^N$, for all $y, z \in (0,1)$. This implies that $\dfrac{\pi^D}{\pi^D - \pi^N} < \dfrac{\pi^C}{y\pi^C - z\pi^N}$. On the other hand, since $z\pi^C\pi^C + x\pi^D\pi^C < y(\pi^C)^2 + x\pi^C\pi^N$, for all $x, y, z \in (0,1)$, implies that $-\frac{\pi^C}{\pi^D - \pi^N} < -\frac{x\pi^C}{y\pi^C - z\pi^N}$. Then, from the previous inequalities we obtain that:

$$\frac{\pi^D - \pi^C}{\pi^D - \pi^N} \leq \frac{(\rho - \bar{x})\pi^C}{\bar{y}\pi^C - \bar{z}\pi^N}.$$

This implies that $\delta < \bar{\delta}^{EP}$. Therefore, the ex-post leniency program negatively affects cartel stability. □

Similar to Proposition 1.11, the sense of cartel destabilization remains invariant when the ex-post leniency program is applied with a total immune penalty rate. The following proposition shows such a result.

Corollary 1.15. *Consider a cartel made up of N symmetrical firms. If the antitrust authority grants full immunity from the fine to whistle-blower cartel members (cartel member applying to ex-post leniency program), then the cartel is strongly destabilized.*

Proof. Consider the following antitrust parameters with total immunity $\bar{x}^* = \rho\beta f - \gamma(1-\alpha)$, $\bar{y}^* = [(1-\beta f) - \gamma(1+(1-\beta f) - \beta(1-0)-0)]$, and $\bar{z}^* = (\beta - \gamma)$, this implies that $\bar{x}^* < \bar{x}$, $\bar{y} < \bar{y}^*$, and $\bar{z} = \bar{z}^*$. To show this proposition, it is enough to verify that $\delta^{EP} < \delta_*^{EP}$, where δ_*^{EP} (δ^{EP}) is the discount factor that sustains the cartel under ex-ante leniency with total (partial) immunity. Since $(\rho - x)\pi^C < (\rho - x^*)\pi^C$, and $y^*\pi^C - z^*\pi^N < y\pi^C - z\pi^N$ we have the following implication:

$$\frac{(\rho - \bar{x})\pi^C}{\bar{y}\pi^C - \bar{z}\pi^N} < \frac{(\rho - \bar{y}^*)\pi^C}{\bar{x}^*\pi^C - \bar{z}^*\pi^N}.$$

Then, we obtain that $\delta^{EP} < \delta_*^{EP}$. Therefore, total immunity from the fine intensifies cartels instability. □

Therefore, from this section, we conclude that the results that hold for an ex-ante leniency program scenario remain invariant in an ex-post leniency program scenario.

1.5 Conclusion

The main question regarding antitrust law is whether the presence of the antitrust authority with different antitrust policy regimes is effective in combating cartels and deterring the formation of new cartels. This article explains in a simple and generalized way the effect of the presence of the antitrust authority with different antitrust policy regimes — antitrust enforcement (antitrust enforcement without leniency program), ex-ante leniency program, and ex-post leniency program — on cartels stability. On one hand, we show that antitrust policies destabilize cartels. However, we show that there are no optimal antitrust enforcement parameters to destabilize cartels. On the other hand, we show that granting full immunity from fines to the whistle-blower cartel member helps to destabilize cartels easily.

Our approach enjoys versatility in terms of cartel stability in different scenarios. For future work, factors intrinsic to the market structure that can dissuade the effects of antitrust policies can be increased in these models. For example, the degree of product differentiation, the difference in production costs, the elasticity of demand, and others. Therefore, this is how we can study: the effects of antitrust enforcement on cartel stability at different levels of elasticity of demand.

Chapter 2

Degree of Product Differentiation, Antitrust Enforcement and Cartel Stability

Abstract: This paper studies the trade-off between the product differentiation degree and cartel stability— cartels made up of two symmetrical firms with collusive strategies of quantity-fixing and price-fixing —, and the effect of antitrust enforcement on cartel stability at different levels of horizontal product differentiation. We corroborate that if the products tend to be homogeneous, then the cartels are less stable. In addition, we show that the quantity-fixing cartel is more stable than the price-fixing cartel — Bertrand competition is more efficient than Cournot competition. Finally, we show that antitrust enforcement affects negatively and with greater intensity on cartels stability if the products are highly differentiated. This result guides the activity of the antitrust authority, indicating that potential cartels with products that tend to be homogeneous should be investigated with greater intensity.

Keywords: Product Differentiation · Quantity-Fixing · Price-Fixing · Antitrust Enforcement · Cartel Stability.

JEL Classification: L13 · L41 · D43 · C73

2.1 Introduction

The literature on cartels'[1] stability under antitrust enforcement is nowadays constantly studied to design optimal antitrust policies to combat cartels and prevent the formation of new cartels. Block et al. (1981); Motta and Polo (2003), each in its way, converge on one result: antitrust enforcement negatively affects price-fixing cartel's stability — antitrust enforcement decreases the collusive price Houba et al. (2015). The primary

[1]Cartels occur when a group of firms in a certain industry agrees on their behavior to obtain supra-competitive profits. Anti-competitive activities such as cartels are considered a type of organized crime that damages the economic order because it mainly harms the well-being of consumers with high prices, restricted production capacity, and others (Spagnolo, 2008; Harrington Jr, 2017).

literature on cartel stability was born with the seminal work of Stigler (1964). Thus, as the literature begins to explore the topic of cartels stability affected by factors intrinsic to the market structure such as the product differentiation degree (Deneckere, 1983; Majerus, 1988; Ross, 1992; Rothschild, 1992; Albæk and Lambertini, 1998), difference in production costs (Rothschild, 1999; Vasconcelos, 2005; Collie, 2006; Miklos-Thal, 2011; Ciarreta and Gutiérrez-Hita, 2012), volatility of the demand system, among others. Here, mainly tried to understand the trade-off between the product differentiation degree and cartel stability. Cartels occur in different industries and at different levels of product differentiation but antitrust authority applies the same antitrust enforcement parameters to destabilize cartels. Not making this difference would cause disparity to destabilize cartels because cartels' stability depends on the collusive strategy and factors intrinsic to the market structure.

The literature on cartel stability under internal (product differentiation degree) and external (antitrust enforcement) factors simultaneously as far as we know unknown. Our goal is to examine the effect of antitrust enforcement on cartel stability at different levels of horizontal product differentiation. Essentially, we answer the question: is antitrust enforcement more effective against cartels when products are horizontally more homogeneous or differentiated? This study can help improve the activity of antitrust authorities and guide antitrust policymakers in the fight against cartels, focusing the investigation on potential cartels according to the product differentiation degree.

To develop the paper we consider two cartels — cartels made up of two symmetrical firms with collusive strategies of quantity-fixing and price-fixing; firms cartelize a linear demand system with horizontally differentiated products — operating in a certain economy with antitrust authority and competition law. The methodology to describe the cartel's stability is that of repeated game theory, more specifically the notion of subgame perfect Nash equilibrium of grim-trigger strategy (Friedman, 1971) and modified grim-trigger strategy (Motta and Polo, 2003; Houba et al., 2015).

The trade-off between the horizontal product differentiation degree and cartel stability has the following relationship: as the products tend to be homogeneous, the cartel is less stable (Majerus, 1988). This trade-off is fulfilled both for cartels with a collusive strategy of quantity-fixing and price-fixing. In our paper, we corroborate these results and we show that for any horizontal product differentiation degree, cartels with a collusive price-fixing strategy are less stable than cartels with a collusive quantity-fixing strategy – i.e., price-fixing cartels are more sensitive to destabilization than quantity-fixing cartels. Therefore, price competition (Bertrand competition) is more efficient than quantity competition (Cournot competition) — i.e., price competition generates more instability in cartels than quantity competition.

On the other hand, under a scenario with the presence of antitrust authority, there is extensive literature. The results presented are on the cartel's stability under different antitrust policy regimes. One of the important results shows that antitrust enforcement (represented by the probability of cartel detection and fine rate applied to cartel members) negatively affects cartel's stability (Block et al., 1981; Motta and Polo, 2003) — i.e., antitrust enforcement impacts the collusive price, causing it to decrease (Houba et al., 2015). Extending to more results, we show that antitrust enforcement negatively affects and with greater intensity on cartel's stability when products tend to be more differentiated. This result is independent of the collusive strategy adopted by the cartel

— it is essentially valid for cartels with a collusive strategy of quantity-fixing and price-fixing.

The rest of the article is organized as follows. Section 2.2 describes cartels stability under antitrust enforcement. Section 2.3 describes a demand system that characterizes a market with horizontally differentiated products. Section 2.4 describes cartel stability with a collusive quantity-fixing strategy at different levels of product differentiation. Section 2.5 describes cartel stability with a collusive price-fixing strategy at different levels of product differentiation. Section 2.6 compares the efficiency of collusive strategies at different levels of product differentiation. Finally, Section 2.7 concludes the paper.

2.2 Cartel Stability Dynamics

Here we conceptualize the notion of cartel stability in two different scenarios using sub-game perfect Nash equilibrium of the grim-trigger strategy (Friedman, 1971), and sub-game perfect Nash equilibrium of the modified grim-trigger strategy (Fudenberg and Tirole, 1991). First, when cartel stability is disturbed by a factor intrinsic to the market structure, such as the horizontal product differentiation degree – the trade-off between the horizontal product differentiation degree and cartel stability. Second, cartel stability under antitrust enforcement — represented by the probability of cartel detection and fine rate applied to cartel members — at different levels of horizontal product differentiation.

2.2.1 Grim-Trigger Strategy

Taking into account the profit relationship according to the behavior of the firms in the market: (i) deviation profit is greater than cooperation profit; (ii) profit cooperation is greater than competition profit; (iii) competition profit is greater than damage profit. The relationship of these profits leads to the cartel dilemma of firms with cooperate and non-cooperate actions[2]. Next, we conceptualize the cartel stability made up of $N = \{1, \cdots, i, \cdots, n\}$ firms. A cartel made up of N firms is said to be stable if for all firm $i \in N$ there exists $\delta \in (0, 1)$, such that $\delta = \max\{\delta_1, \cdots, \delta_i, \cdots, \delta_n\}$, and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \Longleftrightarrow \delta \geq \delta_i = \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^N}, \tag{2.1}$$

where V_i^C is the liquid present value of firm $i \in N$, when it maintains the collusive agreement — in each period firm $i \in N$ takes the cooperate action —, V_i^D is the liquid present value of firm $i \in N$, when it deviates from the agreement — in first period the firm $i \in N$ deviates from the agreement and then decides to compete forever, and

[2]The bimatricial representation of the cartel dilemma of two firms is represented as:

$$\begin{pmatrix} \pi^C, \pi^C & \pi_D, \pi^D \\ \pi^D, \pi_D & \pi^N, \pi^N \end{pmatrix}$$

where the first row (column) are the payoffs of the cooperate action of firm 1 (firm 2) and the second row (column) are the payoffs of the non-cooperate action of firm 1 (firm 2).

