WILLIAN JORGE PEREIRA OLIVEIRA

LAST MILE DELIVERY WITH LOCKERS, FORMULATIONS AND HEURISTICS

Dissertation submitted to the Computer Science Graduate Program of the Universidade Federal de Viçosa in partial fulfillment of the requirements for the degree of *Magister Scientiae*.

Adviser: André Gustavo dos Santos

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I dedicate this work to my wife who has always been with me, for her support, understanding and for the peace she brings me.

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ABSTRACT

OLIVEIRA, Willian Jorge Pereira, M.Sc., Universidade Federal de Viçosa, June, 2022. **Last Mile Delivery with Lockers, Formulations and Heuristics**. Adviser: André Gustavo dos Santos.

The demand for delivering goods to individuals has increased due to several factors such as urbanization, the growth of e-commerce, and the popularization of the internet. There are several challenges to deal and in the face of this, there is a constant search for new alternatives. In this scenario, among other solutions, lockers emerged, which are cabinets equipped with a self-collection interface, strategically positioned to serve customers asynchronously. In this work, we approach different formulations of the last mile delivery problem with lockers, considering the size of parcels and compartments and the probabilistic characteristic of the problem due to uncertainty about users accepting to use the locker. In all formulations, the objective is to decide the location of lockers and vehicle routes in order to minimize the cost of last-mile delivery. For this, we propose heuristics and mathematical models to deal with different problems, we create instances based on real data, and we perform several computational tests. We evaluated different scenarios and estimate how much the use of lockers can generate savings. In the probabilistic problem, we were able to simulate different scenarios, where all users accept or reject the use of lockers. Doing this, we were able to measure the savings that the use of lockers can generate and, thus, make decisions about the investment to be made in the construction and promotion of the use of lockers.

Keywords: Combinatorial optimization. Vehicle routing. Facility location. Location Routing Problem. Last mile delivery. Parcel Lockers.

RESUMO

OLIVEIRA, Willian Jorge Pereira, M.Sc., Universidade Federal de Viçosa, junho de 2022. **Modelagem e Heurísticas para Problema de Última Milha com Utilização de Lockers**. Orientador: André Gustavo dos Santos.

A demanda por entrega de mercadorias para pessoas físicas tem aumentado devido a vários fatores como a urbanização, o crescimento do e-commerce e a popularização da internet e, diante disso, existe uma busca constante por novas alternativas. Neste cenário, dentre outras soluções, surgiram os lockers, que são armários equipados com uma interface de auto coleta, posicionados estrategicamente para atender os clientes assincronamente. Neste trabalho, abordamos diferentes formulações do problema de entrega de última milha com lockers, considerando o tamanho das encomendas e dos compartimentos e a característica probabilistica do problema devido a incerteza quanto aos usuários aceitarem utilizar o locker. Em todas as formulações, o objetivo é decidir a localização dos lockers e as rotas dos veículos de forma a minimizar o custo da entrega de última milha. Para isto, propomos heurísticas e modelos matemáticos para lidar com os diferentes problemas, criamos instâncias baseadas em dados reais, e efetuamos diversos testes computacionais. Avaliamos diferentes cenários e vimos o quanto o uso de lockers pode gerar economias. No problema probabilístico, conseguimos simular diferentes cenários, onde todos usuários aceitam ou rejeitam a utilização dos lockers. Dessa forma, pudemos mensurar a economia que a utilização pode gerar e, assim, tomar decisões acerca do investimento a ser feito na construção e fomento do uso de lockers.

Palavras-chave: Otimização Combinatória. Roteamento de Veículos. Localização de Facilidades. Problemas de Localização-Roteamento. Entrega de Última Milha. Lockers.

LIST OF ILLUSTRATIONS

LIST OF TABLES

SUMMARY

1. INTRODUCTION

Several factors have contributed to the increase in online shopping in recent years, for example, the popularization of the internet and global urbanization.

The world population is projected to reach 8.5 billion in 2030, with approximately 60% living in cities (Zhang, 2008). This crescent urbanization, combined with online shopping, one of the most popular online activities (Statista, 2020), increases the demand for delivery to final customers.

As a result of the demand growth, we can observe some problems like traffic congestion and harmful gas emission.

In the race to deliver faster and more efficiently than competitors, companies investigate and invest in new alternatives to optimize delivery performance.

To analyze deliveries, we can separate the last mile delivery and study it separately. The last mile step consists in delivery goods from the last endpoint to the final customer. This step represents the most expensive part of the delivery (Mangiaracina et al., 2019) with several challenges to be overcome such as the absence of the user at the time of delivery, the small dimension of orders, increased traffic and, consequently, increased pollution, and the large destination dispersion (Macioszek, 2017).

Given this scenario, parcel lockers emerge as an alternative to overcome these challenges. The use of lockers in the delivery process is studied in Wen and Li (2016), Faugere and Montreuil (2016), Rohmer and Gendron (2020), and Iannaccone et al. (2021), among others.

This work contributes to this topic with three articles, related as depicted in Figure 1.1. In the first we propose an integer linear programming model to locate lockers and route deliveries and a VND based heuristic to handle larger instances. In the second we added the size and quantity of slots in the lockers to approximate the problem to the reality and propose a multi-start heuristic to deal with this new scenario. Finally, in the third, we focus in the relevance of user's choice to use or not the lockers. For this last scenario we propose a stochastic definition of the problem and use instances based on real data.

1.1. Objectives

In this section, the general and specific objectives aimed throughout this work are presented.

1.1.1. Overall Objectives

The objective of this work is to model and develop heuristics to optimize the choice of locations for lockers and insert more realistic characteristics, such as order size and

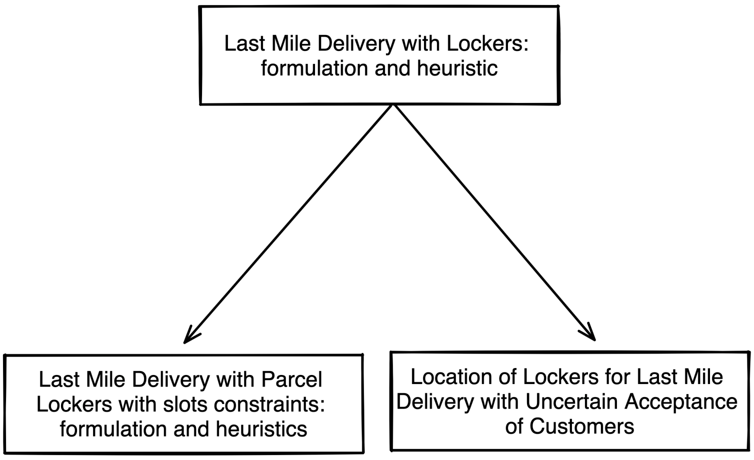


Figure 1.1: Relation between the articles

decision to use the locker by the user.

1.1.2. Specific Objectives

- Develop an integer programming model for the last mile step with lockers
- Develop heuristic to deal with lockers in the last mile step
- Create instances to represent real data to be used in the computational experiments
- Consider the size and quantity of slots in the lockers
- Study an stochastic version of the problem, where the choice of use are made by the customers

1.2. Dissertation Organization

This work is organized as follows: In the following chapters we present the articles produced during our research, attached in chapters 2, 3 and 3.8. Finally, in chapter 4 we discussed the results obtained during the research and propose future works.

The reference for the published paper is listed below:

- Oliveira, W. J. P. and dos Santos, A. G. (2020). Last mile delivery with lockers: Formulation and heuristic. In ICEIS (1), pages 460–467

2. LAST MILE DELIVERY WITH LOCKERS: FORMULATION AND HEURISTIC

2.1. Abstract

The creation of efficient routes is essential for different areas having several practical applications mainly in the transport of goods. With the growth of e-commerce and consequently the increase in demand for delivery to end users, minimizing costs in the delivery process has gained more importance, especially the last mile stage. It is in this context that the use of lockers emerges to optimize last mile deliveries. Lockers have compartments of different sizes, with self-service interface and they can be positioned in supermarkets, parks and other areas that are of interest to customers. The problem addressed in this work is to determine the positioning of the lockers and the necessary routes to supply them and to serve the remaining customers. We present a mathematical model to define the problem, but due to the complexity of the problem obtaining a solution can be very expensive and require a lot of computational effort, therefore we present a heuristic, based on Variable Neighborhood Descent (VND), using a greedy construct inspired by the Clark & Wright savings method. By comparing the results of the heuristic with the Gurobi optimizer, we conclude that the heuristic is capable of obtaining competitive solutions in less time than the exact methods.

2.2. Introduction

The creation of routes and facilities allocation have been the focus of studies for a long time due to its applicability in different areas. Several characteristics can be explored in these problems, such as the homogeneity of the vehicle fleet to be used, delivery time constraints, among others. The growth of e-commerce in recent years and, consequently, the demand for delivery to customers, turned the efficient routes planning an increasingly important problem. Due to the worldwide urbanization trend, traffic problems have increased. There are several challenges to deal with the delivery of goods, among them the last mile problems have gained more attention, mainly due to the traffic of large centers.

The last mile step consists of distributing products from the distribution center to final consumers. This step may have different characteristics from the rest of the delivery process and that is why it is necessary to study it separately.

The problem addressed in this work assumes the inclusion of lockers as an alternative to reduce costs in last mile deliveries. Lockers are kiosks with compartments accessed by self-service interface that can be used to store the goods of customers until they can collect them, reducing completely the fail to delivering caused by the absence of the customer. The model proposed defines the problem mathematically and a heuristic is suggested to obtain satisfactory solutions to the problem. In the next section we detail the use of lockers with real-world examples and related works in the literature.

2.3. The problem and its importance

There are several challenges in last mile deliveries. Among them, we can highlight the increasing in congestion in large centers, the increasing in pollution, the demand of the consumer in relation to services, in addition to restrictions, such as the time window available for delivery. The use of lockers has been shown as an alternative to deal with such challenges due to the flexibility of the collection and supply schedule. The use of several shifts of the same vehicle can help to reduce the size of the fleet. Song et al. (2009) shows that the use of lockers generates savings in the last mile stage, mainly due to the repeated deliveries that need to be made when the consumer is not at the address to receive your order.

Lockers are collection points, where goods are available for a certain period of time and users can collect them on an appropriate time. User interaction with the locker is done through a self-service interface that requires user authentication to ensure the security of orders.

Lockers can be located anywhere that facilitates their access, such as shopping malls, restaurants, gas stations and also in residential areas as long as there is demand from customers.

The installation of the locker in places that have parking, toilets, among other facilities, helps in popularizing the locker. It can also bring advantages to existing businesses close to the locker, as the circulation of customers can generate consumption and consequently profit for traders.

The location is the most important and challenging feature of the implementation of the self-collection, because, in addition to the investment in the construction of the locker, it is essential for its popularization. In this work, we consider that each locker has a coverage distance, that is, we consider that users with addresses in the coverage area would accept that their orders were delivered to the locker.

Wang et al. (2017) highlight the closure of several lockers in Singapore between 2015 and 2017, the main indication being that the location of the lockers did not favor its popularization. The Locker Alliance (LA) was proposed by the government of Singapore in 2019 and consists of a network of lockers distributed across residential areas and metro stations to help to optimize last mile delivery. Lyu and Teo (2019) propose methods to define the location of the network lockers and the demand to be aimed at them.

In door-to-door delivery, it is necessary to consider additional time for interaction with the consumer and the possibility that the consumer is not available to receive his goods. Building lockers, we can reduce operating costs due to the condensation of several deliveries at the same point, reducing the distance covered, eliminating the interaction time with the consumer and consequently reducing the delivery time. The supply of the lockers can also be done at periods with less traffic, using the same vehicle for different work shifts. With the decrease in delivery vehicles circulating during heavy traffic hours, we have a small reduction in the emission of gases harmful to the environment (Edwards et al., 2009).

In this work, the objective is to determine which lockers will be built, the routes to supply them and the routes to meet the demand of customers who have not been allocated to lockers.

We can consider this problem as a variation of the location-routing problem since it is necessary to determine the location of the lockers and create routes to meet the demands. Customers' demands can be met in two different ways: by door-to-door delivery or through the collection from lockers. Given a set of potential locations for building lockers and a set of customers with their respective demands, it is necessary to create two groups of routes: the delivery routes and supply routes, and the associations between customers and lockers. The built-in lockers must also be served through supply routes.

2.3.1. Related works

The last mile problem with the use of lockers is discussed by Wen and Li (2016). The paper presents a model using lockers in vehicle routing, considering relevant aspects such as CO_2 emission, customer time window and congestion. Actual data from Mingguangcun in Beijing is used to apply the proposed model. In this case study, it was possible to conclude that the use of lockers contributes to a better solution, because, relaxes the restriction of time window and reduces traffic congestion.

