

CAMILA RAFAELA ALVARENGA

**THE EFFECTS OF BRAZILIAN GOVERNMENT POLICIES ON
HIGHER EDUCATION**

Dissertation submitted to the Federal University of Viçosa in partial fulfillment of the requirements of the Graduate Program in Applied Economics for the degree *Magister Scientiae*.

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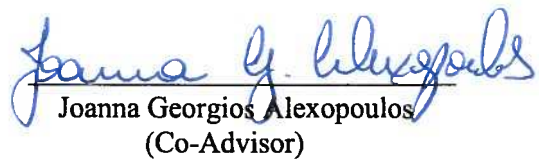
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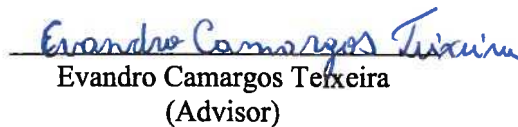
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BIOGRAPHY

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RESUMO

ALVARENGA, Camila Rafaela, M.Sc., Universidade Federal de Viçosa, fevereiro de 2016. **The Effects of Brazilian Government Policies on Higher Education.** Orientador: Evandro Camargos Teixeira. Coorientadora: Joanna Georgios Alexopoulos.

O objetivo principal deste trabalho é desenvolver um modelo de equilíbrio geral dinâmico e estocástico (DSGE) que explique como incentivos governamentais ao ensino superior impactam nas escolhas educacionais de jovens indivíduos. A obtenção de capital humano através da educação superior oferece a indivíduos pobres uma chance de sair da pobreza. No Brasil, incentivos na forma de empréstimos educacionais (FIES), bolsas de estudos (ProUni) e expansão de universidades públicas (REUNI) são alguns dos programas através dos quais o governo pode estimular a acumulação de capital humano. Entre outras coisas, os resultados analíticos indicam que pessoas que possuem recursos suficientes para ter sucesso nos processos seletivos frequentam uma universidade pública, e que indivíduos com restrições financeiras podem preferir trabalhar e poupar em detrimento dos estudos. A análise de equilíbrio parcial revela que o FIES propicia aos jovens sem recursos uma chance de frequentar uma universidade, assim como o faz o ProUni. Entretanto, a análise sugere que o REUNI é o programa mais eficaz na promoção de igualdade de oportunidade no sistema de ensino superior brasileiro.

ABSTRACT

ALVARENGA, Camila Rafaela, M.Sc., Universidade Federal de Viçosa, February, 2016. **The Effects of Brazilian Government Policies on Higher Education.** Advisor: Evandro Camargos Teixeira. Co-Advisor: Joanna Georgios Alexopoulos.

The main goal of this paper is to develop a dynamic stochastic general equilibrium (DSGE) model that explains how governmental incentives for higher education impact on educational choices of young individuals. Human capital attainment through higher education gives poverty-trapped individuals a chance to escape poverty. In Brazil, incentives in the form of educational loans (FIES), college scholarships (ProUni) and expansion of public universities (REUNI) are some of the paths through which the government may affect human capital accumulation. Among other things, the analytical results indicate that agents who have enough resources to succeed in the selection process will further their studies in the public system, and that financially constrained individuals may prefer working and saving in detriment of their studies. The partial equilibrium analysis reveals that the FIES fund allows that some financially constrained individuals be given a chance to attend university, as does the ProUni scholarship. However, the analysis suggests that REUNI is the most effective educational incentive in the sense that it favors equality of opportunity in the higher education system.