$\delta_i \in (0,1)$ is the discount factor with which firm $i \in N$ maintains cartel stability. The minimum discount factor of firm $i \in N$ — discount factor satisfies the equality of the expression 2.1 — is called the critical discount factor of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \delta_i \leq \delta < 1\}$ is called the stability interval of firm $i \in N$ — the cartel member firm that has a smaller stability interval is more likely to destabilize the cartel, i.e., the firms operate at a higher discount factor. Note that the right-hand side of the inequality 1.11 can only be perturbed by factors intrinsic to the market structure. The set defined as $\{\delta \in (0,1) : \max\{\delta_1, \cdots, \delta_i, \cdots, \delta_n\} \leq \delta < 1\}$ is called cartel stability interval. The cartel tends to be less stable if the cartel stability interval tends to decrease.

The dynamics of the cartel stability are simplified when the cartel is made up of symmetrical firms, why, it is enough to analyze the behavior of a single firm. A symmetrical cartel or cartel made up of N symmetrical firms is said to be stable if there exists $\delta \in (0,1)$ such that it satisfies the incentive compatibility constraint:

$$V^C \geq V^D \Longleftrightarrow \delta \geq \frac{\pi^D - \pi^C}{\pi^D - \pi^N}. \tag{2.2}$$

Note that the right-hand side of the inequality 2.2 can only be affected by factors intrinsic to the market structure. A crucial point for there to be instability in the cartel is that the competitive profit approaches the cooperative profit, which implies that the discount factor tends to 1. Thus, greater instability is generated in the cartel.

2.2.2 Modified Grim-Trigger Strategy

The analysis of cartel stability under antitrust enforcement at different levels of product differentiation is modeled using the sub-game perfect Nash equilibrium of the modified grim-trigger strategy. We describe the elements of the modified trigger strategy below.

Denote as V_i^C the expected present value of firm $i \in N$, when it maintains the collusive agreement — in each period firm $i \in N$ maintains the cooperative action knowing that the cartel can be discovered with a certain probability and applied a certain rate of fine — , and define as the following recursive dynamics:

$$V_i^C = \pi_i^C + \beta \left[-f\pi_i^C + \bar{\delta}_i \frac{\pi_i^N}{1 - \bar{\delta}_i} \right] + (1-\beta)\bar{\delta}_i V_i^C, \tag{2.3}$$

where $\beta \in (0,1)$ is the probability of cartel detection, $f \in (0,1)$ is the fine rate applied to cartel members, $\bar{\delta}_i \in (0,1)$ is the discount factor with which firm $i \in N$ maintains the cartel stability. On the other hand, denote as V_i^D the liquid present value of firm $i \in N$, when it deviates from the collusive agreement — firm $i \in N$ deviates in the first period and then competes forever — and define as:

$$V^D = \pi^D + \frac{\bar{\delta}_i \pi^N}{1 - \bar{\delta}_i} = \frac{(1-\bar{\delta}_i)\pi^D + \bar{\delta}_i \pi^N}{1 - \bar{\delta}_i}. \tag{2.4}$$

Since $1 - \bar{\delta}_i > 0$, and $1 - \beta(1 - \bar{\delta}_i) > 0$, the sub-game perfect Nash equilibrium (SPNE) of the modified grim-trigger strategy characterizes cartel stability under antitrust enforcement.

A cartel made up of N firms is said to be stable under antitrust enforcement if for all firm $i \in N$ there exist $\bar{\delta} \in (0,1)$ such that $\bar{\delta} = \max\{\bar{\delta}_1, \cdots, \bar{\delta}_i, \cdots, \bar{\delta}_n\}$, and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \Longleftrightarrow \bar{\delta} \geq \bar{\delta}_i = \frac{\pi_i^D - (1-\beta f)\pi_i^C}{(1-\beta)[\pi_i^D - \pi_i^N]}. \tag{2.5}$$

The minimum discount factor $\bar{\delta}_i$ (discount factor that satisfies equation 2.5) with which firm $i \in N$ maintains cartel stability is called the critical discount factor under antitrust enforcement of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \bar{\delta}_i \leq \delta < 1\}$ is called the stability interval under antitrust enforcement of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \max\{\bar{\delta}_1, \cdots, \bar{\delta}_i, \cdots, \bar{\delta}_n\} \leq \delta < 1\}$ is called the cartel stability interval under antitrust enforcement. If the cartel stability interval tends to decrease, then we say that cartel stability is negatively affected.

The symmetry of firms simplifies the analysis of cartel stability because they all have the same behavior. Therefore, it is enough to analyze a single firm to know the dynamics of its stability. Next, we define the effect of antitrust enforcement on cartel stability when the cartel is formed by symmetrical firms.

Definition 2.1. Consider a cartel made up of N symmetric firms. Antitrust enforcement negatively affects the cartel stability (or destabilizes the cartel) if the critical discount factor is less than the critical discount factor under antitrust enforcement, i.e., $\delta < \bar{\delta}$.

Note that the right-hand side of the incentive compatibility constraint 2.5 can be disturbed by factors intrinsic to the market structure and also by external factors such as the probability of cartel detection and the fine rate of cartel members (antitrust enforcement).

2.3 The Model

Consider an industry made up of two symmetric firms, 1 and 2, that offer horizontally differentiated products and serve an inverse linear demand system (Dixit, 1979).

$$p_1 = a - b(q_1 + \theta q_2), \qquad p_2 = a - b(q_2 + \theta q_1), \tag{2.6}$$

where $\theta \in (0,1)$ indicates the horizontal product differentiation degree between products, 1 and 2. Products q_1 and q_2 tend to be homogeneous if $\theta \in (0,1)$ is close to 1. Products q_1 and q_2 tend to be highly differentiated if $\theta \in (0,1)$ is close to 0. Then, if $\theta = 0$ then both firms are monopolists. The reverse system of linear demand can also be written as a linear demand system as follows:

$$q_1 = \frac{1}{b(1-\theta^2)}(a(1-\theta) - p_1 + \theta p_2), \qquad q_2 = \frac{1}{b(1-\theta^2)}(a(1-\theta) - p_2 + \theta p_1), \tag{2.7}$$

where the parameters a and b are positive and have the following relationship: $a > b > 0$. The different ways of presenting a demand system have a practical character because it adapts to the type of competition that you want to analyze. Demand system 2.6 is for Cournot competition, while inverse demand system 2.7 is for Bertrand competition.

2.4 Quantity-Fixing

Here we capture the different profits of the firms according to their behavior in the market. The firms decide to choose the quantity to be produced, more precisely, the firms attend the linear demand system 2.6 with the Cournot strategy. We also analyze the trade-off between the horizontal product differentiation degree and cartel stability. Subsequently, the impact of antitrust enforcement on cartel stability at different levels of product differentiation.

The strategic interaction of firms, 1 and 2, by quantity to produce for the market consists of: (i) firms 1 and 2 choose quantity to produce $q_1 \in [0, \infty)$, and $q_2 \in [0, \infty)$ respectively; (ii) firms 1 and 2, operating at constant marginal cost $c > 0$, such that $a > b > c > 0$, define their profit functions as:

$$\pi_1(q_1, q_2) = (p_1 - c)q_1, \qquad \pi_2(q_1, q_2) = (p_2 - c)q_2, \tag{2.8}$$

where p_1 and p_2 make up the inverse linear demand system 2.6. Next, we capture the competition profits, cooperation, diversion, and damage (these profits constitute the cartel's dilemma)

Lemma 2.2. *Consider that firms 1 and 2 serve demand system 2.6. The Nash-Cournot equilibrium profile is:*

$$q_1^N = \frac{a-c}{b(2+\theta)}, \qquad q_2^N = \frac{a-c}{b(2+\theta)}. \tag{2.9}$$

Then the competitive profits for firms 1 and 2 respectively are:

$$\pi_1^N = \frac{(a-c)^2}{b(2+\theta)^2}, \qquad \pi_2^N = \frac{(a-c)^2}{b(2+\theta)^2}. \tag{2.10}$$

Proof. The solution for a competitive environment is the individual profit maximization of the firms. Then the situation is solved as a simultaneous game and the Nash equilibrium can be written as the best responses system:

$$q_1 = \frac{a-c-b\theta q_2}{2b}, \qquad q_2 = \frac{a-c-b\theta q_1}{2b}. \tag{2.11}$$

Solving the best responses system 2.11, we obtain the Nash-Cournot equilibrium profile (q_1^N, q_2^N). Then, evaluating (q_1^N, q_2^N) in the profit functions, we obtain the competitive profits, π_1^N and π_2^N, for firms 1 and 2, respectively. □

The following result is due to cooperative behavior, where the maximum joint benefit of the firms in the industry is found.

Lemma 2.3. *Consider that firms 1 and 2 serve demand system 2.6. The optimal cartel quantity vector is:*

$$q_1^C = \frac{a-c}{2b(1+\theta)}, \qquad q_2^C = \frac{a-c}{2b(1+\theta)}, \tag{2.12}$$

Then the cooperative profit for firms 1 and 2 respectively are:

$$\pi_1^C = \frac{(a-c)^2}{4b(1+\theta)}, \qquad \pi_2^C = \frac{(a-c)^2}{4b(1+\theta)}. \tag{2.13}$$

Proof. To capture the prices that induce supra-competitive profits — maximization of joint profits. Then, maximizing the profit function $\pi = \pi_1(q_1, q_2) + \pi_2(q_1, q_2)$ with respect to q_1 and q_2. By first-order conditions we have:

$$a - c - 2bq_1 - 2b\theta q_2 = 0, \qquad a - c - 2bq_2 - 2b\theta q_1 = 0. \tag{2.14}$$

Solving equation 2.14, we obtain the optimal cartel quantity (q_1^C, q_2^C). Then, evaluating (q_1^C, q_2^C) in the profit functions, we obtain cooperative profit, π_1^C and π_2^C, for firms 1 and 2, respectively. □

The following two results explain that the deviation of some firms, given that the others maintain the collusive strategy, obtain greater profits than the cooperative profit and leaves those who maintain the collusive strategy with a profit less than competitive profit.