Faugere and Montreuil (2016) analyzed the business model of twelve companies using lockers and/or access points in order to identify different characteristics in their services. The data were obtained through the companies' websites and online press. The entities involved in the process, and the impacts generated by them, are observed: customers, salespeople, delivery personnel, cities, and the environment. The results consist of the classification of the business models used by the companies and their general characteristics.

Through the observed points, it was possible to conclude that the use of lockers generates a decrease in the delivery time. However, it is a good solution only if they are conveniently located for customers. For logistics providers, they may have a reduced fleet since delivery points are condensed in the lockers, also eliminating multiple trips due to the absence of the customer at the time of delivery. For sellers, the price of deliveries is expected to be reduced in the long term. Finally, reducing gas emissions and congestion is an advantageous consequence for cities and the environment.

Veenstra et al. (2018) proposed to integrate the facility location problem with the vehicle routing problem and lockers service. This integrated problem was applied to the delivery of medicines in the Netherlands. This problem differs from other classic problems due to the possibility of serving a customer in two ways, through the locker or with door-to-door service. A mathematical model and a heuristic are defined to solve the problem. The objective is to minimize total costs, considering the routes and the cost of opening the lockers. Computational tests were performed with two sets of instances: random and based on real data. The branch-and-bound method was applied to the proposed mathematical model, obtaining results for instances with 100 patients and 50 potential lockers with a time limit of 7200 seconds. The proposed heuristic achieves solutions that surpass the optimization software, CPLEX, with only a fraction of the time (4.18% better for the set of random instances and 3.64% better for the instances based on real data). The heuristic proved to be consistent in obtaining the same results considering 10 executions for each instance.

Wang et al. (2017) discussed relevant questions about the use of lockers. The paper uses Singapore's case as an example, where the lockers from POPStation, a leader in this market, are positioned at 2500m from each other. From 2015 to 2017 it was observed that several lockers were closed permanently. According to the authors, the location of the lockers is crucial for its popularization, so they must be positioned in an attractive way for the customer. The competition between various companies and their lockers should also be considered. The article also contributed to the literature because it was the first work related to lockers and your positions, which is based on real and public data. Real distances between locations were also used instead of the Euclidean distance, commonly used in the literature.

Huang et al. (2019) present the problem of vehicle routing using electric cars and stations. The stations can be of three types, the first one, only to recharge cars used on delivery of goods, the second one, only to store goods of customers, such as locker, the third one, a hybrid station, which can be used for both purposes. In the work, two integer programming models are proposed: in the first one, the routes for supplying the lockers and the routes for customer deliveries are separated; in the second one, a hybrid route is allowed. A hybrid heuristic is proposed which, compared to CPLEX, presents good results efficiently and effectively. The results obtained show that the use of lockers combined with door-to-door deliveries can help to reduce the cost of deliveries.

Relevant reasons are presented for calculating the routes separately. Firstly, it is not possible to measure the time that would be spent on each customer. Therefore, on a hybrid route, if a delivery to a customer is delayed, one is forced to delay the delivery to a locker, which will affect multiple users. Second, the hours of interest for deliveries are different: lockers may be located at points with congestion during business hours, so it is interesting to deliver at less busy times, while for door-to-door delivery, deliveries are usually during business hours. Third, the required skills of the driver are different depending on the type of delivery he/she will make: for door-to-door deliveries, the delivery person must have interpersonal skills while for the supply to lockers the driver must have availability in times less conventional.

In this paper, a variation is also proposed that allows for more work shifts. The same vehicle can be used during the day for door-to-door deliveries and, at night, for supplying lockers. In this variation, the vehicle may have more than one route associated with it and the fixed cost of the vehicle is considered only once. In all of the proposed models, the objective is to minimize the total costs including expenses with the construction of the stations and the fixed costs of the vehicles, considered heterogeneous. We do not consider the construction costs, as lockers are built only once and used for years. Considering these costs on a daily delivery would highly overestimate the cost. The instances used to test the methods were created by the authors by combining several instances of the literature. They consider electrical vehicles but characteristics about charging electric cars at stations are not considered, such as charging waiting time, the possibility of changing batteries or charging queue.

As part of The Federated Lockers and Collection Points program, the government of Singapore has proposed the creation of the Locker Alliance (LA) which consists of a network of lockers to complement existing lockers and to improve the performance of deliveries to consumers. Lyu and Teo (2019) aim to determine the best design of the locker network in order to increase its coverage and use. With the increasing in coverage, it is expected that the concentration of existing deliveries in the central business district (CBD) would be reduced by 7.5% due to the possibility of collecting items in locations away from the CBD.

Unlike the current work, the solutions obtained by them are evaluated on the perspective of the attractiveness of the use of lockers, that is, the objective is to optimize the location of the lockers so that the volume of users who choose this type of delivery is maximized. The data collected in the study are from a time when LA did not exist, and therefore, the work takes into account that after the implementation of the network, certain changes will occur in the choice of users. Using real data, a model was developed and calibrated to calculate whether the user would be subject to using the locker, thus, it was possible to measure the effectiveness of the network design concerning the popularity of locker. The supply and delivery routes of users who have not opted for the locker are not addressed in their work.

2.4. Integer programming formulation

We propose here an Integer Linear Programming formulation to formally define the problem.

As input we have the location of the depot, the customers and the candidate positions for lockers. We consider a complete graph, i.e., we know the distance between any pair of locations.

As output we have the the lockers chosen to be used, which customers are associated to each of those lockers, and the routes to delivery items to the lockers and to the customers not served by any locker. All the output is represented by binary variables, but we use also integer variable to keep track of the load in each vehicle, as a way to guarantee that the capacity of lockers and vehicles are satisfied and also avoid isolated sub-tours along the routes.

2.4.1. Input and Decision Variables

The parameters and the decision variables of the proposed model are presented below.

dep: depot
I: set of customers
I': set of nodes, $I' = \{dep\} \cup I$
J: set of candidate location for lockers
J': set of nodes, $J' = \{dep\} \cup J$
n: number of customers, $n = |I|$
m: number of locker candidate locations, $m = |J|$
q_i: demand of customer i
Q: capacity of vehicles for door-to-door delivery
QL: capacity of vehicles that supply lockers
qL_j capacity of locker j
x_{ij}: binary, 1 if arc (i, j) used, $i, j \in I'$, 0 otherwise
xL_{ij}: binary, 1 if arc (i, j) used, $i, j \in J'$, 0 otherwise
f_{ij}: integer, flow on arc (i, j) on customers' route
fL_{ij}: integer, flow on arc (i, j) on lockers' route
l_i: binary, 1 if customer i is served by a locker, 0 otherwise
a_{ij}: binary, 1 if i is served by locker j, 0 otherwise
c_j: binary, 1 if locker j is used, 0 otherwise
dI'_{ij}: distance between nodes i and j, $i, j \in I'$
dJ'_{ij}: distance between nodes i and j, $i, j \in J'$
dIJ_{ij}: distance between customer i and locker j
r_j: cover distance of locker j

2.4.2. Objective Function

The objective function is defined below.

$$\min \sum_{i\in I'} \sum_{j\in I'} x_{ij} * dI'_{ij} + (\sum_{i\in J'} \sum_{i\in J'} x_{ij} * dJ'_{ij}) * 0.8 \tag{2.1}$$

The aim is to minimize the total cost of the routes. We consider the cost proportional to the distance, then the cost of the customers' routes is simply the sum of the distances of the arcs used (first term of (2.1)). For the lockers supply routes (second term), we consider the cost as 80% of the normal cost, as these routes are favourable due to several conditions: higher flexibility in the period of attendance; no temporary absent and no need to return,, which may happen in the customers' route; possibility of night attendance, in period of no traffic congestion; possibility to reuse the same vehicle used in normal routes during the day, so as to reduce fixed costs of vehicles.

2.4.3. Constraints

$$(\sum_{j\in I'} x_{ij}) + l_i = 1, \forall i \in I \tag{2.2}$$

$$(\sum_{j\in I'} x_{ji}) + l_i = 1, \forall i \in I \tag{2.3}$$

$$\sum_{i\in I'} x_{i,dep} = \sum_{i\in I'} x_{dep,i} \tag{2.4}$$

$$(\sum_{j\in I'} f_{ji}) - (\sum_{j\in I'} f_{ij}) + (l_i * q_i) = q_i, \forall i \in I \tag{2.5}$$

$$f_{ij} \leq Q * x_{ij}, \forall i,j \in I' \tag{2.6}$$

$$\sum_{i\in I} a_{ij} * q_i \leq qL_j, \forall j \in J \tag{2.7}$$

$$\sum_{j\in J} a_{ij} = l_i, \forall i \in I \tag{2.8}$$

$$a_{ij} * dIJ_{ij} \leq r_j, \forall i \in I, j \in J \tag{2.9}$$

$$a_{ij} \leq c_j, \forall i \in I, j \in J \tag{2.10}$$

$$\sum_{i\in J'} xL_{ji} = c_j, \forall j \in J \tag{2.11}$$

$$\sum_{i \in J'} xL_{ij} = c_j, \forall j \in J \tag{2.12}$$

$$\sum_{j \in J'} xL_{j,dep} = \sum_{j \in J'} xL_{dep,j} \tag{2.13}$$

$$fL_{ij} \leq QL * xL_{ij}, \forall i, j \in J' \tag{2.14}$$

$$\sum_{i \in J'} fL_{ij} - \sum_{i \in J'} fL_{ji} = \sum_{k \in I} a_{kj} * q_k, \forall j \in J \tag{2.15}$$

$$x_{ij} \in \{0,1\}, \forall i, j \in I' \tag{2.16}$$

$$xL_{ij} \in \{0,1\}, \forall i, j \in J' \tag{2.17}$$

$$f_{ij} \geq 0, \forall i, j \in I' \tag{2.18}$$

$$fL_{ij} \geq 0, \forall i, j \in J' \tag{2.19}$$

$$l_i \in \{0,1\}, \forall i \in I \tag{2.20}$$

$$a_i \in \{0,1\}, \forall i \in I, j \in J \tag{2.21}$$

$$c_j \in \{0,1\}, \forall j \in J \tag{2.22}$$

Routes are modeled using flow variables to control nodes demand service and avoid sub-tours. There are two set of routes, one set for in-door service (constraints (2.2)-(2.6)) and one set for lockers supply (constraints (2.11)- (2.15)). The remaining constraints (2.7)- (2.10) define the assignment of customers to lockers and, finally, (2.16)-(2.22) the range of variables.

Constraint (2.2) and (2.3) define that must be a chosen arc leaving and respectively reaching each customer or else the customer must be served by a locker. Together, they define that each customer must be served by a locker or by exactly one in-door route.

Constraint (2.4) balance the number of arcs leaving and reaching the depot, so that every route starting at the depot must come back to it. Notice that, although a customer may not be served by more than one route, the depot may be in as many routes as needed.

The demand of each customer is guaranteed by constraint (2.5), either by the flow of a route or by assigning it to a locker. The following two constraints establish that the capacity of vehicles may not be exceeded in any point of the route neither the capacity of a locker may be surpassed.

Constraints (2.8)-(2.10) assure that a customer may be assigned only to lockers nearby (within their covering distance) and at most to one locker. If customer i is assigned to a locker j, the corresponding variables l_i (customer served by locker) and c_j (locker used) are set to 1.

Constraints (2.11)-(2.15) control the routes to serve lockers in the same way constraints (2.2)- (2.6) control the door-to-door routes. Particularly, (2.11) and (2.12) define that, if a locker is used, it must be served by a route (arc arriving and leaving the node); (2.13) balances the number of arcs leaving and reaching the depot (the number of routes to supply lockers); (2.14) assure that the capacity of the vehicle is respected and (2.15) that when visiting a locker the demand of all customers assigned to it must be unload there.

Finally, constraints (2.16)-(2.22) define the domain of the decision variables.

2.5. VND

As detailed later in the experiments, optimal solutions using the proposed formulation are reached only for small instances, with up to 75 customers. Heuristic method is needed to handle larger instances.

The proposed heuristic follows the standard Variable Neighborhood Descent (VND) metaheuristic as proposed by Mladenović and Hansen (1997), using a local search with three neighborhood. To provide an initial solution, a random greedy constructive method was used. The construction is followed by a local search and the two steps are performed 200 times, after which the best solution found is returned.

A solution consists of two parts, a set of routes and a set of assignments between lockers and customers. Each route can be of delivery or supply.

An initial solution is generated by a greedy randomized solution: a subset of candidate lockers is chosen at random, and the customers within the coverage distance of these lockers are sorted by distance to the depot. The further away the customer the sooner it is assigned to a locker.

Customers may end up not assigned to any locker, some because they are not within the coverage radius of any locker and others because when they are considered the possible lockers cannot serve their demand anymore due to previously assignments. Routes are built to serve those customers using the Clare and Wright saving heuristic (Clarke and Wright, 1964), satisfying vehicle capacities. The lockers are served by separated routes, created with the same method.