SUMMARY

1	Introduction	1
1.1	Initial Remarks	1
1.2	The Problem and its Importance	5
1.3	Objectives	6
1.3.1	General Objective	6
1.3.2	Specific Objectives	6
2	Theoretical Framework	7
2.1	General Equilibrium Theory and Dynamic Programming.....	7
2.2	Partial Equilibrium.....	11
2.3	The Theory of Search	11
2.3.1	Search Theory and Macroeconomics	12
2.3.2	Search Modeling	13
2.4	Human Capital and Modern Macroeconomics	15
3	The Model	20
3.1	The Basic Model.....	20
3.2	The Production Sector	21
3.3	An Overview of the Agents' Problems	22
3.4	The Adult's Problem.....	23
3.5	The Young Agent's Problem	23
3.6	The Government's Problem.....	24
3.7	Competitive Equilibrium	25
4	Results.....	27
4.1	Solution to the Adult's Problem	27
4.2	Solution to the Young Agents' Problems	29
4.2.1	Agent who chooses a public university ($e = 1$ and $n = 0$).....	29
4.2.2	Agent who chooses a private university ($e = 1$ and $n = 1$).....	30
4.2.3	Agent who chooses not to attend a university ($e = 0$ and $n = 0$).....	31
4.3	Implications	31
4.4	Partial equilibrium analysis	36
5	Concluding Remarks.....	42
	References.....	41

1 INTRODUCTION

1.1 Initial Remarks

Over the last decades, the Brazilian government, through its Ministry of Education (MEC), has designed and implemented incentive programs for higher education, aiming to expand access to universities, in a broad sense, and more specifically, to increase the stock of human capital in Brazil. In view of this goal, the incentives have been directed at young individuals belonging to lower social classes, both through the expansion of public universities (REUNI) and through financing (FIES) and subsidy (ProUni) of private institutions. In the long run, it is expected that these key initiatives lead to increase in economic growth, as well as decrease in wealth inequality. Caselli & Ciccone (2013) have found that investments in human capital through schooling stimulate output. Such results are consistent with the findings of Schultz (1960), Becker (1962), Denison (1985), Romer (1990), Barro (2001) and Becker (2009).

Mincer (1974) demonstrated empirically that investments in human capital through schooling and training result in skill gains, which in turn have a positive and significant effect on labor earnings. Still, it has been questioned whether direct financial incentives for higher education promote the inclusion of less privileged young individuals and the associated increase in human capital. Catani, Hey & Gilioli (2007), raised concerns over ProUni effectiveness as an educational program, seeing that it does

not effectively assist the beneficiaries throughout university. Léba & Mancebo (2009) stated that REUNI is provoking debilitation of the public system of graduate school. What is more, there is also the question of the impact that current human capital investments will have on economic development and growth in the future. As stated in Bourguignon, Ferreira & Leite (2003), “whether more human capital will reduce poverty in the future depends on what happens to the rates of return to it between now and then”.

In order to discuss the matter of formation of (and returns to) human capital in the context of higher education, it is necessary to understand the basic differences between public and private university¹ in Brazil. The more notable distinction is, certainly, the cost – while public institutions of higher education do not require payment of a tuition for attendance, private universities do, which often makes them inaccessible to poor individuals. The rich are also favored in the selection process of public universities in Brazil – in most cases, the ENEM² exam is used to rank candidates, and it comes as no surprise that the people who had the resources to go to private school in the childhood and to take ENEM preparatory courses are the ones who get selected. Araujo pointed that:

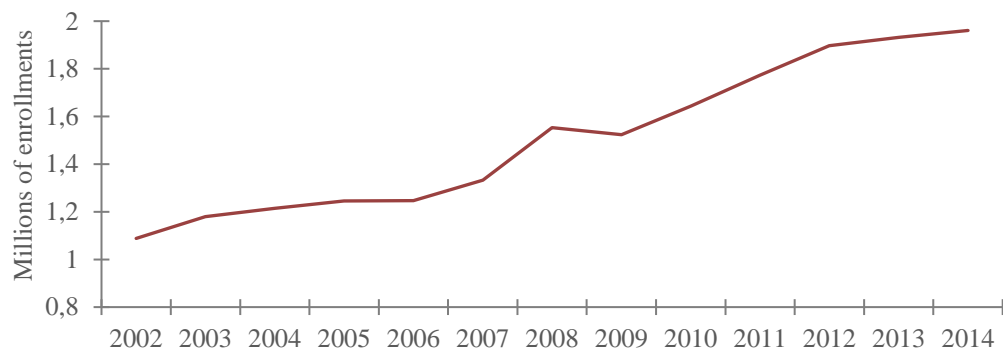
[...] the vast majority of the population could not afford to prepare for the entrance exams and were therefore banned from the good free-of-charge public universities. It was in this context that the country experienced unprecedented college privatization, still in progress today. Until the 1960s, public colleges accounted for more than half of all IHEs [institutions of higher education] in the country. In 1990, according to INEP (2008), out of the 918 IHEs listed, 222 were public and 696, private. (ARAUJO, 2012)

It is crucial to note that, in general, public universities are preferable to private universities in Brazil. Not only are public institutions “free-of-charge”, they are also associated with a higher quality of education, and are responsible for 90% of the scientific production in the country (HILU; GISI, 2011).