Lemma 2.4. *Consider that firms 1 and 2 serve demand system 2.6. Since firm 2 cooperates, firm 1 optimal deviation quantity is:*

$$q_1^D = \frac{(2+\theta)(a-c)}{4b(1+\theta)}, \tag{2.15}$$

Then profit diversion for firm 1 and damage profit for firm 2 respectively are:

$$\pi_1^D = \frac{(2+\theta)^2(a-c)^2}{16b(1+\theta)^2}, \qquad \pi_{2D} = \frac{(2+2\theta-\theta^2)(a-c)^2}{8b(1-\theta)^2}. \tag{2.16}$$

Proof. To find the deviation quantity for firm 1, set the cartel quantity of firm 2. By first-order conditions of the function $\pi_1(q_1, q_2^C)$ with respect to q_1, we obtain:

$$\frac{(2-\theta)(a-c) - 4bq_1(1+\theta)}{2(1+\theta)} = 0 \tag{2.17}$$

Solving equation 2.17, we obtain the optimal deviation quantity q_1^D for firm 1. Then, evaluating the quantity vector (q_1^D, q_2^C) in the profit functions, we obtain deviation profit π_1^D for firm 1 and damage profit π_{2D} for firm 2. □

Lemma 2.5. *Consider that firms 1 and 2 serve demand system 2.6. Since firm 1 cooperates, firm 2 optimal deviation quantity is:*

$$q_2^D = \frac{(2+\theta)(a-c)}{4b(1+\theta)}, \tag{2.18}$$

Then profit diversion for firm 2 and damage profit for firm 1 respectively are:

$$\pi_2^D = \frac{(2+\theta)^2(a-c)^2}{16b(1+\theta)^2}, \qquad \pi_{1D} = \frac{(2+2\theta-\theta^2)(a-c)^2}{8b(1-\theta)^2}. \tag{2.19}$$

Proof. To find the deviation quantity for firm 2, set the cartel price of firm 1. By first-order conditions of the function $\pi_2(q_1^C, q_2)$ with respect to q_2, we obtain:

$$\frac{(2-\theta)(a-c) - 4bq_2(1+\theta)}{2(1+\theta)} = 0. \tag{2.20}$$

Solving equation 2.20, we obtain the optimal deviation quantity q_2^D for firm 2. Then, evaluating the quantity vector (q_1^C, q_2^D) in the profit functions, we obtain deviation profit π_2^D for firm 2 and damage profit π_{1D} for firm 1. □

The profits captured in Lemmas 2.2, 2.3, 2.4, and 2.5 constitute the cartel's dilemma — a cartel with a collusive quantity-fixing strategy. The cartel dilemma generates an infinitely repeated game, in particular, the grim-trigger-trigger strategy.

The following assumption characterizes cartelization with collusive quantity-fixing strategy at different levels of horizontal product differentiation $\theta \in (0,1)$ in an environment free of antitrust enforcement.

Assumption 2.6. For all horizontal product differentiation degree $\theta \in (0,1)$ of the inverse linear demand system 2.6, firms 1 and 2 cartelize the market with a collusive quantity-fixing strategy.

Substituting the profits of the Lemmas 2.2, 2.3, 2.4, and 2.5 in the incentive compatibility constraint 2.2 we obtain the dynamics of the cartel stability. The cartel made up of firms 1 and 2 is said to be stable if it satisfies the incentive compatibility constraint:

$$\delta_q(\theta) \geq \frac{(2+\theta)^2}{\theta^2 + 8\theta + 8}. \tag{2.21}$$

The following result characterizes the cartel stability at different levels of product differentiation.

Proposition 2.7. *Consider that for all horizontal product differentiation degree $\theta \in (0,1)$ of the demand system 2.6, firms 1 and 2 cartilize the market with a collusive quantity-fixing strategy. If the products tend to be homogeneous then the cartel is less stable, i.e., the critical discount factor is increasing over the product differentiation interval.*

Proof. To prove it is sufficient verify that $\delta_q'(\theta) > 0$, for all $\theta \in (0,1)$. Indeed:

$$\delta_q'(\theta) = \frac{4\theta(2+\theta)}{(\theta^2 + 8\theta + 8)^2} > 0, \tag{2.22}$$

for all $\theta \in (0,1)$. □

This result shows the important role that product differentiation plays on cartel stability: as products tend to be homogeneous, firms are more patient, this means that firms value future benefits more than present ones. Therefore, cartel is less stable when the products tend to be more homogeneous.

The following assumption characterizes cartelization with collusive quantity-fixing strategy at different levels of horizontal product differentiation $\theta \in (0,1)$ in an environment under antitrust enforcement.

Assumption 2.8. For all horizontal product differentiation degree $\theta \in (0,1)$ of the linear demand system 2.6, firms 1 and 2 cartelize the market with a collusive quantity-fixing strategy, knowing that there is an antitrust authority that applies a fine rate of $f \in (0, \bar{f}] \subseteq (0,1)$ to cartel members.

Cartel stability — cartel made up of two symmetrical firms, 1 and 2, with a collusive quantity-fixing strategy — under antitrust enforcement with fine parameter $f \in (0, \bar{f}] \subseteq (0, 1)$ is determined by the following incentive compatibility constraint:

$$\bar{\delta}_q(\theta, \beta, f) \geq \frac{(2+\theta)^2(\theta^2 + 4\beta f\theta + 4\beta f)}{(1-\beta)\theta^2(\theta^2 + 8\theta + 8)}. \tag{2.23}$$

The following result shows the antitrust enforcement effect on cartels stability at different levels of horizontal product differentiation.

Proposition 2.9. *Consider that for all horizontal product differentiation degree $\theta \in (0, 1)$ of demand system 2.6, firms 1 and 2 cartilize the market with a collusive quantity-fixing strategy. Antitrust enforcement impacts negatively and with greater intensity on cartel stability if the products tend to be highly differentiated.*

Proof. Since the function $\bar{\delta}_q : (0, 1) \times (0, \bar{\beta}] \times (0, \bar{f}] \subseteq (0, 1)^3 \longrightarrow (0, 1)$ is continuous, it is enough to prove two items. First, let us show that $\lim_{\beta \to 0} \bar{\delta}_q(\theta, \beta, f) = \delta_q(\theta)$, for all $\theta \in (0, 1)$ and $f \in (0, \bar{f}]$. Indeed:

$$\lim_{\beta \to 0} \bar{\delta}_q(\theta, \beta, f) = \lim_{\beta \to 0} \left[\frac{(2+\theta)^2(\theta^2 + 4\beta f\theta + 4\beta f)}{(1-\beta)\theta^2(\theta^2 + 8\theta + 8)} \right] = \frac{(2-\theta)^2}{\theta^2 - 8\theta + 8} = \delta_p(\theta).$$

Second, let us verify that $\lim_{\theta \to 0} \bar{\delta}_q(\theta, \beta, f) = +\infty$, for all $(\beta, f) \in (0, 1)^2$. Indeed:

$$\lim_{\theta \to 0} \bar{\delta}_q(\theta, \beta, f) = \lim_{\theta \to 0} \left[\frac{(2+\theta)^2(\theta^2 + 4\beta f\theta + 4\beta f)}{(1-\beta)\theta^2(\theta^2 + 8\theta + 8)} \right] = \frac{\beta f}{1-\beta}(+\infty) = +\infty.$$

Therefore, this implies that the effect of antitrust enforcement is greater when the products tend to be highly differentiated. □

Figure 2.1 in particular ($\bar{f} = 0.2$), illustrates the proof of the Propositions 2.7 and 2.9. On the one hand, on the left side of Figure 2.1, it can be observed that the critical discount factor that maintains the cartel stability increases over the product differentiation interval (0.1) — i.e., when the products tend to be homogeneous, the cartel is less stable. On the other hand, on the right side of Figure 2.1, it can be seen that antitrust enforcement affects the cartel stability with greater intensity when the products are more differentiated.

Finally, notice in Figure 2.1 —left side— that in the area defined as:

$$1 - \int_0^1 \frac{(2+\theta)^2}{\theta^2 + 8\theta + 8} d\theta, \tag{2.24}$$

cartel stability intervals are found, for each product differentiation degree. Likewise, it is observed that when the products tend to be homogeneous, the cartel stability intervals decrease — i.e., the cartel is less stable when the products tend to be more homogeneous.

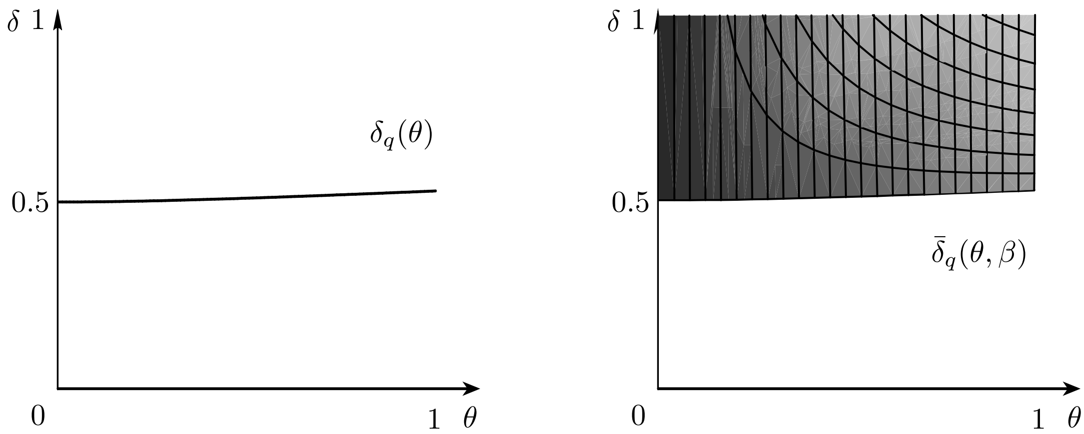

Figure 2.1: Effect of antitrust enforcement on cartel stability at different levels of product differentiation: quantity-fixing case.

2.5 Price-Fixing

Analogously to Section 2.4, we capture the different profits provided by the behavior of the firms in the market when they serve the linear demand system 2.7 with price strategy. We also analyze the trade-off between horizontal product differentiation degree and cartel stability, and the effect of antitrust enforcement on cartel stability at different levels of horizontal product differentiation.

The strategic interaction of firms 1 and 2 by price in the market consists of: (i) firms 1 and 2 choose price to be charged $p_1 \in [0, \infty)$, and $p_2 \in [0, \infty)$ respectively; (ii) firms 1 and 2, operating at constant marginal cost $c > 0$ define their profit function as:

$$\pi_1(p_1, p_2) = (p_1 - c)q_1, \qquad \pi_2(p_1, p_2) = (p_2 - c)q_2, \tag{2.25}$$

where q_1 and q_2 form the demand system 2.7. Next we capture the profits of competition, cooperation, deviations for firms 1 and 2.

Lemma 2.10. *Consider that firms 1 and 2 serve demand system 2.7. The Nash-Bertrand equilibrium profile is:*

$$p_1^N = \frac{a(1-\theta)+c}{2-\theta}, \qquad p_2^N = \frac{a(1-\theta)+c}{2-\theta}, \tag{2.26}$$

Then competitive (non-cooperative) profits for firms 1 and 2 respectively are:

$$\pi_1^N = \frac{(1-\theta)(a-c)^2}{b(1+\theta)(2-\theta)^2}, \qquad \pi_2^N = \frac{(1-\theta)(a-c)^2}{b(1+\theta)(2-\theta)^2}. \tag{2.27}$$

Proof. The solution for a competitive environment is the individual profit maximization of the firms. Then the situation is solved as a simultaneous game and the Nash equilibrium can be written as the best responses system:

$$p_1 = \frac{a(1-\theta)+c+\theta p_2}{2}, \qquad p_2 = \frac{a(1-\theta)+c+\theta p_1}{2}. \tag{2.28}$$

Solving the best response system 2.28, we obtain the equilibrium prices (p_1^N, p_2^N). Then, evaluating (p_1^N, p_2^N) in the profit functions, we obtain the competitive profits, π_1^N and π_2^N, for firms 1 and 2, respectively. □

The following result is due to a cooperative behavior, where the maximum collective profit of the firms and the joint profit of the industry are found.