The routes of the constructed solution is then improved by a local search using three different neighborhoods: 2-swap, 2-opt and exchange. They are used sequentially, as in the VND metaheuristic.

The 2-swap neighborhood consists in swapping two customers inside a route. A best improvement strategy is used in this stage: the best swap is chosen and performed until there is no possible improvement.

For the 2-opt neighborhood, two nodes are randomly chosen and the visit sequence between those nodes is reversed. Successive improvements are made until a stop criteria is reached. We used as stop condition 500 iteration without improvement.

The third and last neighborhood is the exchange. For each node a, the nearest node b in a different route is determined. If moving node a to the route of b just before or after b improves the cost, it is considered for exchange. Then, the best improvement is made. The process continues until no improvement can be made.

The algorithm of VND-based heuristic can be seen below:

```
for(int i=0;i<ITERATIONS;i++){
  s = constructiveMethod();
  int k = 1;
  while(k<3){
    s' = bestOfNeighborhood(k);
    if (f(s') < f(s)){
      s = s';
      k = 0;
    }else{
      k++;
    }
  }
  if (f(s) < f(s*)){
    s* = s;
  }
}
return s*;
```

2.6. Instances

We adapt a set of classic instances from the Vehicle Routing Problem (VRP) (Rochat and Taillard, 1995). We chose 8 instances, with 75, 100 and 150 customers, and adapt them by inserting 10 candidates for lockers. The position of these candidates were manually chosen in areas with many customers. The capacity of each locker was defined randomly in the range of 10 to 20 times the average of customers demand, so a locker may serve around 10 to 20 customers.

Table 2.1 lists some characteristics of each instance: the number of customers (n) and the capacity of the vehicles (Q). We do not impose a limit on the capacity of the vehicles that supply lockers (QL), as the capacity of lockers in the instances used already limit a reasonable capacity.

Table 2.1: Instances

ID	#Customers	#Vehicle Capacity
c_tai75a	75	1445
c_tai75b	75	1679
c_tai75c	75	1122
c_tai75d	75	1699
c_tai100a	100	1409
c_tai100b	100	1842
c_tai100c	100	2043
c_tai150a	150	1544

2.7. Results

In this section, we present the results of the experiments obtained with the ILP formulation and the proposed heuristic. We compare their results and discuss the impact of the use of lockers.

The machine used was an i5-7400 CPU @ 3.00GHz with 8GB RAM. The Integer Linear Programming (ILP) model was implemented in Julia, using JuMP as modelling language, and run on Gurobi solver, with time limit of 1 hour. The heuristic was implemented in C++ and run 3 times for each instance. The average value is reported for analysis.

The results of the ILP model are reported in Table 2.2: solution value, runtime and the linear gap reported by Gurobi. Optimal results were found for only 3 instances, among those with less customers. However, the difficult is not related only to the number of customers. Instance `c_tai75a` has the same number of customers and lockers but the optimal solution was not guaranteed in the time available, finishing with a gap of 3.6%. This may be due to the relative location of lockers and customers. For example, a customer may be located in an area covered by two or more lockers, so may be served by any of them either by a door-to-door route. The higher the alternatives the greater the solution space, which may increase the running time to reach an optimal solution.

Table 2.3 show the results of the proposed heuristic. Column gap^{ILP} represents the percentual difference between the solution of the heuristic and the one reached by the ILP in 1 hour. One may notice that the heuristic found the same result for 4 instances. Column gap^{LB} show the percentual difference to the lowew bound found the ILP model, which is the best known still possible result. Besides the two optimal solutions, for three

Table 2.2: Results of the ILP formulation

Instance	Solution	Runtime(s)	gap
c_tai75a	803.613	3600	3.63
c_tai75b	537.113	25	-
c_tai75c	804.118	194	-
c_tai75d	420.707	55	-
c_tai100a	971.408	3600	3.13
c_tai100b	982.592	3600	4.93
c_tai100c	870.561	3600	2.59
c_tai150a	1097.150	3600	16.01

other the gap was less than 4%.

The time to reach the optimal solution is substantially less in comparison to the ILP model, and for the others the gap was less than 5%.

Table 2.3: Results of the heuristic

Instance	VND	time(s)	gap^{ILP}	gap^{LB}
c_tai75a	803.613	9	-	3.63
c_tai75b	537.113	6	-	-
c_tai75c	813.119	9	1.11	1.11
c_tai75d	420.707	6	-	-
c_tai100a	971.408	22	-	3.13
c_tai100b	1002.820	19	2.02	6.85
c_tai100c	908.177	17	4.14	6.62
c_tai150a	1136.280	54	3.44	18.91

In Figure 2.1 we can see the comparison between the lower bound and the two approaches proposed in this work. The results obtained by ILP and VND are close for most instances, increasing for larger instances, however, as previously mentioned, the time spent by VND to obtain solutions is significantly less.

The instances used in this work, proposed by Rochat and Taillard (1995), are commonly used for the VRP and some of its variations. Table 2.4 show on column VRP the best known results for the classic VRP, which may be used to study and foresee the saving that the use of lockers can bring. Solutions using lockers save in average 52% of the transportation costs. The real saving is not that amount, of course, because we are considering that routes to supply locker are 20% cheaper and we are not considering the costs of building and maintaining the lockers. However, the saving is clear, as the fixed costs of locker are spread over years and the costs of the table are a daily cost.

Table 2.5 shows, for each instance, the number of customers that are served by lockers in the best solution found by each method. One might intuitively think that the optimal solution would include the maximum possible number of customers to lockers. However, this is not always the case, as shown by the results for the instance

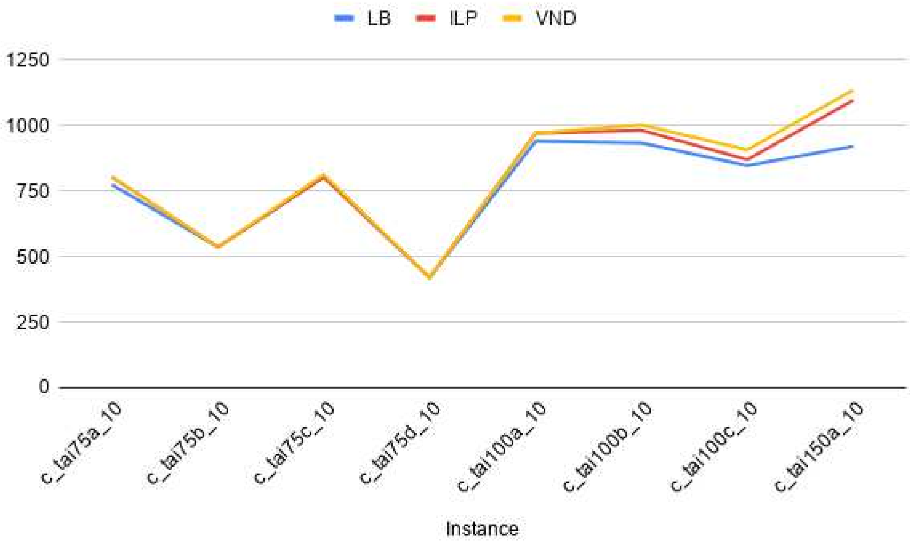


Figure 2.1: Comparison of the Lower Bound reported by CPLEX and the feasible solutions from ILP and VND for each instance

Table 2.4: Impact of the use of lockers

Instance	VRP	ILP	VND
c_tai75a	1618.36	803.613	803.613
c_tai75b	1344.64	537.113	537.113
c_tai75c	1291.01	804.118	813.119
c_tai75d	1365.42	420.707	420.707
c_tai100a	2041.34	971.408	971.408
c_tai100b	1940.61	982.592	1002.820
c_tai100c	1406.21	870.561	908.177
c_tai150a	3055.23	1097.150	1136.280

`c_tai100c`: the heuristic solution is 4% costly, despite having more customers associated to lockers. This happens because, in some scenarios, the cost to visit a locker may be higher than the cost to include the costumers associated to it in other route. Hence, even disregarding installation costs, lockers should not be build deliberately, as there would be still supply costs.

2.8. Conclusion

Due to the increasing popularity of e-commerce the last mile step in the delivery of goods has become more important and relevant in the delivery planning. The use of lockers has gained attention as an alternative to reduce costs, to optimize the delivery time and is also beneficial to the environment, as it contribute to reduce gas emission.

In this work we study methods for a vehicle routing combined with a facility problem for delivering goods using lockers. To define and solve the problem we propose an integer linear programming model, implemented and solved by Gurobi. Due to the

Table 2.5: Customers served by lockers

Instance	Customers	ILP	VND
c_tai75a	75	62	62
c_tai75b	75	69	69
c_tai75c	75	58	57
c_tai75d	75	72	72
c_tai100a	100	82	82
c_tai100b	100	83	83
c_tai100c	100	59	61
c_tai150a	150	125	121

complexity of the formulation, a heuristic is also proposed to handle larger instances. The heuristic is based on the VND metaheuristic and uses 3 neighborhoods to improve a greedy initial solution.

Experimental tests were made with 8 cases adapted from VRP classical instances. The ILP formulation found the optimal solution for three instances and a linear gap below 5% for other four instances within 1 hour of execution time. The heuristic found solutions close to the ones found by the ILP for most of the instances in few seconds.

We show the impact of the use of the lockers and as future works we plan to incorporate time windows and traffic conditions, as the problem in the real context has such characteristics. We also plan to improve the heuristic by adding more neighborhoods and use good solutions to instead of creating new random solutions on each iteration.

3. LAST MILE DELIVERY WITH PARCEL LOCKERS WITH SLOTS CONSTRAINTS: FORMULATION AND HEURISTICS

3.1. Abstract

In this work we consider a set of demands with different sizes that must be served by a fleet of vehicles either by home delivery or to a parcel locker for a later self-collection. The lockers has slots with different sizes and can serve customers located within a coverage area. We present a MILP formulation and a multi-start heuristic and show the results for experiments using instances of different sizes and characteristics. The heuristic has more difficult when a high percentage of customers can be served by lockers.

3.2. Introduction

Demand for efficient logistics solutions is consistently increasing as a consequence of increased sales with direct deliveries to the customer. Online shopping is among the most popular online activities in the world (Statista, 2020) and has experienced a high growth last year, due to the restrictions imposed on the movement of people. Alfonso et al. (2021) analyzed data in 18 countries in the period from September 1st, 2019 to May 30th, 2020. Their data show that 38% of consumers are expecting to shop online more frequently after the pandemic.

A natural consequence of the increase on online shopping is the need of efficient solutions on the supply chain delivery. In this work we focus on the last step of the delivery process, the so called last mile delivery, comprising the transportation of ordered parcels from the last intermediary hub to the final destination. Among the challenges of this step, we cite the absence of customer when the carrier arrives at the destination. To avoid a second visit, in some countries the parcel may be left besides the door, outside the customer's house, but this is subject to robbery or partial damage due to rain. Both problems can be eliminated if the customer allows the access to the own garage, which is another alternative explored by Amazon, but it requires installation of cameras for security purposes. A more popular alternative for customers that are frequently absent during deliveries is the use of parcel lockers. Parcel lockers are facilities where customers can collect their parcels through a self service interface. The demand of customers can then be met by conventional home delivery or by delivery to a locker for further costumers collection at their convenience.

Parcel lockers need to be allocated in order to maximize their use, so part of the decision process for choose a parcel locker is to analyze the demands of a particular region. Parcel lockers in regions with high traffic of people are interesting parcel lockers to choose. Parcel lockers can also increase the traffic of people in certain places, like malls, banks, squares and parks, which can benefit those stores nearby. However not all costumers may be served through lockers. First, because customers are not willing to collect parcels in a locker located far away. Second, the slots of the locker are not suitable for all sizes of parcels, i.e., larger volumes do not fit in the slots and have to be delivered at home. And third, lockers have a limited capacity and can serve a certain number of customers.

The problem we study here is a variation of the problem presented in a previous work (Oliveira and dos Santos, 2020), inspired by other works like Wen and Li (2016), which discussed the use of parcel lockers to improve last mile deliveries, and Wang et al. (2017), to our knowledge.

Given a set of customers with their respective demand and a set of candidate lockers, we have to define which parcel lockers to use, which customers to allocate to parcel lockers, the routes to supply the used parcel lockers and the routes to meet the demand of the remaining customers.

To the current work we add more characteristics to better represent real problems. The main contributions are: (i) we now consider lockers with slots with different sizes, adding a new level of decision, as not only the total weight or size of the demand must be within locker's capacity, but each demand must be allocated individually to a different slot; (ii) we use real travel distances instead of euclidean distance; (iii) we embed an exact method, the VRPSolver (Pessoa et al., 2020), to define the routes to supply the lockers and to visit customers not served by lockers.

We model the problem by a MILP formulation and propose a multi-start heuristic. Both methods are able to reach good quality solutions.