¹ For simplicity, we will use the term *university* to refer to all institutions that provide higher education. But as noted by Macedo et al. (2005), the private system of higher education in Brazil is mostly constituted by small, isolated establishments of education.

² ENEM is a test taken by high school graduates, from both public and private systems, used by higher education institutions to rank candidates, and by the government to select the beneficiaries of higher education incentives.

But how do Brazilian government's key programs for higher education work? Firstly, there is REUNI, a program intended to restructure and expand Brazilian federal universities. MEC states that REUNI aims to raise the number of openings in the public system, by expanding courses, reducing dropout rates, and occupying vacant positions. Furthermore, REUNI intends to restructure the academic environment, as well as renew higher education pedagogically, and improve quality of education, among other goals. The expansion of federal institutions was initiated in the year of 2003, with the inauguration of campuses on smaller cities across the country. As reported by MEC, until 2013 there were created 14 new universities and over 100 new campuses. Additionally, the number of enrolled students in public universities has reached 1.9 million, in contrast to 1.1 million in 2002. Figure 1 illustrates the increase in the enrollment rate:



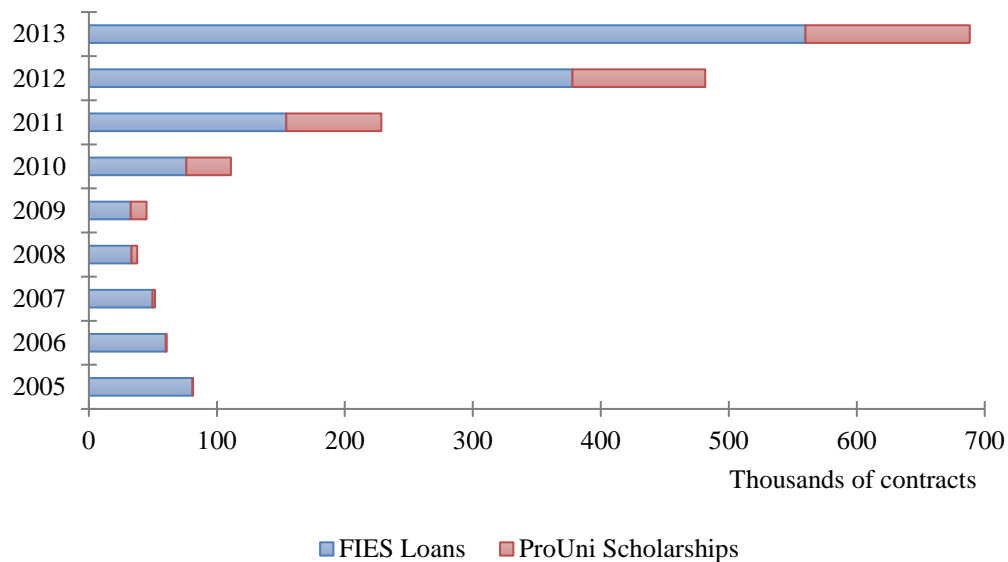
Source: MEC

Figure 1: Number of enrolled students in public university in Brazil, from 2002 to 2014.

The University for All Program, or ProUni, was institutionalized in the year of 2005, with the purpose of expanding access to higher education for financially constrained individuals. If accepted into the program, the student is awarded with a full or a partial scholarship in a private university (MACULAN; RIBEIRO; HADDAD, 2006). According to MEC, a candidate is eligible for a ProUni scholarship if she obtains the minimum of 450 out of 1000 points on the National High School Exam (ENEM), although the score may need to be higher, depending on the institution, the course, and the competition. Furthermore, the candidate must prove that her family does not have the means to afford a private university's tuition. In that interest, the monthly family income has to be less than 3 times the minimum wage, per family member. Moreover, the individual has to satisfy at least one of the following conditions: complete high school in a public school; complete high school in a private school with a full

scholarship; attend part of high school in a public school, and part in a private school with a full scholarship; have a disability; be a teacher in elementary school.