Lemma 2.11. *Consider that firms 1 and 2 serve demand system 2.7. The optimal cartel price vector is:*

$$p_1^C = \frac{a+c}{2}, \qquad p_2^C = \frac{a+c}{2}. \tag{2.29}$$

Then cooperative profits for firms 1 and 2 respectively are:

$$\pi_1^C = \frac{(a-c)^2}{4b(1+\theta)}, \qquad \pi_2^C = \frac{(a-c)^2}{4b(1+\theta)}. \tag{2.30}$$

Proof. To capture the prices that induce supra-competitive profits — maximization of joint profits. Then, maximizing the profit function $\pi = \pi_1(p_1, p_2) + \pi_2(p_1, p_2)$ with respect to p_1 and p_2. By first-order conditions we have:

$$\frac{(1-\theta)(a+c) - 2p_1 + 2\theta p_2}{b(1-\theta^2)} = 0, \qquad \frac{(1-\theta)(a+c) - 2p_2 + 2\theta p_1}{b(1-\theta^2)} = 0. \tag{2.31}$$

Solving equation 2.31, we obtain the cartel optimal prices (p_1^C, p_2^C). Then, evaluating cartel optimal prices (p_1^C, p_2^D) in the profit functions, we obtain cooperative profits, π_1^C and π_2^C, for firms 1 and 2, respectively. □

The following two results explain that the deviation of firms, given that the others maintain the collusive strategy, obtain greater profits than the cooperative profit and leave those who maintain the collusive strategy with profit less than competitive profit.

Lemma 2.12. *Consider that firms 1 and 2 serve demand system 2.7. Since firm 2 cooperates, firm 1 optimal deviation price is:*

$$p_1^D = \frac{a+c}{2} - \frac{\theta(a-c)}{4}. \tag{2.32}$$

Then diversion profit of firm 1 and damage profit of firm 2 respectively are:

$$\pi_1^D = \frac{(2-\theta)^2(a-c)^2}{16b(1-\theta^2)}, \qquad \pi_{2D} = \frac{(2-2\theta-\theta^2)(a-c)^2}{8b(1-\theta^2)}. \tag{2.33}$$

Proof. To find the deviation price of firm 1, set the cartel price of firm 2. By first-order conditions of the function $\pi_1(p_1, p_2^C)$ with respect to p_1, we obtain:

$$\frac{2(a+c) - \theta(a-c) - 4p_1}{2b(1-\theta^2)} = 0 \tag{2.34}$$

Solving equation 2.34, we obtain the optimal deviation price p_2^D. Then, evaluating the price vector (p_1^D, p_2^C) in the profit functions, we obtain deviation profit π_1^D for firm 1 and damage profit π_{2D} for firm 2. □

Lemma 2.13. *Consider that firms 1 and 2 serve demand system 2.7. Since firm 1 cooperates, firm 2 optimal deviation price is:*

$$p_2^D = \frac{a+c}{2} - \frac{\theta(a-c)}{4}. \tag{2.35}$$

Then diversion profit of firm 2 and damage profit for firm 1 respectively are:

$$\pi_2^D = \frac{(2-\theta)^2(a-c)^2}{16b(1-\theta^2)}, \qquad \pi_{1D} = \frac{(2-2\theta-\theta^2)(a-c)^2}{8b(1-\theta^2)}. \tag{2.36}$$

Proof. To find the deviation price of firm 2, set the cartel price of firm 1. By first-order conditions of the function $\pi_2(p_1^C, p_2)$ with respect to p_2, we obtain:

$$\frac{2(a+c) - \theta(a-c) - 4p_2}{2b(1-\theta^2)} = 0. \tag{2.37}$$

Solving equation 2.37, we obtain the optimal deviation price p_2^D. Then, evaluating the price vector (p_1^C, p_2^D) in the profit functions, we obtain deviation profit π_2^D for firm 2 and damage profit π_{1D} for firm 1. □

Similar to Section 2.4, the profits found in Lemmas 2.2, 2.3, 2.4, and 2.5 constitute the cartel's dilemma — we highlight that the payoffs of this dilemma are different from the payoffs of the cartel's dilemma with collusive quantity-fixing strategy.

The following assumption characterizes cartelization with collusive price-fixing strategy at different levels of horizontal product differentiation $\theta \in (0,1)$ in an environment free of antitrust enforcement.

Assumption 2.14. For all horizontal product differentiation degree $\theta \in (0,1)$ of the linear demand system 2.7, firms 1 and 2 cartelize the market with a collusive price-fixing strategy.

Analogously to Section 2.4, substituting the profit of Lemmas 2.10, 2.11, 2.12, and 2.13 in the incentive compatibility constraint 2.2 we obtain the dynamics of the cartel stability. The cartel made up of firms 1 and 2 is said to be stable if it satisfies the incentive compatibility constraint:

$$\delta_p(\theta) \geq \frac{(2-\theta)^2}{\theta^2 - 8\theta + 8}. \tag{2.38}$$

The following result characterizes the trade-off between the product differentiation degree $\theta \in (0,1)$ and cartel stability. For its effect, it is enough to observe the behavior of the function defined on the right side of the inequality 2.38.

Proposition 2.15. *Consider that for all horizontal product differentiation degree $\theta \in (0,1)$ of the demand system 2.7, firms 1 and 2 cartilize the market with a collusive price-fixing strategy. If the products tend to be homogeneous then the cartel is less stable, i.e., the critical discount factor is increasing over the product differentiation interval.*

Proof. To prove it is sufficient verify that $\delta'(\theta > 0)$, for all $\theta \in (0,1)$. Indeed:

$$\delta_p'(\theta) = \frac{4\theta(2-\theta)}{(\theta^2 - 8\theta + 8)^2} > 0, \tag{2.39}$$

for all $\theta \in (0,1)$. □

This result shows that the collusive streak plays an important role in the cartel stability. Proposition 2.7 and 2.15 have the same trade-off between the product differentiation degree and cartel stability — i.e., the trade-off between the product differentiation degree and cartel stability is invariant by the cartel collusive strategy. Cartels with a collusive strategy of quantity-fixing and price-fixing are less stable if the products tend to be homogeneous— i.e., because the cartel members value future benefits more than those present when the products are more homogeneous.

The following assumption to characterize cartelization with a collusive price-fixing strategy under antitrust enforcement at different levels of horizontal product differentiation.

Assumption 2.16. For all horizontal product differentiation degree $\theta \in (0,1)$ of the linear demand system 2.7, firms 1 and 2 cartelize the market with a collusive price-fixing strategy, knowing that there is an antitrust authority that applies a fine rate of $f \in (0, \bar{f}] \subseteq (0,1)$ to cartel members.

Cartel stability — cartel made up of two symmetrical firms, 1 and 2, with a collusive price-fixing strategy — under antitrust enforcement with fine parameter $f \in (0, \bar{f}] \subseteq (0,1)$ is determined by the following incentive compatibility constraint:

$$\bar{\delta}_p(\theta, \beta, f) \geq \frac{(2-\theta)^2(\theta^2 - 4\beta f\theta + 4\beta f)}{(1-\beta)\theta^2(\theta^2 - 8\theta + 8)}. \tag{2.40}$$

The following result shows the effect of antitrust enforcement on cartel stability at different levels of product differentiation.

Proposition 2.17. *Consider that for all horizontal product differentiation degree $\theta \in (0,1)$ of the demand system 2.7, firms 1 and 2 cartilize the market with a collusive price-fixing strategy. Antitrust enforcement impacts negatively and with greater intensity on cartel stability if the products tend to be highly differentiated.*

Proof. Since the function $\bar{\delta}_p : (0,1) \times (0, \bar{\beta}] \times (0, \bar{f}] \subseteq (0,1)^3 \longrightarrow (0,1)$ is continuous, it is enough to prove two items. First, let us show that $\lim_{\beta \to 0} \bar{\delta}_p(\theta, \beta, f) = \delta_p(\theta)$, for all $\theta \in (0,1)$ and $f \in (0, \bar{f}]$. Indeed:

$$\lim_{\beta \to 0} \bar{\delta}_p(\theta, \beta, f) = \lim_{\beta \to 0} \left[\frac{(2-\theta)^2(\theta^2 - 4\beta f\theta + 4\beta f)}{(1-\beta)\theta^2(\theta^2 - 8\theta + 8)} \right] = \frac{(2-\theta)^2}{\theta^2 - 8\theta + 8} = \delta_p(\theta).$$

Second, let us verify that $\lim_{\theta \to 0} \bar{\delta}_p(\theta, \beta, f) = +\infty$, for all $(\beta, f) \in (0,1)^2$. Indeed:

$$\lim_{\theta \to 0} \bar{\delta}_p(\theta, \beta, f) = \lim_{\theta \to 0} \left[\frac{(2-\theta)^2(\theta^2 - 4\beta f\theta + 4\beta f)}{(1-\beta)\theta^2(\theta^2 - 8\theta + 8)} \right] = \frac{\beta f}{1-\beta}(+\infty) = +\infty.$$

Therefore, this implies that the effect of antitrust enforcement is greater when the products tend to be highly differentiated. □

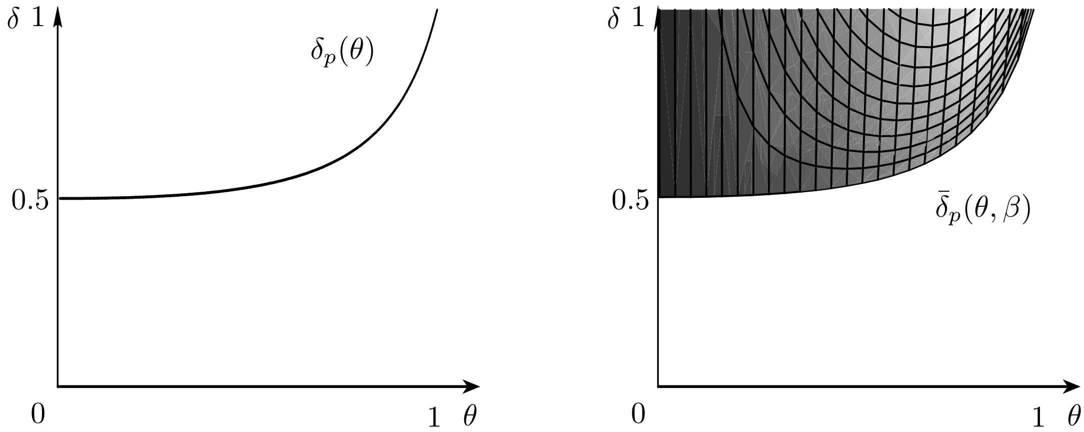

Figure 2.2: Effect of antitrust enforcement on cartel stability at different levels of product differentiation: price-fixing case.