The remainder of the article is organized as follows: In section 3.3, we present the problem definition, our assumptions and overview of the relevant existing literature. In section 3.4, we define the integer programming formulation. In section 3.5, we describe the proposed heuristic method detailed. In section 3.6, we present the instances and discuss the results. Finally, section 3.7 concludes the paper.

3.3. Problem Definition and Assumptions

Given a set of parcel lockers, a set of customers with their respective demands, a matrix of distance from each point to other, the layout of parcel lockers, with the quantity of slots from the each size and the locker coverage it is necessary to determine the allocation of customers to parcel lockers and two groups of routes: the delivery routes and supply routes. The points include customers, parcel lockers and depot. Every tour starts and ends at the depot and there is an unlimited fleet of vehicles to be used.

The used parcel lockers must also be served through supply routes. The demand for parcel lockers corresponds to the sum of the demand from customers allocated to it.

We need to deliver in two types of vertices: parcel lockers and customers not allocated in parcel lockers. Following our previous work (Oliveira and dos Santos, 2020), we use separated routes because of practical advantages presented by Huang et al. (2019): the fleet of vehicles can be minimized due to several shifts of the same vehicle; the use of unconventional times can reduce the traffic congestion. On mixed routes, if home delivery has delay, it can delay parcel locker supply and affect several customers. We did not find strong evidence that motivates mixed routes.

To better represent the real world, our instances does not use the euclidean distance, we use real distance calculated with Open Source Routing Machine (OSRM, 2021), and routing costs are proportional to the distance travelled.

Each locker has a certain number of slots of small, medium and large sizes. The lockers have a coverage distance measured from the locker to the customer. If the distance the customer needs to travel is greater than the limit, he cannot receive his parcels in the parcel locker, but if within the limit the company is free to decide to deliver to the locker instead at home. In this work we consider a coverage distance of 3 km for the lockers based on Wang et al. (2017) slightly adjusted to our instances.

3.3.1. Related Works

Wen and Li (2016) used data from Mingguangcun in Beijing and discussed the use of parcel lockers to improve solutions to last mile delivery problem. The study proposed a vehicle routing optimization model with parcel lockers, moreover, aspects such as CO_2 emission, customer time window and congestion are considered. This study concluded that the use of parcel lockers can contribute to better solutions, due to relaxing the delivery time window and reducing traffic congestion.

Faugere and Montreuil (2016) collect data from twelve companies and analyzed the impacts of use parcel lockers to customers, salespeople, delivery personnel, cities and environment. Their results showed: decrease in the delivery time, because the parcel lockers avoid multiple deliveries due to absence of customer in delivery address; reduced fleet due the condensation of many customers in the parcel lockers; the price of deliveries is expected to be reduced in the long term and reducing gas emissions and congestion due to reduce fleet.

Wang et al. (2017) was the first work related to parcel lockers and your positions based on real and public data. This work studied the Singapore's case, with data from POPStation, a leader in this market. They observed that several lockers was close between 2015 and 2017, they argued that localization of parcel lockers is the mainly causes to this closes due to non-popularization of their use.

Huang et al. (2019) presented the problem of vehicle routing using electric cars and multi types stations. The stations can be used to store parcels, to recharge electric cars or to both purposes. They proposed two integer programming models, the first with separated routes and other without this restriction. Their results showed that parcel lockers can be a good alternative to improve the last mile delivery.

In addition, they presented practical advantages for considering the use of separate routes: the time spent in each customers cannot be determined precisely and affects the next deliveries, in the hybrid route, delay the delivery to parcel locker can affect many customers; routes to supply parcel lockers can be carried out during non-commercial hours, avoiding heavy traffic hours; the required profile of drivers is different, for to home deliveries are necessary interpersonal skills while to supply routes the driver must have availability in non-commercial hours.

Pick up points are staffed collection points, usually integrated with shops and limited to opening hours of shop. Rohmer and Gendron (2020) analyzed different business models, including the use of parcel lockers compared with home delivery and pick up points. With their analysis we can highlight the advantages of using parcel lockers instead of pick up points and home deliveries. Although parcel lockers have a high initial investment, the number of failed deliveries are close to none, besides they allow collecting and delivery at any time. Consequently they have a delivery cost less than home delivery.

Schwerdfeger and Boysen (2020) presented the problem with mobile lockers applied to last mile delivery. Mobile parcel lockers can attend demands of customers in different regions during the planing horizon. Their problem consists to define the stopovers of each mobile locker along the day and designate which customer will be served by which parcel locker. Among the restrictions about their problem, we can highlight the use of a maximum walking range, similar to coverage adopt in our work. Their results compare only fleet sizes of mobile and traditional stationary lockers.

Wang et al. (2020) also used mobile lockers, to improve the weaknesses of fixed parcel lockers in the last mile delivery. In this work, they name from *aggregation problem* the decision of assumes that a demand point you receive a mobile locker that will attend a set of neighbors demand points. They tested two schemes, with and without *aggregation problem*. Mobile locker operate similar to home delivery in the schema without *aggregation problem*. The results of two schemes showed that with use of *aggregation problem* reduce the delivery time and labor costs, the scheme without *aggregation problem* reduces the number of mobile lockers.

Lin et al. (2020) used a multinomial logit to model the customer's choice. They called *service level* the decreasing function relative to the walking distance. The objective of problem adopted for this work is maximize the service level. The problem was modeled as multi-ratio linear-fractional 0-1 program (MLFP) and two approaches to solving it were proposed. To solve small instances was proposed a MILP with McCormick inequalities and to large instances was proposed a Quadratic Transform with Linear Alternating. Although there are limitations, the work highlights the importance of customer's choice in the problem.

Grabenschweiger et al. (2021) introduce the vehicle routing problem with heterogeneous lockers boxes and an metaheuristic to resolve the proposed problem. In addition, different configurations of lockers were tested and concluded that configurations that use most of their space in large slots achieve, on average, better results. To motivate the users to use lockers a compensation value is paid to customers that accept to use lockers.

3.4. Integer Programming Formulation

The problem is formally defined as follows. There is a set I of customers to be served and a set J of parcel lockers. All customers must be served, either directly by a vehicle departing from the company or indirectly by having their parcel delivered to a nearby parcel locker. There are two types of vehicles, the ones visiting customers for home delivery and the ones that supply the parcel lockers. Both types have a limited capacity but there is no limit in the number of routes performed by them. All lockers are identical, all with a predefined number of slots of each of 3 sizes: small, medium and large. The objective is to minimize the delivery costs by assigning customers to parcel lockers and by defining routes to supply the parcel lockers and routes to serve customers not assigned to parcel lockers.

3.4.1. Input and Decision Variables

The input of the problem is comprised of:

- dep: the depot
- I: set of customers
- J: set of parcel lockers
- $I' = I \cup \{dep\}$: points visited by the vehicles for home delivery
- $J' = J \cup \{dep\}$: points visited by the vehicles that supply lockers
- d_{ij} distance between locations of i and j, for $i, j \in I' \cup J'$
- q_i demand of customer $i \in I$
- r maximum distance allowed from a customer to the designated locker
- Q: capacity of vehicles for home delivery
- Q^L: capacity of vehicles that supply lockers

- Q_s, Q_m, Q_l capacity of slots of size small, medium and large, respectively
- S_s, S_m, S_l number of slots of each size
- δ and λ: cost per unit of distance traveled by the home delivery and the parcel locker supply vehicles.

and the decision variables are the following:

- $x_{ij}, \forall i, j \in I'$: binary, 1 if arc (i, j) is used for home delivery, 0 otherwise
- $x^L_{ij}, \forall i, j \in J'$ binary, 1 if arc (i, j) is used for parcel locker supply, 0 otherwise
- f_{ij}: integer, flow on arc (i, j) on a delivery route
- f^L_{ij}: integer, flow on arc (i, j) on a supply route
- $a_{ij}, \forall i \in I, j \in J$: binary, 1 if customer i is assigned to parcel locker j, 0 otherwise
- $l_i, \forall i \in I$: binary, 1 if customer i is assigned to a parcel locker, 0 otherwise
- $c_j, \forall j \in J$: binary, 1 if parcel locker j is used, 0 otherwise

3.4.2. Objective Function

The objective is to minimize the overall delivery cost:

$$\min \ \delta \cdot \sum_{i \in I'} \sum_{j \in I'} d_{ij} x_{ij} + \lambda \cdot \sum_{i \in J'} \sum_{i \in J'} d_{ij} x^L_{ij} \tag{3.1}$$

The first term sums the total distance traveled by the vehicles in the delivery routes and the second the distance traveled in the supply routes. We consider the transportation costs proportional to the distance traveled, but the costs may be different for the two types of vehicles. In the experiments we use $\lambda < \delta$, due to various factors that make the supply routes more favourable in terms of costs: several deliveries in each point visited; no delay waiting the customer and no failure due to customers absence; possibility to use periods of the day without traffic congestion, like very early in the morning or very late in the night.

3.4.3. Constraints

The constraints assure the customer's service and that all transportation and storage capacities are respected.

Customer's service:

$$l_i + \sum_{j \in I'} x_{ji} = 1, \qquad \forall i \in I \tag{3.2}$$

$$l_i + \sum_{j \in I'} x_{ij} = 1, \qquad \forall i \in I \tag{3.3}$$

$$\sum_{j \in J} a_{ij} = l_i, \qquad \forall i \in I \tag{3.4}$$

$$a_{ij} = 0 \qquad \forall i \in I, \forall j | d_{ij} > r \tag{3.5}$$

Delivery routes:

$$\sum_{i \in I} x_{i,dep} = \sum_{i \in I} x_{dep,i} \tag{3.6}$$

$$\sum_{j \in I'} f_{ji} - \sum_{j \in I'} f_{ij} = q_i(1 - l_i), \qquad \forall i \in I \tag{3.7}$$

$$f_{ij} \leq Q x_{ij}, \qquad \forall i, j \in I' \tag{3.8}$$

Supply routes:

$$a_{ij} \leq c_j, \qquad \forall i \in I, j \in J \tag{3.9}$$

$$\sum_{j \in J'} x^L_{ji} = c_i, \qquad \forall i \in J \tag{3.10}$$

$$\sum_{j \in J'} x^L_{ij} = c_i, \qquad \forall i \in J \tag{3.11}$$

$$\sum_{i \in J} x^L_{i,dep} = \sum_{i \in J} x^L_{dep,i} \tag{3.12}$$

$$\sum_{j \in J'} f^L_{ji} - \sum_{j \in J'} f^L_{ij} = \sum_{k \in I} q_k a_{ki}, \qquad \forall i \in J \tag{3.13}$$

$$f^L_{ij} \leq Q^L x^L_{ij}, \qquad \forall i, j \in J' \tag{3.14}$$

Locker's storage:

$$\sum_{i | q_i \leq Q_s} a_{ij} \leq S_s + S_m + S_l, \qquad \forall j \in J \tag{3.15}$$

$$\sum_{i | Q_s < q_i \leq Q_l} a_{ij} \leq S_m + S_l, \qquad \forall j \in J \tag{3.16}$$

$$\sum_{i | Q_m < q_i \leq Q_l} a_{ij} \leq S_l, \qquad \forall j \in J \tag{3.17}$$

$$l_i = 0, \qquad \forall i \in I | q_i > Q_l \tag{3.18}$$

Variables domain:

$$x_{ij} \in \{0,1\}, f_{ij} \geq 0, \qquad \forall i,j \in I' \tag{3.19}$$

$$x^L_{ij} \in \{0,1\}, f^L_{ij} \geq 0, \qquad \forall i,j \in J' \tag{3.20}$$

$$l_i, c_j, a_{i,j} \in \{0,1\}, \qquad \forall i \in I, j \in J \tag{3.21}$$

Constraints (3.2)-(3.5) assure that all customers are served, either by a parcel locker or by a vehicle. In case of service by a vehicle, the route must arrive (3.2) and leave (3.3) the customers' location. In case of service by a parcel locker, the customer must be assigned to one of the parcel lockers (3.4), but not for the noes located further from the allowed distance (3.5).

Constraints (3.6)-(3.8) define the delivery routes and (3.9)-(3.14) the supply routes using flow variables to eliminate sub-tours and to assure the demand of each point visited. For the delivery routes: (3.6) assure the all routes finish where started, the depot; (3.7), together with (3.2) and (3.3), guarantee that customers not assigned to parcel lockers have their demand served by a vehicle; and (3.8) that the capacity of the vehicles is not exceeded anywhere in the route. For the supply routes, every parcel locker used by a customer (3.9) must be visited by the supply vehicles (3.10)-(3.11) and the vehicles must deliver the demand of all customers assigned to the parcel locker (3.13). Moreover, the vehicles of they supply routes must depart and come back to the depot (3.12) and their capacity may not be exceeded (3.14).