Another program designed by MEC to incentive higher education in Brazil is FIES, a student finance fund that makes loans to private university attendees, or candidates, at lower rates than those practiced by commercial banks – MEC stipulates 3.4% per year in 2015. Such as ProUni, FIES’ goal is to reach less privileged high school graduates, which is why eligibility is conditional to a set of prerequisites relating to family income. The Brazilian government, through FIES, finances from 50 up to 100% of tuition costs, and the beneficiary has to start paying her debt 18 months after graduation, given that she completes the course. The program was institutionalized in 1992, but it went through several changes over the years. Nowadays, it is mandatory that the candidate take the ENEM exam, and the attainment of a low score may be disqualifying – a recent change in legislation determines that the minimum score must be 450, the same as ProUni. While the selection criteria for FIES have become more rigorous, the offer of loans has increased substantially in the last few years. Figure 2 depicts the trajectory of both FIES and ProUni over the last decade:



Source: MEC

Figure 2: Private university incentive, in number of contracts, from 2005 to 2013.

As illustrated by Figure 2, the number of educational loans has increased over 6 times since 2005, reaching almost 560 thousand contracts in 2013. The number of ProUni scholarships has also augmented significantly – in 2005, 475 scholarships were offered, in contrast with the 128 thousand contracts in the year of 2013. Those 688

thousand beneficiaries of private university incentives in 2013 represent almost 13% of enrollments in the Brazilian private university system that year³.

While both programs – FIES and ProUni – aim to give financial aid to young individuals who cannot pay for private university, they differ on their forms of incentive, as well as on their selection process. If all private institutions' candidates could choose, unconstrained, which of these programs they would participate in, they would surely choose ProUni, because it does not require acquirement of debt, as opposed to FIES. The specification of 450 as the minimum score on ENEM, along with the income constraint requirements, are the main tools used by the government to select which candidates will benefit from each program. If, as instance, a candidate who scores 500 on ENEM does not get a ProUni scholarship because competition was high, she is still eligible for FIES. If she meets the other requirements, she may be contemplated with a loan.

1.2 The Problem and its Importance

The recent debate around economic growth and development has attracted attention to distributional issues (GALOR, ZEIRA; 1993), and one of the ways through which these issues manifest themselves is human capital formation and returns. Kochar (2008) stated that “the level of schooling in an economy affects the relative wages of skilled and unskilled labor, and educational choices in turn reflect the returns to schooling”. Therefore, it is important to understand how agents make their choices when they're young.

That being the case, this paper aims to contribute to the existing literature on DSGE models and the role of governmental policy on human capital formation. More than that, it aims to build on the debate on financial and structural incentives to higher education in Brazil. Are they effective for human capital accumulation? Do they allow the mobility of financially constrained individuals?

In terms of modeling, we intend to build a model economy where young individuals make educational choices, given their financial constraints, and investigate how these choices impact on human capital formation. In particular, we intend to apply my model to the case of Brazil, in order to evaluate the effectiveness of higher education incentive programs, such as ProUni, FIES, and REUNI. While there are

³ In 2013, there were registered in Brazil 5,373,450 enrollments in private universities.

several other works that make investigations on human capital accumulation for other countries, such as Schultz (1960), Mincer (1974), Becker (2009) and Kochar (2008), this is, to the best of our knowledge, the first theoretical model built to explain the trade-offs between different governmental policies for human capital formation.

The paper is organized as follows. After presenting the objectives of the research, we develop the theoretical framework in chapter 2, which is divided in a technical section (in which we put forward the methods for building and solving a DSGE model of the sort we have chosen to work with) and an overview of the literature on human capital and DSGE models in macroeconomics. Chapter 3 contains the model economy we have built to achieve the goals set for the research, and chapter 4 contains the results. Finally, we present the concluding remarks in chapter 5.