Similar to Section 2.4, Figure 2.2 in particular ($\bar{f} = 0.2$), illustrates the proof of Propositions 2.15 and 2.17. On the one hand, on the left side of Figure 2.2, it is observed that the critical discount factor that maintains the cartel stability is increasing over the product differentiation interval. On the other hand, on the right side of Figure 2.2, it is observed that antitrust enforcement affects the cartel stability with greater intensity when the products tend to be more differentiated. Therefore, on the left sides of Figures 2.1 and 2.2, it is observed that antitrust enforcement affects the cartel stability with greater intensity if the products tend to be homogeneous, that is for cartels with a collusive strategy of quantity-fixing and price-fixing.

Finally, notice in Figure 2.2 —left side— that in the area defined as:

$$1 - \int_0^1 \frac{(2+\theta)^2}{\theta^2 - 8\theta + 8} d\theta \tag{2.41}$$

cartel stability intervals are found, for each product differentiation degree. Likewise, it is observed that when the products tend to be homogeneous, the cartel stability intervals decrease. Therefore, the trade-off between the product differentiation degree and cartel stability has the same dynamics when the cartel adopts a collusive strategy of either quantity-fixing or price-fixing.

2.6 Comparing Cartels

This section analyzes the effect of collusive strategy —quantity-fixing and price-fixing — on cartel stability at different levels of horizontal product differentiation $\theta \in (0,1)$ in an environment free of antitrust enforcement. The left side of Figures 2.1 and 2.2 illustrate that for all degrees of differentiation, the cartel with a collusive price-fixing strategy is less stable than the cartel with a collusive quantity-fixing strategy.

The following result diagnoses which of the cartels is more susceptible to destabilizing at different levels of horizontal product differentiation.

Proposition 2.18. *Consider that for all horizontal product differentiation degree* $\theta \in (0,1)$ *of the demand system 2.6 and 2.7, firms 1 and 2 cartilize the market with a collusive* *quantity-fixing and price-fixing strategy. For all degree of horizontal product differentiation* $\theta \in (0,1)$*, the cartel with a collusive quantity-fixing strategy is more stable than the cartel with a collusive price-fixing strategy.*

Proof. To prove it is sufficient verify that $\delta_p(\theta) > \delta_q(\theta)$, for all $\theta \in (0,1)$. Indeed: for all $\theta \in (0,1)$, satisfies that, $(2+\theta)^2 > (2-\theta)^2$ and $\theta^2 + 8\theta + 8 > \theta^2 - 8\theta + 8$. Then $\frac{(2-\theta)^2}{\theta^2 - 8\theta + 8} > \frac{(2+\theta)^2}{\theta^2 + 8\theta + 8}$. Therefore, $\delta_p(\theta) > \delta_q(\theta)$, for all $\theta \in (0,1)$ □

This result shows that for each product differentiation degree $\theta \in (0,1)$, the cartel with a collusive price-fixing strategy is less susceptible to destabilizing than the cartel with a collusive quantity-fixing strategy — in Figures 2.1 and 2.2 (left side), it is observed that for each product differentiation degree, the critical discount factor with which the quantity-fixing cartel operates is lower than that with which the price-fixing cartel operates. Therefore, price-fixing cartels are less stable than quantity-fixing cartels.

It is important that this result considered by the antitrust authorities because by using the same antitrust parameters to combat the cartels, in some cases they are more effective than in others, for example, we have just shown that the collusive strategy adopted by the cartels is capable of neutralizing the effect of antitrust enforcement.

2.7 Conclusion

This paper analyzes two aspects of a cartel made up of two symmetrical firms with a collusive strategy of quantity-fixing and price-fixing in a market with horizontally differentiated products. First, we study the trade-off between the horizontal product differentiation degree and cartel stability. Second, the effect of antitrust enforcement on cartel stability at different levels of horizontal product differentiation. The results are valid for cartels with collusive strategies of quantity-fixing and price-fixing. We corroborate that when the products tend to be homogeneous, the cartels are less stable. Besides, we proved that cartels with collusive quantity-fixing strategy are more stable than cartels with collusive price-fixing strategy — Bertrand competition is more efficient than Cournot competition. On the other hand, we show that antitrust enforcement affects negatively and with greater intensity on cartels stability when the products are more differentiated — it is valid for quantity-fixing and price-fixing cartels. Therefore, there is a disparity in the sense of destabilization of cartels according to the product differentiation degree.

Finally, to guide the activity of the antitrust authority and antitrust policy-makers, we recommend: i) more severe fines for firms participating in a price-fixing (quantity-fixing) cartel with moderately differentiated products (products that tend to be homogeneous) because they enjoy excessive cooperative profit relative to competitive profit; ii) intensifying research on potential cartels with a lesser product differentiation degree.

Chapter 3

Cartel Stability under Antitrust Enforcement and Product Quality Differentiation

Abstract: Cartels occur in different markets and with different levels of product quality differentiation. This paper examines the trade-off between the product quality differentiation degree and cartel stability — a cartel made of a firm that produces low-quality products and one that produces high-quality products —, and the effect of antitrust enforcement on cartel stability at different levels of product quality differentiation. We show that the firm that produces low-quality products is more likely to destabilize the cartel. Thus, we show that cartel stability is quadratic over product quality differentiation interval — when products are moderately differentiated, the cartel is more stable. On the other hand, we show that antitrust enforcement uniformly and negatively affects cartel stability, for every product quality differentiation degree — i.e., antitrust enforcement does not strongly affect the stability of a cartel that has a specific product quality differentiation degree. These results can help in the activity of the antitrust authorities indicating that potential cartels with moderately differentiated products should be investigated with more emphasis.

Keywords: Product Quality Differentiation · Price-Fixing · Antitrust Enforcement · Cartel Stability.

JEL Classification: L13 · L41 · D43 · C73

3.1 Introduction

Cartels occur in markets with vertically differentiated products and at different levels of product quality differentiation (Häckner, 1994; Andaluz, 2010; Bos and Marini, 2019; Bos et al., 2020). The literature continually explores the cartels' stability in markets with differentiated product quality. Thus, cartel stability essentially depends on factors intrinsic to the market structure. For example, the difference in production costs (Bos and Marini, 2019; Bos et al., 2020), product quality differentiation degree (Häckner, 1994;

Andaluz, 2010) (trade-off between the product quality differentiation degree and cartel stability), and others. On the other hand, currently, the literature has gained strength in the study of the cartels' stability under antitrust enforcement. The literature generally discusses the effects of antitrust enforcement on cartels' stability. The main result obtained is that antitrust enforcement negatively affects cartels' stability and stops the formation of new cartels.

The literature on cartels' stability under antitrust enforcement in the case of price-fixing cartels is constantly explored (Block et al., 1981; Motta and Polo, 2003; Houba et al., 2015). However, the effects of antitrust enforcement on cartel stability at different levels of product quality differentiation are not known. Contributing to the economic collusion literature, our objective is to examine the cartel stability under antitrust enforcement at different levels of product quality differentiation. Essentially answering the questions: (i) what is the dynamic of the trade-off between the product quality differentiation degree and cartel stability?, (ii) is antitrust enforcement more effective against cartels when the products are more differentiated or homogeneous?. Analyzing this topic has the utmost importance because the cartels can be constituted in different industries and with differentiated products. Therefore, this analysis helps guide the activity of the antitrust authority to focus its investigation on more stable cartels.

To develop our questions, we consider a cartel —a cartel made up of a firm that produces low-quality products and another of high-quality with a collusive price-fixing strategy— operating in a market with vertically differentiated products under an economy with antitrust law. Thus, we consider the scenarios: cartel stability under the absence of antitrust law and cartel stability under antitrust enforcement. To analyze stability, sub-game perfect Nash equilibrium (SPNE) of the grim-trigger strategy Friedman (1971) and modified grim-trigger strategy (Fudenberg and Tirole, 1991) is used, respectively.

The trade-off between the product quality differentiation degree and cartel stability has been and is constantly studied. The results of Häckner (1994), and Andaluz (2010) converge on some points: cartel stability does not have a clear behavior over product quality differentiation interval. However, they make it clear that the behavior is not the same as in the case of horizontally differentiated products. In contrast, we show that cartel stability is quadratic over product quality differentiation interval — i.e., the cartel is more stable when products are moderately differentiated. Furthermore, we show that cartel stability is more dependent on the firm than it produces low-quality products because it is the most sensitive to destabilizing the cartel — its competitive profit is close to its competitive profit when the products tend to be homogeneous and highly differentiated. On the other hand, our result has different dynamics than the case of horizontally differentiated products. Majerus (1988), and Huamani and Braga (2022) show that cartel stability is increasing over horizontal product differentiation interval— i.e., the cartel is more stable when horizontal products are highly differentiated.

The effect of antitrust enforcement on cartel stability is extensively explored in the literature, particularly for price-fixing cartels. Block et al. (1981), Motta and Polo (2003), and Houba et al. (2015) approach in their way but converge to a common result: antitrust enforcement negatively affects cartel stability (antitrust enforcement destabilizes cartels) — i.e., antitrust enforcement lowers the collusive price (Houba et al., 2015). Moving on to more results, we diagnose the effect of antitrust enforcement at different levels of product quality differentiation: antitrust enforcement has a negative and uniform effect on cartel stability — antitrust enforcement does not affect with greater intensity a product quality differentiation degree in specific. Here, we note that our result is not similar to the case of horizontally differentiated products. Huamani and Braga (2022) show that antitrust enforcement negatively and intensely affects cartel stability when horizontal products are highly differentiated — the result is independent of the collusive strategy adopted by the cartel (quantity-fixing or price-fixing).

The rest of the paper is organized as follows. Section 3.2 describes a demand system with product quality differentiation. Section 3.3 describes the capture of profits that constitute the cartel dilemma. Section 3.4 describes the trade-off between the product quality differentiation degree and cartel stability. Section 3.5 describes the dynamics of cartel stability under antitrust enforcement at different levels of product quality differentiation. Section 3.6 describes a numerical simulation of the impact of antitrust enforcement with different levels of fines on cartel stability at different levels of product quality differentiation. Finally, Section 3.7 concludes and includes recommendations to guide antitrust authority activity.

3.2 Model

Consider a market with vertically differentiated products[1] — products differentiated by quality — served by a duopoly, firms 1 and 2, such that firm 2 produces high-quality products and firm 1 produces low-quality products, and the product differentiation parameters have the following relation respectively: $s_2 \geq s_1$. The valuation of the products and the distribution structure of consumers consists of: (i) the consumers value the products with the parameter $\omega \in [a, b] \subset [1, \infty)$; (ii) consumers are evenly distributed in $[a, b]$ with mass normalized to one; (iii) consumers hardly buy a product. The consumer located in $\omega \in [a, b]$ has the utility function defined as:

$$U(\omega) = \omega s_i - p_i, \tag{3.1}$$

where $p_i \in [0, s_i b]$ are the prices of the products $i \in \{1, 2\}$. The existence of the indifferent consumer between consuming product 1 or product 2 leads to obtaining the same level of utility, that is:

$$\omega s_1 - p_1 = \omega s_2 - p_2 \Longrightarrow \omega = \frac{p_2 - p_1}{s_2 - s_1}. \tag{3.2}$$

Since any consumer located in $\omega \in [a, b]$ can be indifferent between consuming the product 1 and product 2, it generates a demand system that is served by firms 1 and 2, that is:

$$D_1(p_1, p_2) = \omega - a = \frac{p_2 - p_1}{s_2 - s_1} - a, \qquad D_2(p_1, p_2) = b - \omega = b - \frac{p_2 - p_1}{s_2 - s_1}. \tag{3.3}$$

Expressing in this way the demand system 3.3 that characterizes a vertically differentiated product market structure is conducive to Bertrand's competition.