The set of constraints (3.15)-(3.18) controls the occupancy of the parcel lockers. If a demand fits in a small slot, it may be stored in any slot, small, medium or large (3.15). Demands larger than the size of a small slot may be stored in medium or larger slots (3.16), or only in larger slots if does not fit in medium slots (3.17). Demands larger than the large slot cannot be allocated to parcel lockers (3.18). The combination of these constraints and the fact that variables a and l are binary guarantees that each slot is used for at most one customer and each customer uses at must one parcel locker and one slot.

3.5. Proposed Heuristic

As detailed later in the experiments, the proposed formulation can handle small instances of the problem. As the size of the instance increases, state-of-the-art solvers fail to obtain optimal solutions or to establish good bounds. Therefore, we propose an heuristic approach, as described below.

The solution to the problem consists in: i) a set of parcel lockers to be used and the respective customers allocated to each of them; ii) two set of routes, one for home delivery, to serve customers not allocated to the lockers, and the other to supply the lockers with the demand of the allocated customers.

The proposed method is a multi-start heuristic, which consists of two stages repeated for t iterations. The first stage is responsible for the part of the solution, the lockers and the allocated customers are chosen by a constructive heuristic. The second completes the solution by constructing the set of routes for locker supply and home delivery. The best solution obtained after t iterations is returned.

The heuristic for the first stage is detailed in Algorithm 1. It is a greedy randomized constructive heuristic.

Algorithm 1 Allocate customers to lockers

```
$Input : I, J, d_{ij}, r, q_i$
$\hat{J} \leftarrow$ random subset of $J$
for each $j$ in $\hat{J}$ do
    $\hat{I} \leftarrow$ customers around $j$ that fits in $j$, i.e, $i \in I | d_{ij} \leq r \wedge q_i \leq Q_l$
    while $\hat{I} \neq \emptyset$ do
        roulette $\leftarrow$ build a greedy roulette for $\hat{I}$
        $i \leftarrow$ random selection from roulette
        remove $i$ from $\hat{I}$
        if there is free slot in $j$ for $q_i$ then allocate $i$ in $j$  remove $i$ from $I$
        end if
    end while
end for
return $\hat{J}$ and customers of each $j \in \hat{J}$
```

Firstly, a subset of parcel lockers is chosen, and one by one is filled with customers. A set of candidates is created on the parcel locker, each candidate representing a customer that can be served by the locker according to the size of the demand and the distance to the locker.

Given the set of candidates a roulette is created, where each candidate has a slice proportional to the distance to depot. This is the greedy criteria used in the first stage, the further away from the depot a customer is, the greater may be its impact on delivery routes, so that customer is prioritized for allocation. From the created roulette, a random customer is selected. As further customers have larger slices they are more likely to be selected.

A new check is performed because during the iterations the parcel locker can be filled completely and cannot serve the selected customer, or the available slots cannot accommodate the customer's parcel anymore. If it is possible the selected customer is allocated to the locker. The selected candidate is removed from the set of candidates, a new roulette is created and this step is repeated until the candidate set is empty.

After iterating through all chosen lockers, the method returns a partial solution that contains the chosen parcel lockers to be used and the customers allocated to them.

The second stage completes the solution by building the routes to supply the selected parcel lockers and the routes for home delivery to customers not allocated to lockers. Each set of routes can be defined by solving a *Capacitated Vehicle Routing Problem*. For the supply routes, the demand of each locker is the total demand of customers allocated to them. For both type of routes we use the demo provided in the VRPSolver package from Pessoa et al. (2020), adapting the code to read the matrix of driving distances instead of the standard Euclidean distance. The VRPSolver is a very efficient exact solver, but it must be used twice in each iteration, for each solution constructed in the first stage. Then, in the experiments, we limit each VRPSolver call to 2 minutes.

By default the modeling of VRPSolver use euclidean distance, we customize the data read to supports a distance matrix and changes to accept directed edges. The output file was also customized for easy reading and also to return the time spent by the optimizer.

Each iteration of the heuristic builds a complete feasible solution using the aforementioned two stages. The heuristic finishes after t iterations and returns the best solution found. In the experiments we used $t = 100$ for all instances.

3.6. Computational Experiments

The experiments were conducted on an i5-7400 CPU @3.00GHz machine with 8GB RAM. The Integer Linear Programming (ILP) model was implemented in C++, using Concert Technology and solved by CPLEX, with time limit of 1 hour. The heuristic was implemented in C++ and integrated with VRPSolver that use Julia language.

3.6.1. Instances

The instances contain the parcel lockers, the customers and their demand, the capacity of the home delivery vehicle, the capacity of the slots used (small, medium and large) and the distance matrix.

The distance matrix has the distance between all points of the instance: parcel lockers, customers and deposit. Instances can be represented by a directed graph, so that the distance from x to y may be different from y to x.

Instances have names with the following pattern c_slots_{n}c_{m}l where n is the number of customers and m the number of parcel lockers available. The number of customers ranges from 20 to 250 and of lockers from 3 to 20. For instances with the same dimensions a letter was added to the end to differentiate them.

Some instances were adapted from the previous work (Oliveira and dos Santos,

2020) including the 3 different slot's sizes.

For the new instances, the position of customers and parcel lockers were generated randomly inside a predefined polygon. The polygon used represents the city of Viçosa, Minas Gerais, Brazil. Some adjusts was made in position of parcel lockers to represent better the real world problem.

The parcel lockers of all instances are the same, they have 12 small, 6 medium and 4 large slots, a division inspired by the Amazon lockers. The capacity of the slots is 100, 200 and 350 respectively. To new instances the size of parcels is randomly generated within a range, described on Table 3.1 and vehicle capacity is generated randomly ranging from 5 to 10 times the average size of parcels.

Table 3.1: Range of parcel size from new instances.

Name	Min Size	Max Size
c_slots_20c_3l	50	500
c_slots_30c_3l	50	500
c_slots_150c_10l	30	500
c_slots_250c_20l	10	500

The demands of adapted instances remain the same, the lowest and highest parcel size is described in the Table 3.2. In column *%Suitable* we show the percentage of parcels that are smaller than the size of the largest slot, we can see that, although the largest parcel is much larger than the largest slot, on average 83% of the orders fit in the slots.

Table 3.2: Demands from adapted instances.

Name	Lowest	Highest	%Suitable
c_slots_75c_5l_a	3	1085	81.33
c_slots_75c_5l_b	3	1066	77.33
c_slots_75c_5l_c	3	1061	90.66
c_slots_100c_15l	3	1023	84.00
c_slots_100c_10l	3	1085	80.00
c_slots_150c_15l	3	1023	85.33

Table 3.3 shows the number of customers that can be served by just one locker, many lockers and that cannot be served by lockers. We can see that the instances have very different distributions. Some instances with low availability of using lockers and others where most customers have the possibility of using lockers and even with more than one locker available.

3.6.2. Results and Discussion

In this section, we present the results of the experiments obtained with the ILP formulation and analyze the components of the solution. We compare the results of the

Table 3.3: Availability lockers by customers.

Instance	Percentage of customers served by		
	no locker	just 1 locker	2 or more
c_slots_20c_3l	75	25	0
c_slots_30c_3l	67	30	3
c_slots_75c_5l_a	36	44	20
c_slots_75c_5l_b	44	36	20
c_slots_75c_5l_c	47	52	1
c_slots_100c_10l	38	45	17
c_slots_100c_15l	30	39	31
c_slots_150c_10l	59	21	20
c_slots_150c_15l	29	21	50
c_slots_250c_20l	40	24	36

proposed heuristic and ILP formulation and discuss the impacts of the percentage of customers served by lockers.

The results of the proposed model are reported in Table 3.4. The column *Total* shows the evaluation of the solution, which can be broken down into two values, shown in the *Home Delivery* and *Supply Routes* columns. The last columns show the runtime (limited to 1h) and the linear gap reported by Cplex.

In the experiments we use $\delta = 1$ and $\lambda = 0.8$ and for the weight costs respectively of the home delivery and the supply routes, due to the mentioned factors that make the supply routes more favorable. But the main reason for the total cost of home delivery be higher than those of the supply routes in all instances is the amount of points visited by the first set of routes, while the supply routes visit only the lockers.

Table 3.4: Results of the ILP formulation.

Instance	Total	Home Delivery	Supply Routes	Runtime(s)	gap
c_slots_20c_3l	67.672	64.24	3.432	62	0
c_slots_30c_3l	82.102	73.07	9.032	82	0
c_slots_75c_5l_a	129.944	111.80	18.144	3600	6.90
c_slots_75c_5l_b	128.924	119.54	9.384	3600	5.85
c_slots_75c_5l_c	164.060	140.70	23.360	3600	3.37
c_slots_100c_10l	226.786	213.01	13.776	3600	9.92
c_slots_100c_15l	176.734	139.47	37.264	3600	3.74
c_slots_150c_10l	350.114	320.37	29.744	3600	11.74
c_slots_150c_15l	211.042	173.85	37.192	3600	7.51
c_slots_250c_20l	526.342	470.47	55.872	3600	15.64

The optimal solution was found only for the two smallest instances within the time limit. The first instance contains 20 customers and the second 30 customers, both having 3 parcel lockers. For the remaining instances, the linear gap tends to grow as the number of customers and lockers grows reaching 15% in the largest instance tested.

Table 3.5 shows information about solutions found by the ILP and heuristic methods.

The columns *Total* show the evaluation of the solution returned by each method, the column *Customers* show the number of customers allocated to parcel lockers and the column *Lockers* show the number of parcel lockers used in the solution.

Table 3.5: Usage of parcel lockers.

	Exact			Heuristic		
Instance	Total	Customers	Lockers	Total	Customers	Lockers
c_slots_20c_3l	**67.672**	3	1	**67.672**	3	1
c_slots_30c_3l	**82.102**	8	2	**82.102**	8	2
c_slots_75c_5l_a	**129.944**	46	5	130.574	46	5
c_slots_75c_5l_b	128.924	29	3	**128.274**	30	3
c_slots_75c_5l_c	**164.060**	34	4	**164.060**	34	4
c_slots_100c_10l	226.786	35	5	**222.604**	35	4
c_slots_100c_15l	**176.734**	60	10	178.808	69	11
c_slots_150c_10l	350.114	53	10	**338.194**	53	10
c_slots_150c_15l	**211.042**	102	10	212.674	107	14
c_slots_250c_20l	526.342	113	19	**512.608**	114	18

Notice that the heuristic allocate customers to lockers whenever a chosen customer fits an available spot, without considering the costs of the supply routes. Despite the lower cost of supply routes that aggregate the delivery of several customers in one visit, minimizing the total delivery cost does not necessarily implies maximizing the number of customers assigned to parcel lockers. According to Table 3.5, comparing the solutions obtained, one can see that not always solutions with more customers using parcel lockers are better than solutions with less customers allocated. For example, for instances `c_slots_100c_15l` and `c_slots_150c_15l` the ILP solution allocate less customers to lockers than the heuristic solution (respectively 60 and 102 instead of 69 and 107) but has a better overall cost. On the other hand, solutions for instances `c_slots_250c_20l` and `c_slots_75c_5l_b` of the heuristic are better, with more customers allocated to lockers.

A general conclusion is that in all cases the best solutions use a smaller number of parcel lockers compared to the other solution. However, we can also observe that it is not enough to stop using all lockers to obtain the best solution, as for the smaller instances, the optimal solution uses at least one locker.

Analyzing this characteristic together with the previous one, we can conclude that although it is interesting to increase the number of customers, the number of parcel lockers used to allocate customers must also be observed. Lockers should not be used in all situations because there will still be supply costs.

Therefore, we conclude that it is not enough to analyze the two criteria separately. It is necessary to find solutions that manage to increase the number of customers not served by home delivery but also maintain an effective use of parcel lockers to serve them.

Table 3.6: Results of the heuristic.

Instance	Total	Runtime(s)	gap^{ILP}	gap^{LB}
c_slots_20c_3l	67.672*	689	0	0
c_slots_30c_3l	82.102*	139	0	0
c_slots_75c_5l_a	130.574	2778	0.48	7.34
c_slots_75c_5l_b	**128.274**	3113	-0.51	5.37
c_slots_75c_5l_c	*164.060*	3413	0	3.37
c_slots_100c_10l	**222.604**	3195	-1.88	8.22
c_slots_100c_15l	178.808	2631	1.16	4.85
c_slots_150c_10l	**338.194**	3115	-3.52	8.62
c_slots_150c_15l	212.674	1774	0.76	8.21
c_slots_250c_20l	**512.608**	3214	-2.68	13.37

The results of heuristic are detailed in the Table 3.6, where besides showing total cost of the solution and the runtime, also shows the gap of the total cost compared to two values reported by CPLEX, the value of the feasible solution (column *gap*^{ILP}) and the lower bound (column *gap*^{LB}). The heuristic method found better solutions than the ILP constrained to 1 hour for 4 instances, highlighted in bold in the first column. For the 2 smaller instances marked with * the heuristic also found the optimal solution. For one instance (highlighted in italic), the heuristic found a solution with the same value as the ILP, but the optmality of this solution is not known (gap$^{LB} > 0$). For the other 3 instances CPLEX found better solutions, despite that, the gap^{ILP} of the heuristic solutions is less than 1.2%.