1.3 Objectives

1.3.1 General Objective

The general objective of this research is to investigate the effects of governmental incentives to higher education on human capital investment and other related variables such as consumption level, bequest mechanisms and educational choices of heterogeneous agents, using a partial equilibrium analysis.

1.3.2 Specific Objectives

Specifically, we intend to:

- i. Develop a theoretical DSGE model for human capital investment.
- ii. Investigate which type of incentive – expansion of public universities or subsidy and financing of private universities – is the most effective in increasing human capital investment, through a partial equilibrium analysis.
- iii. Analyze the trade-off between saving and investments on human capital for young agents, using a partial equilibrium analysis.

2 THEORETICAL FRAMEWORK

2.1 General Equilibrium Theory and Dynamic Programming

As opposed to econometric analysis, which consists of an inductive inference method, in general equilibrium a model is an instrument used to deduce the implication of theory⁴ (PRESCOTT, 1998). The model (or law) is not the product of inference, and that is convenient for policy evaluation in macroeconomics, not least because it recognizes that optimal decisions made by economic agents are subject to change when relevant conditions for the decision-making process are modified (LUCAS, 1976).

There exists various modeling techniques within the general equilibrium framework. To fulfill the purposes of this paper, the emphasis will be given to dynamic stochastic general equilibrium (DSGE) models. While this branch of contemporary macroeconomics has been subject to critique over the last decades – as has any other method for explaining aggregate economic phenomena – DSGE models have been widely used in policy analysis and in forecasting by central banks⁵ all over the world (SCHORFHEIDE, 2011). Such models express the general equilibrium allocations and prices of an economy, relying on the rationality of agents. They seek to dynamically

⁴ According to Prescott (1998), a theory is a set of instructions for constructing an instrument to measure something or predict the consequences of a policy.

⁵ See Tovar (2009) for a summary of applications of DSGE models by central banks.

maximize their utility (families), profits (firms) and so on, subject to financial or resource constraints. (TOVAR, 2009). According to Ireland (2004):

DSGE models [...] are firmly grounded in economic theory. These models draw tight links between the structural parameters describing private agents' tastes and technologies and the time-series behavior of endogenous variables such as aggregate output and employment (IRELAND, 2004).

Dynamic optimization problems can be expressed and solved with the help of *dynamic programming*, introduced by Bellman (1954). This method usually makes it easier to solve dynamic problems than the direct approach, primarily because it allows us to rewrite the objective function in a way that the agents' choice of a sequence of controls⁶ is expressed as a sequence of choices for the control variables.

In order to make clear what the applications for dynamic programming are, and how to solve a problem of the sort, we will use an example model by Brock and Mirman (1972). In this one-sector model of economic growth, a benevolent planner chooses sequences of controls $\{c_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize the logarithmic preferences:

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t \ln(c_t) \\ & s.t. \\ & k_0 \in \mathbb{R}^n \\ & k_{t+1} + c_t = Ak_t^\alpha \end{aligned}$$

Where $k_{t+1} + c_t = Ak_t^\alpha$, and c_t is consumption at time t , k_{t+1} is physical capital at time $t+1$, $A > 0$, $\alpha \in (0,1)$ and $\beta \in (0,1)$ is the subjective discount rate. As expressed above, the maximization is subject to the initial stock of capital and to a transition law.

In this context, dynamic programming consists of finding a time-invariant consumption policy by mapping the state – the historically given capital stock – into the

⁶ As explained by Gozzi (2012), when working with an economic system that changes over time, we can describe it with a dynamic variable $x(\cdot)$ such that $x(t)$ explains the state of the system at time t . The state at time t cannot be influenced by the decision-making of agents in the same point of time. On the contrary, agents take the state at time t as given, and they make informed decisions through the control variable $z(t)$, which, in combination with $x(t)$, may change the state in the next period, $x(t+1)$.

sequence of controls $\{c_s\}_{s=0}^{\infty}$ so as to maximize the expected sum of discounted utilities. A solution to such a problem is said to be *recursive*, given that in order to find an optimal consumption policy it is necessary to know a *value function*, which expresses the optimal value of the original problem, starting from an arbitrary initial condition $k_0 \in K$:

$$V_0(k) = \max_{\{c_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

s.t.