[1] See Wauthy (1996).

3.3 Firms Behavior

This section describes the profit capture of firms in the market according to their behavior: diversion profit is greater than cooperative profit, cooperative profit is greater than competitive profit, and competitive profit is greater than damage profit. These profits constitute the cartel dilemma — a strategic game with cooperate and non-cooperate actions. The strategic interaction via Bertrand of a vertically differentiated duopoly consists of:

- Firms: 1 and 2;
- Prices choice: $p_i \in [0, p_{max}]$, for all $i \in \{1, 2\}$;
- Payoff functions: $\pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j)$, for all $i, j \in \{1, 2\}$ with $i \neq j$.

where $c = 0$ is the marginal cost with which firms operate in the market, $D_i(p_i, p_j)$ is the demand system 3.3 for all $i, j \in \{1, 2\}$ with $i \neq j$. The behavior of firms in the market leads to the capture of different profits: diversion, cooperative, competitive, and damage. Next, we capture the profits of the firms according to their behavior in the market.

Lemma 3.1. *Consider that firms 1 and 2 serve demand system 3.3. The Nash-Bertrand equilibrium profile is:*

$$p_1^N = \frac{(b-2a)(s_2-s_1)}{3}, \qquad p_2^N = \frac{(2b-a)(s_2-s_1)}{3}. \tag{3.4}$$

Then competitive (non-cooperative) profits for firms 1 and 2 respectively are:

$$\pi_1^N = \frac{(s_2-s_1)(b-2a)^2}{9}, \qquad \pi_2^N = \frac{(s_2-s_1)(2b-a)^2}{9}. \tag{3.5}$$

Proof. The solution for a competitive environment is the individual profit maximization of the firms. Then the situation is solved as a simultaneous game and the Nash equilibrium can be written as the system of best responses:

$$p_1 = \frac{p_2 - a(s_2-s_1)}{2}, \qquad p_2 = \frac{p_1 + b(s_2-s_1)}{2}. \tag{3.6}$$

Then, solving the system of best responses, the Bertrand-Nash equilibrium profile (p_1^N, p_2^N) is obtained. Consequently, evaluating (p_1^N, p_2^N) in the profit functions, the competitive profits are π_1^N and π_2^N. □

The following result explains how cooperative behavior (firm joint profit maximization) leads to greater profitability.

Lemma 3.2. *Consider that firms 1 and 2 serve demand system 3.3. The optimal cartel price vector is:*

$$p_1^C = as_1, \qquad p_2^C = as_1 + \frac{b}{2}(s_2 - s_1). \tag{3.7}$$

Then cooperative profits for firms 1 and 2 respectively are:

$$\pi_1^C = \frac{a(b-2a)s_1}{2}, \qquad \pi_2^C = \frac{2bas_1 + b^2(s_2-s_1)}{4}. \tag{3.8}$$

Proof. To capture the prices that induce supra-competitive profits — maximization of joint profits. Häckner (1994) sets the price of firm 1 with value $p_1^C = as_1$. Then, maximizing the profit function $\pi = \pi_1(p_1^C, p_2) + \pi_2(p_1^C, p_2)$ with respect to p_2. By first-order conditions we have:

$$\frac{2p_2 - 2as_1 + bs_1 - bs_2}{s_1 - s_2} = 0. \tag{3.9}$$

Solving equation 3.9, we obtain the cartel price p_2^C of firm 2. Then, evaluating the cartel price vector (p_1^C, p_2^C) in the profit functions, we obtain the cooperation profits, π_1^C and π_2^C, for firms 1 and 2, respectively. □

The last two results to follow explain how cooperative action by one firm can be disadvantageous given that another firm is projected to deviate from cooperative action.

Lemma 3.3. *Consider that firms 1 and 2 serve demand system 3.3. Since firm 2 cooperates, firm 1 optimal deviation price is:*

$$p_1^D = \frac{b(s_2 - s_1)}{4} - \frac{a(s_2 - 2s_1)}{2}, \tag{3.10}$$

Then diversion profit for firm 1 and damage profit for firm 2 are:

$$\pi_1^D = \frac{(b(s_2 - s_1) - 2a(s_2 - 2s_1))^2}{16(s_2 - s_1)}, \qquad \pi_{2D} = \frac{(2as_2 - 3b(s_2 - s_1))(2as_1 + b(s_2 - s_1))}{8(s_2 - s_1)}. \tag{3.11}$$

Proof. To find the deviation price of firm 1, set the cartel price of firm 2. By first-order conditions of the function $\pi_1(p_1, p_2^C)$ with respect to p_1, we obtain:

$$\frac{4p_1 - 4as_1 + 2as_2 + bs_1 - bs_2}{2(s_1 - s_2)} = 0 \tag{3.12}$$

Solving equation 3.12, we obtain the optimal deviation price p_1^D — since $\pi_1(p_1, p_2^C)$ is concave, then p_1^D is an optimum point. Therefore, evaluating the price vector (p_1^D, p_2^C) in the profit functions, we obtain deviation profit π_1^D for firm 1 and damage profit π_{2D} for firm 2. □

Lemma 3.4. *Consider that firms 1 and 2 serve demand system 3.3. Since firm 1 cooperates, firm 2 optimal deviation price is:*

$$p_2^D = \frac{as_1 + b(s_2 - s_1)}{2}, \tag{3.13}$$

Then diversion profit for firm 2 and damage profit for firm 1 are:

$$\pi_2^D = \frac{(as_1 + b(s_2 - s_1))^2}{4(s_2 - s_1)}, \qquad \pi_{1D} = \frac{as_1(b(s_2 - s_1) - a(2s_2 - s_1))}{2(s_2 - s_1)}. \tag{3.14}$$

Proof. To find the deviation price of firm 2, set the cartel price of firm 1. By first-order conditions of the function $\pi_2(p_1^C, p_2)$ with respect to p_2, we obtain:

$$\frac{2p_2 - as_1 + bs_1 - bs_2}{s_1 - s_2} = 0. \tag{3.15}$$

Solving equation 3.15, we obtain the optimal deviation price p_2^D — since $\pi_2(p_1^C, p_2)$ is concave, then p_2^D is an optimum point. Therefore, evaluating the price vector (p_1^C, p_2^D) in the profit functions, we obtain damage profit π_{1D} for firm 1 and damage profit π_2^D for firm 2. □

The profits captured in Lemmas 3.1, 3.2, 3.3, and 3.4, constitute the cartel dilemma[2] — stage game with cooperate and non-cooperate actions. This stage game generates infinitely repeated game, particularly grim-trigger strategy.

3.4 Cartel Stability

This section describes trade-off between the product quality differentiation degree and cartel stability. Previously we diagnose which of the firms is more sensitive to destabilize the cartel because the cartel stability depends on the more sensitive firm.

Taking into account the relationship of the profits that constitute the cartel dilemma — diversion profit π^D greater than cooperation profit π^C, cooperation profit π^C greater than competition profit π^N, and competition profit π^N greater than damage profit π_D —; and finding the sub-game perfect Nash equilibrium (SPNE) of the grim-trigger strategy (Friedman, 1971) we characterize the cartel stability. A cartel made up of $N = \{1, \cdots, i, \cdots, n\}$ firms is said to be stable if for all $i \in N$ there exists $\delta \in (0, 1)$ such that $\delta = \max\{\delta_1, \cdots, \delta_i, \cdots, \delta_n\}$ and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \iff \delta \geq \delta_i \geq \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^N}, \tag{3.16}$$

where V_i^C is the liquid present value of firm $i \in N$, when it maintains the collusive agreement — in each period firm ii takes the cooperate action —, V_i^D is the liquid present value of firm $i \in N$, when it deviates from the agreement — in the first period the firm $i \in N$ deviates from the agreement and then decides to compete forever, and $\delta_i \in (0, 1)$ is the discount factor with which firm $i \in N$ maintains cartel stability. The minimum discount factor of firm $i \in N$ — the discount factor satisfies the equality of the expression 3.16 — is called the critical discount factor of firm $i \in N$. The set defined as $\{\delta \in (0, 1) : \delta_i \leq \delta < 1\}$ is called the stability interval of firm $i \in N$ — the cartel member firm that has a smaller stability interval is more likely to destabilize the cartel, i.e., the firms operate at a higher discount factor[3].

Definition 3.5. Consider a cartel made up of N asymmetric firms. Firm $j \in N$ is more susceptible to destabilizing the cartel if for all firm $i \in N - \{j\}$ with $i \neq j$, $\delta_i < \delta_j$, i.e., firm j maintains the cartel stability at a higher critical discount factor than firms $i \in N - \{j\}$.

[2] The bimatricial representation of the cartel dilemma is:

$$\begin{pmatrix} \pi_1^C, \pi_2^C & \pi_{D1}, \pi_2^D \\ \pi_1^D, \pi_{D2} & \pi_1^N, \pi_2^N \end{pmatrix}$$

where the first row (column) are the payoffs of the cooperate action of firm 1 (firm 2) and the second row (column) are the payoffs of the non-cooperate action of firm 1 (firm 2).

[3] See Bruttel (2009).

To see the dynamics of cartel stability —a cartel with a collusive price-fixing strategy— made up of firms 1 and 2, in a market with differentiated quality products. Without loss of generality, we assume that $\frac{b}{a} = 2.01$ and $\frac{s_1}{s_2} = \theta \in (0,1)$. Formalizing, we have the following assumption.

Assumption 3.6. For all product quality differentiation degree $\theta = \frac{s_1}{s_2} \in (0,1)$ of the demand system 3.3, firms 1 and 2 cartelize the market, knowing that there is no antitrust enforcement.

Evaluating the profits obtained from the Lemmas 3.1, 3.2, 3.3, and 3.4, in the incentive compatibility constraint 3.16 we characterize the cartel stability. The cartel made up of firms, 1 and 2, is said to be stable if there exists $\delta \in (0,1)$ such that $\delta = \max\{\delta_1, \delta_2\}$ and satisfies the following inequalities:

$$\delta_1(\theta) \geq \frac{9(2.01\theta - 0.01)^2}{35.639\theta^2 + 0.3614\theta - 0.0007}, \quad \delta_2(\theta) \geq \frac{9\theta^2}{-27.301\theta^2 + 36.421\theta - 0.1207}, \tag{3.17}$$

where the inequality on the left side is the ICC of the firm that produces low-quality products (firm 1), and the inequality on the right side is the ICC of the firm that produces high-quality products (firm 2).

The trade-off between the product quality differentiation degree and cartel's stability is essentially linked to the firm that is more likely to destabilize the cartel. Then, we diagnose which of the firms is more sensitive to destabilizing the cartel.