Considering only instances for which the optimal solution was not found, the average gap^{LB} is 7.41%, i.e., compared to the lower bound found during the optimization of the ILP, the heuristic found a solution is only about 7% above a proved limit.

For the small instances our heuristic also found the optimal solution but spent a little more time than the exact method. For the others, the average execution time was 2206 seconds, most of it consumed by VRPSolver. However, temptations to substitute it by heuristics degraded the quality of the solutions.

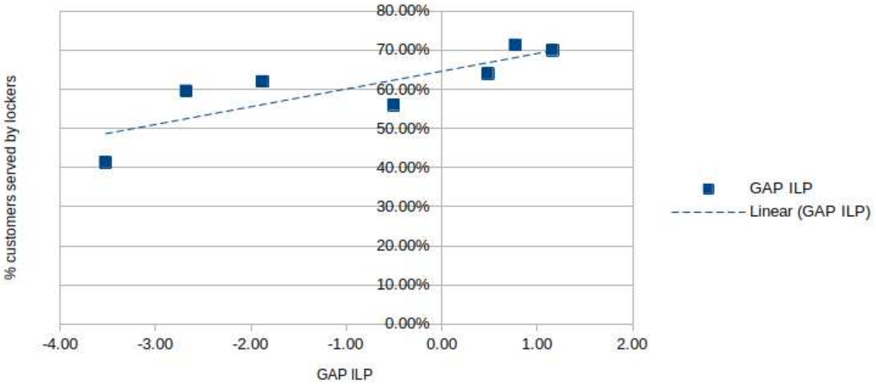


Figure 3.1: Comparison of gap^{ILP} and the percentage of customers served by lockers

In Figure 3.1 are present a comparison between the two approaches proposed in this work and the percent of customers that can be served by lockers per instance. The instances who have the same result in two approaches are not presented. Observing the trend line we can see that as the percentage of customers that can be served by lockers increases, the gap from the heuristic to the exact method also increases. Based on this observation, we can highlight that the heuristic presents difficulty in the allocation stage.

3.7. Conclusion

The problem of last mile delivery has become more relevant as a consequence of increase of sales with direct deliveries to the customer. Online shopping has increased in the last year and part of consumers tends to keep the frequency even after the pandemic.

In this work we proposed a multi-start heuristic to optimize the last mile delivery considering parcel lockers and the size of the parcels. It is an assignment-routing problem where some customers are assigned to lockers and the remaining are served by routes directly from the depot.

The heuristic is compared to solutions found solving a MILP formulation using instances of different sizes and characteristics, with real distances between locations, and has be shown competitive.

Although good results were found by both methods, the computational time is still to be improved. The heuristic spends around 1 hour for instances with 100 customers, mainly due to the exact method used to optimize the routes.

3.8. LOCATION OF LOCKERS FOR LAST MILE DELIVERY WITH UNCERTAIN ACCEPTANCE OF CUSTOMERS

3.8.1. Abstract

Given a sequence of D days, with n_d customers per day, and a set of m potential locations to build lockers, we considered the problem of Last Mile Delivery with Uncertain Acceptance of Customers such that the cost to deliver the goods to all customers in that period, through a locker or home delivery, is minimized. The problem addressed in this work is to determine the subgroup of locations where lockers should be built. The decision is taken considering historical data, deciding for each day of a long time horizon the routes to supply them and the routes to supply the remaining customers. Each customer has a predetermined probability to accept to use each locker. We propose a probabilistic hill-climbing heuristic and analyze how the users' acceptance influence the delivery costs.

3.8.2. Introduction

The growth of e-commerce (Deloison et al., 2020) in the last years, and prevision of the next years, increase the challenge to attain a cheap, reliable, and sustainable way to supply the customers.

This study focus on the last mile step of the delivery process. This step has an important role, which consists of the delivery of the goods from the last endpoint to the final customer.

The traditional way of delivery in the last mile use vehicles to transport the goods to the customer. The increase of vehicles to do these deliveries create many problems.

From the point of view of the environment, for example, delivery vehicles increase significantly the traffic in large centers and contribute much more than passenger cars to global gas emissions in the cities(Deloison et al., 2020).

The home delivery model has intrinsic problems, like the absence of customers at the moment of delivery. To deal with this, a second or even a third try of delivery is needed. Such increases the cost of delivery and consequently a loss of effectiveness. Some alternative approaches are used, like to deliver to a neighbor or leave in front of the door, but this cannot be done in many countries due to the loss of security.

Parcel lockers can be used as an alternative to home delivery in some cases. They are cabinets with compartments where customers can pick up their goods. With a self-service interface, they dispense human interaction and contain an authentication method to ensure security. Due to this autonomy, they can be located in different places and can be accessed at different hours of the day. They make it possible for the delivery person and the customer not to be at the same location at the same time and thus eliminate the need for retries to deliver goods. They are already widely used in more than 20 countries, including the UK, US, Canada, and some European countries(Deutsch and Golany, 2018).

There are many studies about the integration of parcel lockers into the last mile delivery ((Zurel et al., 2018), (Seghezzi et al., 2022), (Schwerdfeger and Boysen, 2020), (Yuen et al., 2019)) that state their contribution in the costs and effectiveness of the delivery while also contributing to a more environment friendly delivery, since several deliveries may be made in a single visit. However, most of the studies assume that the company decides which parcels go to the lockers and which ones are delivered at home. This work considers that the decision to use lockers is up to the customer. We do not assume that the customer will accept to use locker using only a subset of premises. There are always unknown reasons that the customers consider when deciding to accept, or not, to use a locker. We then consider that each customer has a given probability to accept to use each locker.

We believe that the aspect stochastic of the customers' willingness makes the problem closer to reality, despite not considering other characteristics such as time windows, installation, and maintenance costs. As can be seen in the computational experiments presented ahead, we hope that the analysis of the results can help to support decisions on the costs and necessity of building new lockers.

The goal of the proposed problem is to select the best locations to build lockers. To achieve this goal, we propose a heuristic that simulates and evaluates many scenarios, using historical data from deliveries provided by a start up company in Italy.

The remainder of the article is organized as follows: In subsection 3.8.3, we present the problem definition, the main characteristics and the evaluation method. In subsection 3.8.4 are presented a overview of the literature about the topic. In subsection 3.8.5, we describe the probabilistic hill-climbing heuristic detailed. In subsection 3.8.6 we describe the data used and the instances created. In subsection 3.8.7 we discuss the results. Finally, subsection 3.8.8 concludes the paper.

3.8.3. Problem Definition

Given a set M of potential locations to build lockers, the problem is to define p locations such that the expected cost to deliver goods is minimized. In order to evaluate the costs, we use historical data of customers for a period of D days: for each day d we have a set C_d of customers, and each customer i has a probability p_{ij} to accept to use locker j. Moreover, each locker has slots for up to W customers and there is a fleet of vehicles with a capacity of Q units for both home and locker delivery. For the routing cost, a matrix of distances d_{ij} between each entity of the problem (lockers, customers, and depot) is provided.

Note that we assume that the capacity of lockers and vehicles are measured in units of delivery, not in size or weight. We also assume that each customer has only one order to be delivered on a certain day. If a customer has more than one good to be delivered, these are unified as one single order. Moreover, if a customer has orders in different days, we consider as different customers, since the delivery on each day is independent. Thus, for this work, the number of customers and deliveries are equivalent.

To evaluate a single solution, we estimate the overall cost using historical data. For that, we simulate the delivery in the planning horizon firstly randomly choosing, for each customer, if the delivery is to be made on a locker or at home, according to the probability of acceptance. We then deliver to the lockers the goods of those who accepted and deliver the remaining ones to the customers' locations. The delivery costs are the routing cost, considering the distance travelled from the depot to lockers and customers and back to depot. There is no additional cost for customers who accepted to use the lockers, except the cost to supply the used locker. As the outcome is stochastic, several simulations are done for each solution in order to have a good estimate. A more precise algorithm is presented in subsection 3.8.5 when the overall method is presented. If the historical data provide information similar to the future orders, the choice of locations that minimizes the expected delivery cost for past orders is expected to be a good choice to serve the orders in a long planning horizon.

3.8.4. Related Works

In this subsection we present related works about the main concepts and characteristics of the problem: last mile delivery, parcel lockers and preference of customers. Moreover, we discuss the method used to solve the routing step of the problem.

Last mile step consists in the delivery to final customers in urban areas. Boysen et al. (2021) lists some reasons for the last mile's gain in importance: increasing demand, sustainability, costs, time pressure, and aging workforce.

Compared to 2020, a 78% increase globally in last-mile delivery demands is expected by 2030 (Deloison et al., 2020). This growth can be justified by a lot of factors, for example: The unprecedented increasing urbanization, arriving to 60% of the worldwide population living in cities; Customers more motivated to buy online, mainly, from their mobile devices; The rising of new categories online, like e-grocery, reaches more customers and, consequently, creates more demand for deliveries; Faster delivery needs, as same-day and instant delivery are the types of delivery with the fastest-growing, increasing 36% and 17% annually. Furthermore, it is expected that, in the next years, technologies like electric vehicles, machine learning, autonomous vehicles, drones, among others, become more and more explored to improve last mile transport.

Deloison et al. (2020) state that nowadays, cities are responsible for 70% of global emissions and delivery vehicles contribute much more than passenger cars to this, demanding urgent interventions to the last mile delivery. They also present an overview of 24 interventions to improve some aspects of last mile delivery. One of them is the use of parcel lockers, which can reduce the delivery costs by 2% to 12% and affect the traffic congestion by 5% to 18%, depending on the scenario.

We refer to Boysen et al. (2021) and Deloison et al. (2020) for comprehensive research about last mile step.

Deutsch and Golany (2018), Zurel et al. (2018), and Rohmer and Gendron (2020) highlight some advantages of using lockers, like a reduction in traffic congestion, reducing in failed deliveries, reducing the size of the fleet to cover a region, flexibility in collection hours, among others.

In the literature one can find different approaches to integrate parcel lockers in the last mile delivery. Deutsch and Golany (2018), for example, want to maximize the profit as a whole, even allowing the loss of customers, while Orenstein et al. (2019) consider that the delivery of goods may be delayed, although penalized in the solution's evaluation. More details of these and other works are presented below.

Deutsch and Golany (2018) intend to improve profit, calculated by the following aspects: the costs of delivery, maintenance, and installation of lockers, discounts in delivery costs for customers who collect your own goods, revenue with sales, and loss of potential customers who are not willing to collect their goods on lockers. In their work, not all demands need to be satisfied, but the loss of potential customers are penalized in the profit.

Orenstein et al. (2019) study a scenario where not all demands need to be met in the current shift. However, in this case, each customer has a penalty for failed delivery during the next shift. Such penalty can increase after each shift. Differently from the previous work, where a delivery may not be made at all (thus loosing a sale), in this case, this delivery will be made at some moment.

Most of the works does not handle the dynamic aspects of the problem, like a variation on the volume of deliveries and the variability of customers' willingness to accept to use lockers.

In order to address this question, Iannaccone et al. (2021) restrict the study to a specific audience and region. They focused their study on the young people, between 18 and 30 years old, from the city of Rome.

Through a survey applied, the results collected indicate that lockers have good acceptance, and some characteristics of lockers to make these more attractive are: short distance to home/work (less than 500m), 24h accessibility, and money incentive. They also conclude that some personal characteristics affect the chance of acceptance to use lockers. For example, customers that are graduated, who do not stay at home to receive goods, or who already use lockers before, are more inclined to accept to use lockers.

As a result, the work suggests three strategies to implements the use of lockers: a widespread locker network; a less-diffused lockers network, combined with compensation fees; focusing on specific groups, like students, people who lives alone, among others.

These results can not be generalized to all ages and cities. For future studies, they propose focusing on other generations and expanding to a representative sample.

Our work explores the aspect stochastic of the customers' willingness to accept use a locker. Gdowska et al. (2018) proposed a bi-level methodology to deal with the stochastic aspect of occasional couriers(OCs) bringing attention to the probabilistic characteristics that exist in decisions made in solving the last mile step. They consider a probability to represent the will of the OCs to accept perform a delivery to the final customer. The study uses random data to generate instances and the probability of OCs' acceptance. Their approach solves the total cost without crowdshipping in the first step, and gradually, the cost is reducing, due to the assignment between OCs and customers. The results analyzed are obtained from 25 random seeds and show a 9% reduction, compared to the initial cost, due to crowdshipped. In their case, a final customer is served by an OC with a given probability, and the remaining ones are served by vehicles from the company. In our case, a customer is served by a locker with a given probability. The remaining ones are served by vehicles of the company, which also are used to bring the parcels to lockers, for further collect by customers. In order to reduce the costs, our objective is to decide where is more convenient to build the lockers.