$$k_0 \in \mathbb{R}^n$$

$$k_{t+1} + c_t = Ak_t^\alpha$$

If we knew the value of $V_0(k)$, finding the optimal consumption policy would be simple. But since we cannot know the solution to the value function until after solving the problem, we need to tackle the issue from another perspective – by solving the problem for both the value *and* the policy functions, simultaneously. These functions are linked by the *Bellman equation*, which can be expressed as:

$$V'(k) = \max_c \{\ln(c) + \beta V(k)\}$$

Where $V'(k)$ is the value function of the problem when the end of the economy is s periods away, and $V(k)$ is the value function of the previous period, or when the end of the economy is $s-1$ periods away (note that variables with prime denote the future). The structure of the Bellman equation makes explicit the recursive nature of the problem. For the case at hand, it consists of a functional equation to be solved for the pair of unknown functions $V(k)$ and c .

As stated in Ljungqvist & Sargent (2004), there exists three main methods to solve functional equations: iteration on the Bellman equation, guess and verify method, and Howard's improvement algorithm (also known as policy function iteration). The example problem above can be solved with any of the described methods, given its relative simplicity. We are going to use the *iteration on the Bellman equation*, which will give us enough insight into solution techniques to proceed to the model developed in this paper. Following the steps in Ljungqvist & Sargent (2004), the method consists

of iterating the Bellman equation until it converges, starting from $V_0(k) = 0$, so that we have:

$$V_1(k) = \max_c \{ \ln(c) + \beta V_0(k) \}$$

$$V_1(k) = \max_c \{ \ln(c) \}$$

Here we have a simple maximization problem, subject to the same restrictions as the original problem:

$$\begin{aligned} & \max_c \{ \ln(c) \} \\ & \text{s. t.} \\ & c + k' = Ak^\alpha \end{aligned}$$

The solution is the pair $k' = 0$, $c = Ak^\alpha$. If we replace those results in the Bellman equation we get the optimal $V_1(k)$ function:

$$V_1(k) = \ln(Ak^\alpha)$$

$$V_1(k) = \ln A + \alpha \ln k$$

In the next step we have to find the optimal value for:

$$V_2(k) = \max_c \{ \ln(c) + \beta V_1(k) \}$$

$$V_2(k) = \max_c \{ \ln(c) + \beta (\ln A + \alpha \ln k) \}$$

Again, the maximization problem is subject to the same restrictions as before. The idea is to continue to iterate until we get to the limiting policy functions and the optimal value function:

$$c^* = (1 - \beta\alpha) Ak^\alpha$$

$$k'^* = \beta\alpha Ak^\alpha$$

$$V^*(k) = \frac{1}{1-\beta} \left\{ \ln[A(1-\beta\alpha)] + \frac{\beta\alpha}{1-\beta\alpha} \ln(A\beta\alpha) \right\} + \frac{\alpha}{1-\beta\alpha} \ln k$$

We conclude, for the equation of k' , as instance, that k_t converges as t approaches infinity, given that $\alpha < 1$ and $k_0 > 0$ (LJUNGQVIST; SARGENT, 2004). The technique presented above will be used in our model economy to structure the problems of our agents – adults and young individuals – in chapter 3. Our approach to solving the problems are detailed in chapter 4.

2.2 Partial Equilibrium

While the solution of a general equilibrium model results in equilibrium allocations and prices of an economy that may contain a large number of markets, in a partial equilibrium model the analysis is limited to a small number of them (in some cases, only one market). Notwithstanding such a limitation, the analytical simplicity of partial equilibrium analysis is valuable in some cases of policy evaluation. In the words of Mas-Colell, Whinston & Green (1995):

The partial equilibrium approach [...] envisions the market for a single good (or group of goods) for which each consumer's expenditure constitutes only a small portion of his overall budget. When this is so, it is reasonable to assume that changes in the market for this good will leave the prices of all other commodities approximately unaffected and that there will be, in addition, negligible wealth effects in the market under study (MAS-COLELL; WHINSTON; GREEN, 1995)

This means that all economic agents take prices as given and make the best possible choices considering their preferences. This approach to market equilibrium will be useful afterward for analyzing the response of heterogeneous individuals to various forms of educational incentive, all other things being equal. In the next section, we will review an application of dynamic programming that will be helpful to understand the model we will present in chapter 3.