Proposition 3.7. *Consider that for all product quality differentiation degree $\theta \in (0,1)$ of the demand system 3.3, firms 1 and 2 cartilize the market with a collusive price-fixing strategy. The firm that produces low-quality products (firm 1) is more likely to destabilize the cartel than the firm that produces high-quality products (firm 2).*

Proof. To show this conjecture, it is enough to prove that for every product quality differentiation degree $\theta \in (0,1)$, the critical discount factor of firm 1 is greater than that of firm 2. Indeed: since $9(2.1\theta - 0.01)^2 > 9\theta^2$, and $-27.301\theta^2 + 36.421\theta - 0.1207 > 35.639\theta^2 + 0.3614\theta - 0.0007$, for all $\theta \in (0,1)$,

$$\frac{9(2.01\theta - 0.01)^2}{35.639\theta^2 + 0.3614\theta - 0.0007} > \frac{9\theta^2}{-27.301\theta^2 + 36.421\theta - 0.1207}. \tag{3.18}$$

Therefore, $\delta_1(\theta) > \delta_2(\theta)$, for all $\theta \in (0,1)$. □

Once the firm most sensitive to destabilizing the cartel is diagnosed — Proposition 3.7 informs us of this result — then we proceed with the trade-off analysis between the product quality differentiation degree and cartel stability. Thus, we evaluate the behavior of the incentive compatibility constraint (ICC) of the firm that produces low-quality products.

Proposition 3.8. *Consider that for all product quality differentiation degree $\theta \in (0,1)$ of demand system 3.3, firms 1 and 2 cartilize the market with a collusive price-fixing strategy. The cartel is less stable when the products are almost homogeneous and highly differentiated, i.e., the critical discount factor function is quadratic and reaches its minimum over the interior of the product differentiation interval.*

Proof. To prove this proposition, it is sufficient to show that the critical discount factor function $\delta_1 : (0,1) \longrightarrow (0,1)$ defined as:

$$\delta_1(\theta) = \frac{9(2.01\theta - 0.01)^2}{35.639\theta^2 + 0.3614\theta - 0.0007}, \tag{3.19}$$

it is quadratic. Indeed: By first-order conditions (FOC) we have:

$$\frac{1.3018 \times 10^9\theta^2 - 2.866 \times 10^7\theta + 1.1037 \times 10^5}{M} = 0, \tag{3.20}$$

where $M = 6.3507 \times 10^{10}\theta^4 + 1.2880 \times 10^9\theta^3 - 1.8417 \times 10^7\theta^2 - 2.5298 \times 10^5\theta + 2450.0$. Solving FOC we obtain the root $\bar{\theta} = 4.9753 \times 10^{-3}$. Then substituting $\bar{\theta} = 4.975310^{-3}$ in $\delta_1(\theta)$ we obtain 5.6633×10^{-10}. Therefore, δ_1 reaches a minimum in the product differentiation interval $(0,1)$. □

The trade-off between the product quality differentiation degree (vertical product differentiation) and cartel stability has been widely discussed in literature. In the case of markets with vertically differentiated products (or products differentiated by quality), there is no clear behavior of the trade-off between the product quality differentiation degree and cartel stability[4]. We have just shown in Proposition 3.8 that this trade-off is quadratic over the product quality differentiation interval — the cartel is less stable when the products tend to be homogeneous and are highly differentiated. Figure 3.1 illustrates the proofs of Propositions 3.7 and 3.8. On one hand, it is observed that the critical discount factor of the firm that produces high-quality products (firm 1) is greater than that of the firm that produces low-quality products over the product differentiation interval. On the other hand, it is observed that the critical discount factor (incentive compatibility constraint of firm 1) that supports the cartel stability is quadratic over product differentiation interval — i.e., the cartel is more stable when the products are moderately differentiated. Finally, note that in areas defined as:

$$1 - \int_0^1 \frac{9(2.01\theta - 0.01)^2}{35.639\theta^2 + 0.3614\theta - 0.0007} d\theta, \quad 1 - \int_0^1 \frac{9\theta^2}{-27.301\theta^2 + 36.421\theta - 0.1207} d\theta, \tag{3.21}$$

the levels of discount factors that maintain the cartel stability are found. Illustrating Proposition 3.7, it is observed that for all product differentiation degree $\theta \in (0,1)$, we have the relationship: $[\delta_1(\theta), 1) \subset [\delta_2(\theta), 1)$ — i.e., firm 1 is more likely to destabilize the cartel (cooperation profit is close to competition profit).

3.5 Cartel Stability under Antitrust Enforcement

This section describes cartel stability (cartel with collusive price-fixing strategy) under antitrust enforcement (represented by the probability of cartel detection and finely applied to cartel members) at different levels of product quality differentiation. Essentially, we will see the effect of antitrust enforcement on cartel stability at different levels of product quality differentiation.

[4]See Häckner (1994), and Andaluz (2010).

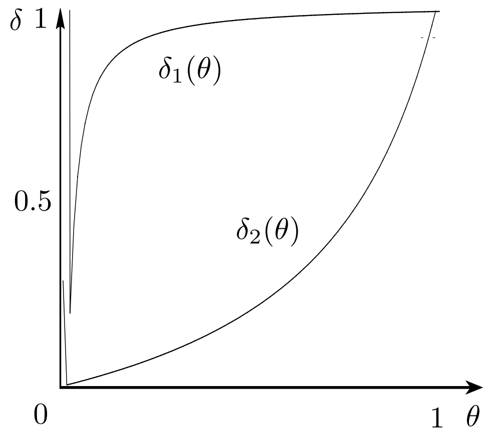

Figure 3.1: Trade-off between the product quality differentiation degree and cartel stability: case price-fixing.

The elements that characterize cartel stability are described below. Denote as V_i^C the expected present value of firm $i \in N$, when it maintains the collusive agreement — in each period firm $i \in N$ maintains the cooperative action knowing that the cartel can be discovered with a certain probability and applied a certain rate of fine — , and define as the following recursive dynamics:

$$V_i^C = \pi_i^C + \beta \left[-f\pi_i^C + \bar{\delta}_i \frac{\pi_i^N}{1-\bar{\delta}_i} \right] + (1-\beta)\bar{\delta}_i V_i^C, \tag{3.22}$$

where $\beta \in (0,1)$ is the probability of cartel detection, $f \in (0,1)$ is the fine rate applied to cartel members, $\bar{\delta}_i \in (0,1)$ is the discount factor with which firm $i \in N$ maintains the cartel stability. On the other hand, denote as V_i^D the liquid present value of firm $i \in N$, when it deviates from the collusive agreement — firm $i \in N$ deviates in the first period and then competes forever — and define as:

$$V_i^D = \pi_i^D + \frac{\bar{\delta}_i \pi_i^N}{1-\bar{\delta}_i} = \frac{(1-\delta)\pi_i^D + \bar{\delta}_i \pi_i^N}{1-\bar{\delta}_i}. \tag{3.23}$$

Since $1-\bar{\delta}_i > 0$, and $1-\beta(1-\bar{\delta}_i) > 0$, the sub-game perfect Nash equilibrium (SPNE) of the modified grim-trigger strategy characterizes cartel stability under antitrust enforcement. A cartel made up of N firms is said to be stable under antitrust enforcement if for all firm $i \in N$ there exist $\bar{\delta} \in (0,1)$ such that $\bar{\delta} = \max\{\bar{\delta}_1, \cdots, \bar{\delta}_i, \cdots, \bar{\delta}_n\}$, and satisfies the following inequality (incentive compatibility constraint):

$$V_i^C \geq V_i^D \Longleftrightarrow \bar{\delta}_i \geq \frac{\pi_i^D - (1-\beta f)\pi_i^C}{(1-\beta)[\pi_i^D - \pi_i^N]}. \tag{3.24}$$

The minimum discount factor $\bar{\delta}_i$ (discount factor that satisfies equation 3.24) with which firm $i \in N$ maintains cartel stability is called the critical discount factor under antitrust enforcement of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \bar{\delta}_i \leq \delta < 1\}$ is called the stability interval under antitrust enforcement of firm $i \in N$. The set defined as $\{\delta \in (0,1) : \max\{\bar{\delta}_1, \cdots, \bar{\delta}_i, \cdots, \bar{\delta}_n\} \leq \delta < 1\}$ is called the cartel stability interval under antitrust enforcement. If the cartel stability interval tends to decrease, then we say that cartel stability is negatively affected.

To measure the effect of antitrust enforcement on cartel stability, we compare the critical discount factors of both scenarios—under free antitrust enforcement and free antitrust enforcement. The following definition characterizes this effect.

Definition 3.9. Consider a cartel made up of N firms. Antitrust enforcement negatively affects cartel stability if the critical discount factor under antitrust enforcement is greater than the critical discount factor, i.e., $\delta < \bar{\delta}$.

We now make an assumption to analyze the dynamics of cartel stability (cartel with collusive price-fixing strategy) under antitrust enforcement at different levels of product quality differentiation.

Assumption 3.10. For all degrees of product quality differentiation $\theta = \dfrac{s_1}{s_2} \in (0,1)$ of the demand system 3.3, firms 1 and 2, cartelize the market with a collusive price-fixing strategy, knowing that there is an antitrust authority that applies a fine rate of $\bar{f} = \min\{f : 0.01 \leq f \leq 0.20\}$ to cartel members.

The cartel made up of firms 1 and 2 is stable under antitrust enforcement, if for each firm $i \in \{1,2\}$ there exist $\bar{\delta} \in (0,1)$, such that $\bar{\delta} = \max\{\bar{\delta}_1, \bar{\delta}_2\}$ and satisfies the incentive compatibility constraints:

$$\bar{\delta}_1(\theta,\beta) = \frac{(3.6361 \times 10^5\theta^2 - 36180\theta + 9) - 72\beta\theta(1-\theta)}{(3.5639 \times 10^5\theta^2 + 36140\theta - 7) - \beta(3.5639 \times 10^5\theta^2 + 36140\theta + 7)}, \tag{3.25}$$

$$\bar{\delta}_2(\theta,\beta) = \frac{(90450\theta^2 - 1.8181 \times 10^7\theta + 90450) - \beta(1809\theta^2 - 3.6542 \times 10^5\theta + 3.6361 \times 10^5)}{(3.621 \times 10^7\theta^2 - 1.8241 \times 10^5\beta + 30250.0) + \beta(3.621 \times 10^7\theta^2 - 1.8241 \times 10^7\theta + 30250)}, \tag{3.26}$$

where inequality 3.25 (3.26) is the incentive compatibility constraint of the firm that produces low-quality (high-quality) products.

The following result explains the effect of antitrust enforcement on cartel stability at different levels of product quality differentiation. Since Proposition 3.7 diagnoses that the firm that produces low-quality products (firm 1) is more likely to destabilize. Then to capture such an effect it is only conducive to look at the dynamics of incentive compatibility constraints of firm 1.