We then need to solve a Vehicle Routing Problem (VRP) to route home deliveries to the remaining customers, who do not accept to collect their goods in the locker and to lockers for those who accepts. For this task, we use the VRPSolver (Pessoa et al., 2020) an exact solver for VRP and other related problems by a Branch-Cut-and-Price (BCP) algorithm. The generic solver can reach better solutions than a specific algorithm for the majority of the experimented problems.

In summary, the work presented in this paper brings the last mile problem closer to reality by adding user acceptance to the use of lockers. Furthermore, we propose a heuristic to solve the problem. We expect that results obtained from the heuristic can support decisions about the demand to construct more lockers and of the viable amount to be invested to encourage the use of lockers.

3.8.5. Heuristic

Due to the probabilistic characteristic of the problem and the amount of data, exact methods cannot explore the search space at an acceptable time. Therefore, we propose a probabilistic hill-climbing heuristic to explore the solution space by simulating several scenarios, several times and, based on the information gathered from these simulations, return the best subset of locations found to build lockers.

First, we get an initial set of lockers that follows greedy criteria, as explained in subsection 3.8.5.

With the set of lockers, we simulate and get the solution value to this set. In the next steps, the algorithm tries to improve the solution, obtaining a neighbor candidate set of lockers and evaluating them.

The candidate is a neighbor from the best set found so far. The neighbor is obtained by applying movements that exchange the least used locker for another one chosen at random. If this candidate is better than the best set, then the best is replaced. The neighborhood used is explained better in subsection 3.8.5.

These steps are carried out until a stop criteria is reached (see subsection 3.8.5) and return the best set of locations to build lockers.

The algorithm 2 shows the overview of the heuristic, where the initial solution is evaluated and improved.

Algorithm 2 heuristic()

```
$s* \leftarrow setInitialLockers()$
$i \leftarrow 0$
while $i < k$ do
    $s \leftarrow getNeighbor(s^*)$
    $i \leftarrow i + 1$
    if $eval(s) < eval(s^*)$ then
        $s^* \leftarrow s$
        $i \leftarrow 0$
    end if
end while
return $s^*$
```

Evaluation a solution

To evaluate a set of lockers we simulate a horizon of D days. For each day, we allocate users to lockers using their probability of acceptance and planning delivery routes to remaining customers and used lockers. The A_d customer set is a subset of the C_d customers that have been allocated to lockers. The evaluation is done by simulation as shown in the Algorithm 3. The simulation is not a deterministic process, because due to the acceptance probabilities, we can have different values for each execution.

To obtain the evaluation of a subset of lockers we repeat the simulation $t = 10$ times and consider the average of the simulations as the evaluation value.

Algorithm 3 simulation()

```
$totalValue = 0$
for $d \in D$ do
    $A_d \leftarrow alocateCustomersToLockers(C_d)$
    $totalValue \leftarrow totalValue + routing(C_d \backslash A_d)$
end for
return $totalValue$
```

Each day is evaluated separately, then all costs are added and we have the cost for the simulation. The cost of each day is composed by the cost of delivery routes.

Customer allocation to lockers

Each day has a set of customers sorted in the order in which orders were placed. To each customer is offered the nearest locker that has space. The customer is assigned to the locker if he accepts, otherwise, he is added to a set of customers that will be served by home delivery. The algorithm is detailed in Algorithm 4.

Algorithm 4 alocateCustomersToLockers(C_d)

```
$remainingCustomers \leftarrow \{\}$
for $c \in C_d$ do
    $nearestLocker \leftarrow getNearestAvailableLocker(customer)$
    $hit \leftarrow rand(0, 100)$
    if $hit > p_{c,nearestLocker}$ then
        insert $customer$ in $remainingCustomers$
    end if
end for
return remainingCustomers
```

Route remaining customers and used lockers

The routing step aims to attend two sets of vertices: all customers that were not attended by lockers; and all lockers used to attend customers. However, for routing, these sets are undifferentiated and mixed routes are generated to serve them. A locker occupies q positions in the transport vehicle, as many as the q customers allocated to it.

The routing is made in two steps, a heuristic, and an exact method. First, we obtain an upper bound value through a saving-based heuristic, the Clark & Wright saving(Clarke and Wright, 1964). After, we used this upper bound to pass to the VRP-

Solver, a Branch-Cut-and-Price-based exact solver proposed in (Pessoa et al., 2020). VRPSolver is limited by 60 seconds. If it does not return a solution in the time limit, we use the upper bound as a result.

The overview of route step as showed in algorithm 5.

Algorithm 5 routing(vertices)

```
$upperBound \leftarrow routingHeuristic(vertices)$
$routingValue \leftarrow vrpSolver(upperBound, vertices)$
return routingValue
```

Initial Lockers

An initial set is chosen in a greedy way, considering the distance between each locker and the customers. The initial set is decided by Algorithm 6, that computes the average distance between each locker to all customers. The value of average distance is sorted and the p lockers with lowest average are chosen. In the results, we analyze the effectiveness of the greedy criteria used in this algorithm.

Algorithm 6 setInitialLockers(p)

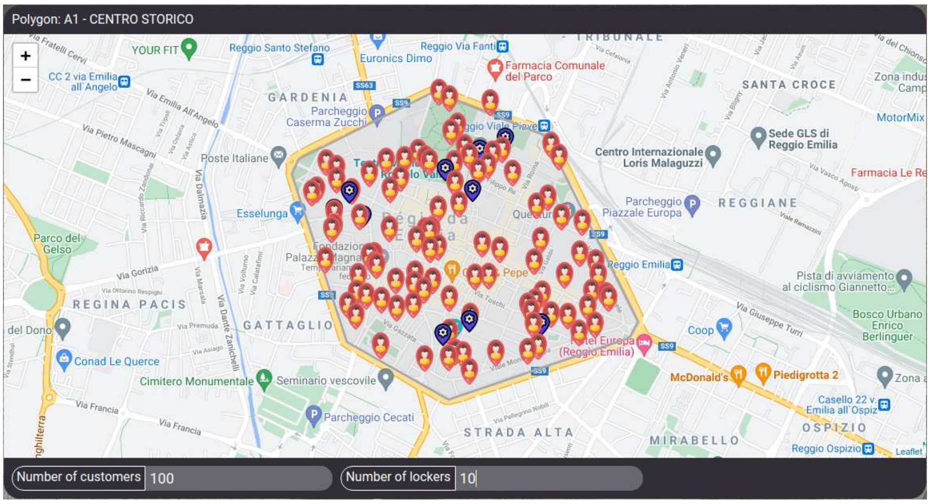
```
for $m \in M$ do
    $distance \leftarrow 0$
    for $d \in D$ do
        for $c \in C_d$ do
            $distance \leftarrow distance + d_{c,l}$
        end for
    end for
end for
return $p$ lockers with min $total\_distance$
```

Neighborhood

The algorithm consists in replacing the least used locker with another locker. In this way, there are $|M| - p$ neighbors, one for each non-used locker. A random locker is chosen to replace the less used locker. Not all neighbors are evaluated due to the time required to evaluate a solution. However, the same neighbor can be chosen more than one time, as the evaluation of a set of lockers can result in different results because of the probabilistic characteristic of the problem. Therefore the neighborhood is explored by random improvement. The algorithm used to obtain a neighbor set of lockers is shown in Algorithm 7.

Algorithm 7 getNeighbor(currentLocations)

```
$lessUsedLocation \leftarrow getLessUsedLocker()$
$neighboor \leftarrow currentLocations \setminus lessUsedLocation \cup getRandomLocation()$
return $neighboor$;
```



Number of customers 100

Number of lockers 10

Figure 3.2: One day region representation

Stop criteria

At each iteration, the heuristic checks a candidate subset of locations. If the candidate has a better evaluation than the current best subset, it replaces the best subset. Otherwise, it is discarded and a new candidate is chosen from the neighborhood of the best set.

The stopping criterion is a pre-defined k iterations without improvement.

3.8.6. Data used and instances created

Instances were created by the author based on realistic instances obtained from a start up company in Italy. The data provided contains information about customers location in 52 regions in Emilia-Romagna. Figure 3.2 shows one of the regions (gray area) with their customers (red marks) and potential locations to build lockers (blue marks).

The distances between locations were calculated using Haversine formula as in (Sinnott, 1984), that returns the shortest distance between two points, given their latitude and longitude values.

Based on these data, we selected 5 regions with different sizes and created one instance per region, and some instances combining up to three neighboring regions. Table 3.7 presents the characteristics of the instances used in the experiments. The first five use data of selected regions located in Emilia-Romagna province, F3, D8, B2,

Table 3.7: Characteristics of the instances used in the experiments

ID	Description	n	m	p	W_t
F3	Reggio - Canali	32	10	5	110
D8	Via Adua - Ospizio	75	10	5	110
B2	Zona Annonaria - Kennedy	85	10	5	110
A1	Centro Storico	100	10	5	110
C9	Buco del Signore	133	10	5	110
C9_F3	C9 + F3	165	20	10	220
A1_D8	A1 + D8	175	20	10	220
A1_B2	A1 + B2	185	20	10	220
C9_D8_F3	C9 + D8 + F3	240	30	15	330
A1_B2_D8	A1 + B2 + D8	260	30	15	330

A1 and C9. The next five combine data of the selected instances, grouping together regions geographically near. For each instance the table lists the number of customers per day (n), number of potential locations to build lockers (m), number of locations to be selected (p) and total number of locker slots (W_t). Notice that each region has $m = 10$ potential locations to choose to build $p = 5$ lockers, with different number n of customers. When combining regions, these values are summed and data are not considered individually, i.e., customers of one region may use lockers of other regions and a region may have more lockers than others, if that is profitable.

The probability of a customer to accept to use a locker is generated in the range 10% to 90%. Considering a coverage of β, if a costumer i is in the coverage distance of a locker j, the probability p_{ij} varies linearly from 10% to 90% for distances β to 0, respectively. If the distance between the customer and the locker is greater than β, the probability is $p_{ij} = 10\%$.

The storage capacity of each locker is $W = 22$ slots. Amazon lockers contain 44 slots, we use half the size to adapt to our instances. We do not differentiate goods by size, meaning they are all “standard” sizes and can be placed in any of the locker slots. The vehicles used to home delivery and supply routes has a capacity of $Q = 5$.

3.8.7. Experimental results

In this subsection, we explain the calibration of parameters and present the results of the proposed heuristic. In addition, we discuss two other scenarios and show how they can be used to support decisions.

The machine used was an i5-10400 CPU @ 2.90GHz with 16GB RAM. The heuristic was implemented in C++ integrated with Julia.

Parameter calibration

The calibration was performed using the data provided by the company, i.e., 52 instances considering only 1 day.

The only parameter of our heuristic is k, the number of iterations without improvement, used in the stop criteria. Recall that k is not limited to the number of neighbors $m - p$ due to the stochastic evaluation of a solution, that may return a different value for the same neighbor. To decide this value we conducted a statistic analysis with values $k = 1, 5, 10$. Values greater than 10 have not been tested due to the impact in the time to obtain a solution.

Table 3.8 lists the results found. For each region, we present the lowest cost obtained among all different values of k, and for each k, the ratio between its result and this lowest value. Figure 3.3 plots these results as an Empirical Cumulative Distribution Function (ECDF): in the x-axis the ratio and in the y-axis the percentage of instances with a ratio equal to or less than x.

Analyzing the results we can observe that $k = 10$ presents better results than the other values: the ratio of $k = 10$ is less than 1.4 for all instances and 1.0 to more than 50% of the instances. We can reject the null hypothesis that there is no significant differences between the results, through the Friedman test, with $p < 0.05$ reliability. Therefore we chose $k = 10$ in all the following experiments.

Heuristic results

As mentioned in the previous subsection, in order to execute computational experiments we use a set of created instances that extends the real data, grouping regions and generate data for $D = 5$ days (Table 3.7).

The proposed heuristic improves the solution iteratively, changing the subset of locations in each iteration using a neighborhood structure and simulating a scenario with lockers built in these locations. To verify the effectiveness of the heuristic we compare in Table 3.9 the evaluation of the initial subset generated by the greedy algorithm to the final subset found by the heuristic. For the smaller instances, there is no improvement at all, the heuristic does not find a better subset in the neighborhood of the initial subset. However, for larger instances, there is an improvement of about 30%.

Other scenarios

Here we test two different scenarios to study the influence of the use of lockers in the delivery cost. In the first, we increase the use by supposing that all customers wish to use the locker. In the second we consider that all lockers are built (thus, more customers are in the coverage distance of a locker, increasing the probability to use it.

Table 3.8: Ratio between results for each value k and the lowest one.