2.3 The Theory of Search

Search models were introduced by McCall (1970), and since then they have been widely used in the investigation of labor markets, in particular for matters related

to job search – hence the designation. This section will be divided in two subsections. In the first part, we will explain what is search theory and what are some of its applications within macroeconomics, and in the second section we will exemplify a search model using a recursive framework.

2.3.1 Search Theory and Macroeconomics

Lucas (1987) described search theory as being a “real social science: an attempt to model, to understand, human behavior by visualizing the situations people find themselves in, the options they face and the pros and cons as they themselves see them”. Search theory has been extensively used in job search environments, by placing unemployed workers in a setting where they are faced with work opportunities that arrive randomly, and where they have the option to reject an available offer if they think they will receive a better offer in the future. (LJUNGQVIST; SARGENT, 2004). The assumptions made by search models differ from the ones in the complete markets⁷ competitive equilibrium model, seeing that search models assume that agents have to put effort into the search for commodities or work, instead of imagining that there is a centralized market place where they make exchanges (LJUNGQVIST; SARGENT, 2004).

The pioneering model of job search designed by McCall (1970) consists of an unemployed individual who can choose between accepting a job offer (and work forever receiving the same wage) or wait for a better offer to come. In this case, the individual receives an unemployment insurance. Whether the agent will take a work opportunity or not depends on the *value of searching* for a job which, in turn, depends on the *reservation wage* equation for the problem at hand. A high reservation wage (which may be observed when the unemployment rate is high, for example) makes it more rewarding for an unemployed agent to remain unemployed (LJUNGQVIST; SARGENT, 2004). Neal (1999) constructed a simplified version of the model by McCall (1970) to study the occupational choices of recent high school graduates. This work became a normative model for young agents. Among other things, he has found that young people should not go job searching before finding a career they actually like.

⁷ A market (or system of markets) is complete when there exists a market for every good (FLOOD, 1991). Goods are state-contingent. In a complete market setting, agents are able to exchange the economy goods with any other agents without transaction costs.

Another interesting application of search models within the labor market consists of Phelps (1970) “island economy”. In contrast to the models reviewed above, this approach does not focus on job search decisions made by *individuals*. In the “island economy”, where each spatially separated *island* contains a labor market with its own characteristics, a *continuum of individuals* interact and decide whether they will take the job opportunity that is available in their current island (which pays the market-clearing wage at said island) or decline that offer and try their luck at another island – that is, another job market (LJUNGQVIST; SARGENT, 2004). Aspects such as the tightness of labor markets – the proportion of people looking for jobs to available spots – and the existence of transaction costs in the search process (MERZ, 1995) influence the equilibrium wage and thus the flow of individuals between islands.

Although search models are more commonly applied to labor market problems, the method may also be applied to several other situations that involve an individual (or a group of individuals) choosing between two different courses of action. One example is the search model of money by Kiyotaki & Wright (1993), in which agents choose trading strategies – they can be either money or commodity traders – in order to maximize their discounted lifetime utility. In the model developed in the present paper, heterogeneous agents who just finished high school choose whether they will take a job right away or further their education before starting a career. Moreover, those who decide against working have yet to choose between public or private university. The technique necessary to develop this concept into a model (and solve it) will follow in the next section.