Proposition 3.11. *Consider that for all degree of quality differentiation product $\theta = \dfrac{s_1}{s_2} \in (0,1)$ of demand system 3.3, firms 1 and 2 cartilize the market with a collusive price-fixing strategy. Antitrust enforcement affects negatively and uniformly on cartel stability, i.e., antitrust enforcement does not specifically strongly affect a product quality differentiation degree.*

Proof. Since the function $\bar{\delta} : (0,1) \times [0,\bar{\beta}] \longrightarrow (0,1)$ is differentiable. To prove it is sufficient verify that $\bar{\delta}(\theta,\beta)$, for all $\beta \in (0,\bar{\beta}]$, reaches its minimum at $(0,1)$. Indeed: By first-order conditions (FOC) we have:

$$\bar{\delta}_1'(\theta,\beta) = \left(\frac{(3.6361 \times 10^5\theta^2 - 36180\theta + 9) - 72\beta\theta(1-\theta)}{(3.5639 \times 10^5\theta^2 + 36140\theta - 7) - \beta(3.5639 \times 10^5\theta^2 + 36140\theta + 7)}\right)' = 0.$$

Solving F.O.C., we obtain that the critical point $\bar{\theta}$ is a point of minimum in $(0,1)$. Therefore, for all $\beta \in (0,\bar{\beta}) \subset (0,1)$, the function $\bar{\delta}_1(-,\beta)$ is convex quadratic over $(0,1)$. □

Proposition 3.11 shows us that the effect of antitrust enforcement on cartel stability at different levels of product quality is not the same as for horizontally differentiated products. In the case of markets with horizontally differentiated products, it is shown that antitrust enforcement negatively and more intensely affects cartel stability when products are highly differentiated[5]. However, for the case of products vertically differentiated, we obtain: antitrust enforcement negatively and uniformly affects on cartel — antitrust enforcement does not affect the cartel stability with greater intensity on a specific product quality differentiation degree.

3.6 Numerical Simulation

This section presents a numerical simulation of cartel stability under antitrust enforcement — varying the levels of fine rates applied to cartel members — at different levels of product quality differentiation. To do the simulation, guaranteed by Proposition 3.7, it is enough to work with the incentive compatibility constraint of firm 1 (the firm that produces low-quality products).

$$\bar{\delta}_1(\theta,\beta,f) = \frac{\pi_1^D - (1-\beta f)\pi_1^C}{(1-\beta)[\pi_1^D - \pi_1^N]}. \tag{3.27}$$

where the critical discount factor under antitrust enforcement depends on the probability of cartel detection and the rate of fine applied to cartel members. To simulate we consider three penalty rates: minimum $\bar{f} = 0.01$, average $\bar{f} = 0.105$, and maximum $\bar{f} = 0.20$. The description of the double-entry tables is: fine rate level is specified in the table number, cartel detection probabilities are found in the first column of the table, and product quality differentiation levels (degrees of product quality differentiation) are found in the first row of the table.

The double-entry tables have the following description: (i) the column with the probability of cartel detection $\beta = 0$, describes the critical discount factor levels that maintain cartel stability in a scenario free of antitrust enforcement (base discount factor levels to measure the impact of antitrust enforcement); (ii) the column with the probability of cartel detection $\beta > 0$, describes the critical discount factors levels that maintain the cartel stability with the antitrust enforcement parameter $(\beta, f) \subset (0,1)^2$, (iii) spaces where there is a trace, there is no more cartel (there is no critical discount factor that supports cartel stability).

In Tables 3.1, 3.2, and 3.3 it can be seen that the sense of instability of the cartel is uniform — as the probability of cartel detection and fine rate increase, the levels of critical discount factors increase but in a small proportion and uniformly. However, the first affected are the cartels that tend to have homogeneous products. This result is contrary to what

[5]See Huamani and Braga (2022).

Table 3.1: Simulation of the effect of antitrust enforcement with a minimum fine rate $\bar{f} = 0.01$ on cartel stability at different levels of product quality differentiation.

β	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.00	0.8379	0.9237	0.9546	0.9705	0.9802	0.9868	0.9915	0.9950	0.9978
0.02	0.8551	0.9426	0.9741	0.9904	-	-	-	-	-
0.04	0.8729	0.9623	0.9944	-	-	-	-	-	-
0.06	0.8915	0.9827	-	-	-	-	-	-	-
0.08	0.9109	-	-	-	-	-	-	-	-
0.10	0.9312	-	-	-	-	-	-	-	-

Table 3.2: Simulation of the effect of antitrust enforcement with a average fine rate $\bar{f} = 0.105$ on cartel stability at different levels of product quality differentiation.

β	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.00	0.8379	0.9237	0.9546	0.9705	0.9802	0.9868	0.9915	0.9950	0.9978
0.02	0.8554	0.9428	0.9742	0.9904	-	-	-	-	-
0.04	0.8736	0.9626	0.9946	-	-	-	-	-	-
0.06	0.8925	0.9832	-	-	-	-	-	-	-
0.08	0.9123	-	-	-	-	-	-	-	-
0.10	0.9330	-	-	-	-	-	-	-	-

Huamani and Braga (2022) found for the case of cartels with a collusive strategy of price-fixing in a market with horizontally differentiated products: antitrust enforcement affects negatively and with greater intensity the cartel stability when the products are highly differentiated. Therefore, the intensity of the effect of antitrust enforcement on cartel stability depends on the type of product differentiation (horizontal or vertical).

Table 3.3: Simulation of the effect of antitrust enforcement with a maximum fine rate $\bar{f} = 0.20$ on cartel stability at different levels of product quality differentiation.

β	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
0.00	0.8379	0.9237	0.9546	0.9705	0.9802	0.9868	0.9915	0.9950	0.9978
0.02	0.8557	0.9429	0.9743	0.9905	-	-	-	-	-
0.04	0.8742	0.9629	0.9948	-	-	-	-	-	-
0.06	0.8935	0.9837	-	-	-	-	-	-	-
0.08	0.9137	-	-	-	-	-	-	-	-
0.10	0.9347	-	-	-	-	-	-	-	-

3.7 Conclusion

This paper examines three aspects of a cartel made up of a duopoly with a collusive price-fixing strategy in a market with vertically differentiated products. On the one hand, we show that cartel stability is quadratic over product quality differentiation interval—i.e., if products are highly differentiated and tend to be homogeneous, then the cartel is less stable. In addition, we find that the cartel stability depends more on the firm that produces low-quality products because it is more likely to destabilize the cartel — cooperative profitability is greater than competitive profitability when products are moderately differentiated. On the other hand, we show that antitrust enforcement negatively affects cartel stability but does not strongly affect a specific product differentiation degree — i.e., antitrust enforcement negatively affects cartel stability uniformly.

This topic can help as a guide to antitrust policy-makers and antitrust authority activity: (i) focus the investigation on potential cartels according to the product quality differentiation degree — intensify investigation of potential cartels with moderately differentiated products and punish with more severe fines because their cooperative profitability significantly exceeds their competitive profitability, (ii) encourage the cartel member firm more sensitive (sensitive to destabilizing the cartel) to denounce their anti-competitive activity.

Concluding Remarks

This thesis presented three essays on cartels' stability under antitrust policies. The first essay describes in general terms the cartels' stability under different antitrust policy regimes. The second and third essays focus on analyzing the effect of antitrust enforcement on cartels' stability at different levels of product differentiation, horizontally and vertically, respectively.

The first essay explains in a general and simple way the effect of antitrust policies — antitrust enforcement without leniency program, ex-ante leniency program, and ex-post leniency program—on cartels stability. We also conclude that antitrust policies negatively affect the cartels' stability — regardless of the collusive strategy adopted by the cartels. Furthermore, we show that there are no optimal antitrust enforcement parameters without a leniency program to destabilize cartels. On the other hand, leniency policies —ex-ante and ex-post— as instruments to combat cartels have a greater effect when total immunity from fines is guaranteed to the firm that signs the leniency agreement.

Our versatile approach allows us to insert factors intrinsic to the market structure (degree of product differentiation, differentiation of production costs, volatility of the demand system, and among others) in the modeled scenarios — cartel stability under antitrust enforcement without leniency program, ex-ante leniency program, and ex-post leniency program. Incorporating these factors is appropriate to observe that factors intrinsic to the market structure can mitigate the effect of antitrust policies. Therefore, our approach allows us to extend the analysis of cartel stability, and as a consequence, it can help make the best decisions when reformulating the parameters of sanctions for cartel members according to the cartelized market.

The second essay aimed to analyze the effect of antitrust enforcement on cartels at different levels of horizontal product differentiation. The question to be answered was: is antitrust enforcement more effective against cartels when products are horizontally more homogeneous or differentiated? Using Cournot's (Bertrand's) horizontally differentiated duopoly model, we conclude that antitrust enforcement is more effective against cartels when products are more differentiated. This result is invariant by the collusive strategy adopted by the cartel (collusive strategy of quantity-fixing and price-fixing).

The third essay aimed to examine the effect of antitrust enforcement on cartels at different levels of product quality differentiation. Essentially, we answered the question: is antitrust enforcement more effective against cartels when products are vertically more homogeneous or differentiated? Based on Bertrand's vertically differentiated duopoly model, we conclude: antitrust enforcement uniformly negatively affects cartel stability — there is no strong impact on cartel stability with a specific degree of product quality differentiation.

The literature has results on how to destabilize price-fixing cartels through the implementation of antitrust policies. The antitrust authority is also known to use the same antitrust parameters to destabilize all cartels. However, the effects of antitrust policies on cartels' stability at different levels of product differentiation are unknown. Our second and third essays conclude that more specific antitrust policies are needed to destabilize cartels with moderately differentiated products: cartels with moderately differentiated products (horizontally and vertically) under antitrust enforcement are the most stable.

The results report how general antitrust enforcement affects the cartels' stability in different industries — i.e., the same antitrust enforcement parameters for all types of cartels. For instance, antitrust authorities should focus their investigation on more stable potential cartels and could establish harsher penalties — more stable cartels enjoy greater cooperative profitability relative to competitive profitability. Therefore, our second and third essays indicate that more specific antitrust policy parameters are needed so that there is no disparity in cartel destabilization across different industries and different levels of product differentiation. Ideally, antitrust parameters should be reformulated to combat cartels according to the type of industry and the level of product differentiation. Thus, the results of our essays suggest that members of more stable cartels should be fined more severely because they enjoy excessive cooperative profit relative to competitive profit. Therefore, we recommend that antitrust authorities focus their investigation on potential price-fixing cartels that have moderately differentiated products (horizontally and vertically) because they enjoy greater stability. On the other hand, it is also recommended to focus the investigation on potential quantity-fixing cartels that have products (horizontally differentiated) that tend to be homogeneous because they also enjoy greater stability.

Finally, the versatility of our first essay to analyze cartel stability allowed us to examine cartel stability under an internal (degree of product differentiation) and external (antitrust enforcement) factor in the second and third essays. Thus, our approach allows us to examine the cartels' stability under internal and external factors simultaneously. For example, in future work, it would be interesting to analyze cartel stability under antitrust enforcement at different levels of elasticity of demand. Essentially, the effect of antitrust enforcement on cartel stability at different levels of elasticity of demand.

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