ID	Lowest	1	5	10
A1	4090	1.218	1.000	1.157
A2	28441	1.170	1.099	1.000
A3	135095	1.000	1.003	1.032
A4	326252	1.071	1.000	1.015
A5	337270	1.015	1.042	1.000
A6	391431	1.026	1.000	1.004
A7	176394	1.092	1.097	1.000
A8	546950	1.001	1.001	1.000
B1	130596	1.119	1.000	1.085
B2	11768	1.275	1.000	1.081
B3	151693	1.097	1.000	1.002
B4	65835	1.443	1.238	1.000
B6	16908	1.096	1.000	1.083
B8	234983	1.048	1.004	1.000
B9	8642	1.301	1.181	1.000
C1	347491	1.048	1.066	1.000
C2	145590	1.053	1.032	1.000
C4	173970	1.097	1.053	1.000
C5	101072	1.026	1.076	1.000
C6	171900	1.000	1.149	1.034
C7	57065	1.481	1.199	1.000
C8	52728	1.042	1.000	1.005
C9	45710	1.047	1.000	1.019
D2	107516	1.010	1.000	1.032
D3	28874	1.088	1.000	1.009
D4	67277	1.197	1.000	1.109
D5	300778	1.057	1.073	1.000
D6	129309	1.500	1.499	1.000
D7	242394	1.085	1.030	1.000
D8	8451	1.612	1.505	1.000
D9	79149	1.316	1.062	1.000
E7	107693	1.593	1.443	1.000
F1	17416	1.007	1.340	1.000
F2	11465	1.020	1.000	1.007
F3	18228	1.022	1.000	1.013
F4	73078	1.136	1.000	1.010
F5	43510	1.044	1.000	1.014
F7	37235	1.000	1.000	1.011
F8	28249	1.041	1.015	1.000
M1	288155	1.116	1.042	1.000
M2	103411	1.010	1.000	1.003
M3	176839	1.498	1.496	1.000
M4	166292	1.573	1.000	1.377
M5	145844	1.059	1.005	1.000
M6	156744	1.181	1.210	1.000
N1	220204	1.076	1.036	1.000
N2	43888	1.041	1.486	1.000
N3	91918	1.217	1.000	1.202
N4	68411	1.000	1.094	1.022
N5	118893	1.307	1.064	1.000
N6	158042	1.219	1.000	1.056
N8	51834	1.278	1.234	1.000

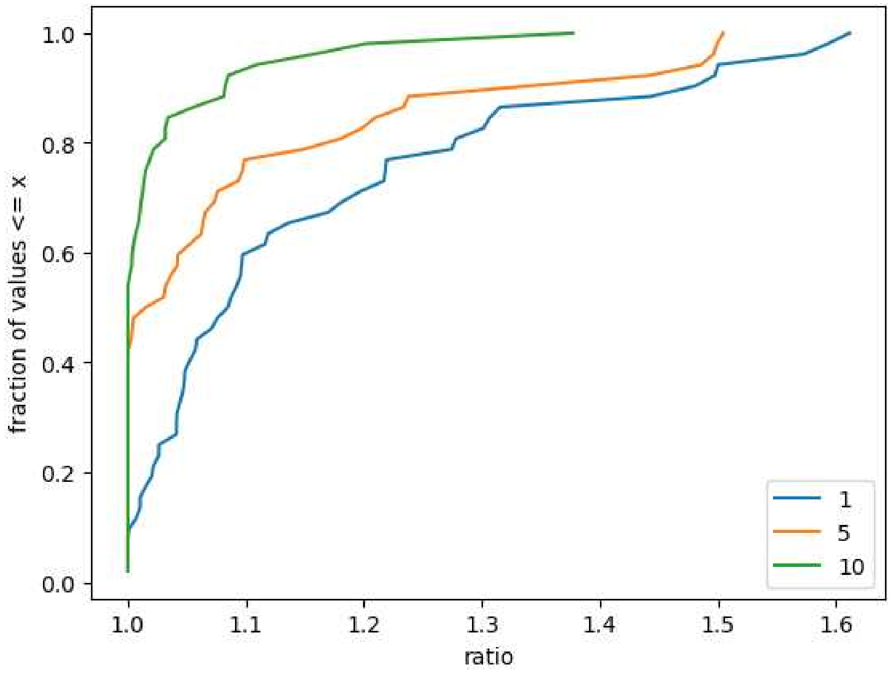


Figure 3.3: Empirical Cumulative Distribution Function for stop criteria $k = 1, 5, 10$ iterations without improvement

Table 3.9: Effectiveness of iteratively steps

	Cost		
ID	Initial subset	Heuristic	Improvement
F3	93670	93670	00.00%
D8	60239	60239	00.00%
B2	83076	83076	00.00%
A1	34847	27398	21.38%
C9	274625	212059	22.78%
C9_F3	435998	286612	34.26%
A1_D8	109627	70207	35.96%
A1_B2	117020	74518	36.32%
C9_D8_F3	498832	352125	29.41%
A1_B2_D8	231106	153041	33.78%

Table 3.10: Resultsfor the stochastic scenario vs a deterministic scenario where all customers wish to use the lockers

	% Customers using lockers		Cost		
ID	$p_{ij} \in [10, 90\%]$	$p_{ij} = 100\%$	$p_{ij} \in [10, 90\%]$	$p_{ij} = 100\%$	Savings
F3	87.50	100.00	93670	46924	49.90%
D8	90.13	100.00	60239	23485	61.01%
B2	85.18	100.00	83076	22015	73.50%
A1	86.60	100.00	27398	9380	65.76%
C9	81.65	82.71	212059	214652	-1.22%
C9_F3	88.48	100.00	286612	80490	71.92%
A1_D8	88.00	100.00	70207	27510	60.82%
A1_B2	90.49	100.00	74518	26100	64.97%
C9_D8_F3	85.08	100.00	352125	103120	70.71%
A1_B2_D8	85.85	100.00	153041	42580	72.18%

Table 3.10 compares the results of the proposed scenario (with p_{ij} ranging from 10 to 90%) with the results considering that all customers accepts to use lockers, i.e., $p_{ij} = 100\%$ for all i and j. In the later case, there is no home delivery in most of the instances, i.e. all deliveries are made to the lockers. The exception is in instance C9, where only ≈ 82 of customers use lockers. That happens because $W_t < n$ in this instance, i.e. the total capacity of the lockers is less than the number of customers, thus there is no slots available for all customers.

Comparing the results we can evidence a saving greater than 50% for the majority of instances when 100% of customers accept to use lockers. The instance C9 does not present a significant difference because the instance has a locker capacity under the number of customers and even in the stochastic scenario the full capacity of the locker was almost reached. The small negative difference may be due the heuristic solution of the routing part or by the order customers are allocated to the locker, which is not optimized in any case.

Table 3.11: Results for proposed instances, when $p = 5$ lockers are chosen for each $m = 10$ candidates, compared with $p = m$, i.e. all the lockers are build.

	Cost		
ID	Choose $p = m/2$	Choose all, $p = m$	Savings
F3	93670	92867	0.86%
D8	60239	46865	22.20%
B2	83076	44245	46.74%
A1	27398	17190	37.26%
C9	212059	80490	62.04%
C9_F3	286612	171144	40.29%
A1_D8	70207	64055	8.76%
A1_B2	74518	61435	17.56%
C9_D8_F3	352125	218009	38.09%
A1_B2_D8	153041	108300	29.23%

This saving may be viewed as the cost of customers' decisions and can be used to guide initiatives to encourage the use of lockers, like bonuses, advertising, and others.

From these results we can observe that the use of lockers strongly influences the cost: the more the lockers are used the greater is the saving. This reinforces the premise used to propose the greedy algorithm to obtain an initial set of locations to build lockers.

The table 3.11 shows the result if all lockers are built, that is, $p = m$.

Analyzing the results we can observe that region C9 presents the greatest saving, which is again a consequence of not having slots available for all customers for the standard case $p = 5$. When all lockers are build there is such capacity. Furthermore, it shows how much the C9 region is impaired by the excess of customers. Even when the region is mixed with other regions (C9_F3, C9_D8_F3) a great saving occurs when more lockers are used. This type of information is valuable to guide stakeholder decisions.

In all cases, the information of the total savings may guide the decision of the number of lockers to build. The heuristic may be executed with different values of p and the results can help do determine the acceptable value to build a new locker and where to build it. Here the savings are for $D = 5$ days, but a simulation for 1 year, for example, indicates how much the company may invest on a new locker to use in a long-horizon.

3.8.8. Conclusions

With the rise of e-commerce in recent years, it has become more important for companies to deliver goods efficiently and effectively in order to remain competitive in the marketplace.

Considering the last mile step, the use of lockers is one of the alternatives to deal with the challenges of delivery to end consumers. The locker solves the problem of the user’s absence at the time of delivery, it can reduce the size of the fleet of vehicles used in deliveries, and consequently generate a decrease in the emission of polluting gases.

In this work, we address the last-mile delivery problem using lockers with uncertain acceptance of customers. We focus on the relevance of the probability that the user has to accept using a locker to receive their parcels and on how this affects delivery costs.

To solve the problem, we propose a probabilistic hill-climbing heuristic, which, based on historical sales data from a region, determines the set of lockers to be built in order to minimize the delivery costs.

Experimental tests were performed using instances based on real data from the region of Reggio-Emilia in Italy.

Analyzes were made on how the probability of user acceptance affects delivery costs, this information can help in decision making about values to be invested in bonuses, advertisements, and other strategies that encourage users to use lockers.

Analysis was done on how the probability of user acceptance affects delivery costs, this information can help in making decisions about the amounts to be invested in bonuses, advertisements, and other strategies that encourage users to use lockers.

For future work, we intend to consider the probability of using lockers not only by their distance but in previous evidence obtained by historical data mining, for example, the profile of customers who use lockers based on types of products and specific customer data. It is also necessary to improve the computational time of the heuristic, mainly in the solution evaluation step, because the exact method used, although efficient, is used many times, so it had limited time in the experiments performed.

4. CONCLUSION

Several factors contribute to the increase in demand for deliveries to end consumers, among them we can highlight urbanization, globalization, and the popularization of e-commerce. In this way, research is increasingly needed to improve the performance of deliveries to deal with the problems generated by this increase.

In this work, we consider the last mile step, which is final part of the supply chain, responsible for deliver the parcels for final customers. We focus on the use of lockers as an alternative to address certain characteristics of the problem, for example, the user's absence at the time of delivery, the high level of dispersion of delivery locations and, consequently, other indirect problems, like traffic congestion and harmful gas emission.

The problems addressed in this work are the delivery of orders to a set of users, which consist of home delivery and supply routes, with the aim of reducing delivery costs. Given a set of possible locations, the choice of lockers to build is also part of the problem-solving process. The cost of building the lockers is not considered, as this value represents a long-term investment, while the cost of the routes considers only a limited horizon. We consider that all users need to receive their deliveries, either at home or through locker collection.

We propose an Integer Linear Programming formulation to define the problem, that can handle small instances. For larger instances, we proposed a multi-start heuristic.

We can see, by the results, that the problem has two relevant factors: the number of lockers built and the number of customers who use the lockers. Although it is interesting to increase the number of customers that use lockers, the number of lockers used to allocate customers should also be noted due to costs to build and supply the lockers, among other constraints. Therefore, it is necessary to analyze both aspects together.

The creation of instances that represent the problems addressed was a constant part of this work. In the first article instances from the literature were adapted, inserting in them unique characteristics of the problem.

To bring the problem addressed closer to reality, for the second article we considered the size of the slots of the lockers. Some instances of the first article were adapted, adding new features such as the size of the slots and other adaptations such as the distance, which is no longer Euclidean and becomes the real distance between two coordinates.

For this new problem, we propose another integer linear programming and to deal with larger instances a multi-start heuristic.

The last version of the problem addressed was based on the first problem and does not consider the size of the locker slots. In this version, we focus on the impact of user choice on problem resolution. In the two previous versions, the decision if a customer is assigned to a locker or served by home delivery is made by the model/heuristic, i.e., decided by the company. But in a real context this decision is up to customer. To bring the problem closer to reality, we propose a version in which the customer has a probability to accept to collect his order in a locker. In this way, the problem becomes stochastic, and to evaluate a solution we run a series of simulations to obtain its evaluation value.

For this last problem, we used instances based on real data provided by a start-up company in Italy. Based on the data received, we projected more days to build instances that had a broader horizon of days.

With this work we propose a MILP formulation for the problem of last mile deliveries using lockers, highlight the impact of lockers on costs and provide algorithms that can be used to support business decisions. We also emphasize that the user's decision is extremely relevant for lockers to be an efficient alternative in minimizing delivery costs.

For future works, we propose the union of the second and third problems defined in this work, i.e., considering both the size of the drawers and the probability of user acceptance. Another suggestion is to improve the evaluation of the last problem: instead of using a simulation, compute the expected cost based on the probabilities.

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