2.3.2 Search Modeling

In this section we will use the model designed by McCall (1970) as a foundation to explain how to set up and solve a search model. In this problem, an unemployed individual receives a job offer every period she is without a job. She can accept the offer and receive wage $w \in [0, w_{\max}]$ forever, or she can reject the offer and receive unemployment insurance b . The preferences μ are linear and correspond to:

$$\mu = \sum_{t=0}^{\infty} \beta^t y_t$$

Where $0 < \beta < 1$ is the discount factor and y_t is the worker's income at time t . In order to set up this problem recursively, we start by establishing the *value function*, which differs from the functions we have already worked with – in the problem at hand, the value function is defined over two other value functions:

$$v(w) = \max_{\{A,R\}} \{v^e(w); v^u\}$$

Where $\eta = \{A, R\}$ is the discrete control variable, with A denoting “accept job offer” and R denoting “reject job offer”. Here we have a set of choices that is non-convex, so that the value function $v(w)$ is non-differentiable. The wage w is the state variable, and the value functions $v^e(w)$ and v^u correspond to circumstances of employment and unemployment, respectively. Note that the value function for a situation of unemployment does not depend on the wage. The value functions $v^e(w)$ and v^u can be expressed as:

$$v^e(w) = \sum_{t=0}^{\infty} \beta^t w$$

or

$$v^e(w) = \frac{w}{1-\beta}$$

$$v^u = b + \beta \int_0^{w_{\max}} v(w') f(w') dw'$$

or

$$v^u = b + \beta E[v(w')]$$

Where $f(w)$ is a probability distribution function and $E[v(w')]$ is the expected value function for the next period. Replacing the expressions above into the initial value function we have that:

$$v(w) = \max_{\{A,R\}} \{v^e(w); v^u\}$$

$$v(w) = \max \left\{ \frac{w}{1-\beta}; \beta \int_0^{w_{\max}} v(w') f(w') dw' \right\}$$

The solution to the problem above is the *reservation wage* \bar{w} . Given \bar{w} , the value function $v(w)$ takes the forms:

$$v(w) = \begin{cases} A & \text{if } w \geq \bar{w} \\ R & \text{if } w < \bar{w} \end{cases}$$

$$v(w) = \begin{cases} \frac{w}{1-\beta} & \text{if } w \geq \bar{w} \\ \frac{\bar{w}}{1-\beta} & \text{if } w < \bar{w} \end{cases}$$

Solving the problem requires finding the value of \bar{w} , that is, the lowest wage at which an unemployed worker would be willing to accept a job offer. Seeing that an individual faced with a wage offer that is lower than her reservation wage will prefer to remain unemployed, we can find the value of \bar{w} if we equalize $\bar{w}/(1-\beta)$ and v^u :

$$\frac{\bar{w}}{(1-\beta)} = b + \beta \int_0^{w_{\max}} v(w') f(w') dw'$$

$$\bar{w} - b = \beta \int_{\bar{w}}^{w_{\max}} \frac{w - \bar{w}}{1-\beta} f(w') dw'$$

The left side of the equation corresponds to the cost of searching for a job when the wage offered is \bar{w} , while the right side of the equation is the benefit of looking for a job (or *value of searching*) when $w = \bar{w}$. From the equation above, we can observe, as instance, that the reservation wage is affected positively by increases in the unemployment insurance. We could go further and obtain more results, but keeping in mind the purposes of this research, we have obtained enough understanding of search models to proceed. In the next section, we will review the literature on human capital, growth and inequality, after which we will be ready to be introduced to the model economy of this paper.

2.4 Human Capital and Modern Macroeconomics

Among the challenges of contemporary macroeconomics, empirically confirmed facts such as the growing inequality in income and wealth, and the substantial increases in the wealth-income ratio (STIGLITZ, 2014) are worthy of extensive investigation.

Stiglitz (2014) argues that “overall wealth inequality is related both to the transmission mechanisms for human and financial capital across generations and to life cycle savings” and stresses the role of advantages passed through generations in the distribution of wealth, as well as the political aspects of inequality.

There is a good deal to be said about the reasons behind investment decisions of each individual agent, as well as the governmental decisions for allocation of resources, and the essential role of interactions between economic agents – and just as importantly, of the economy’s starting point – in future distributions of wealth and income. And there is quite a lot to be said about the theory of human capital in this context.

Insofar as we intend to examine the workings of these interactions for the case of Brazil, it is useful to start with some empirical works that bring human capital investments and returns in the country (and developing economies in general) into scrutiny. Resende & Wyllie (2006) estimate the returns to education⁸ for Brazil to be around 12.6% for women and 15.9% for men, which is consistent with earlier estimations such as Sachida, Loureiro & Mendonça (2004) – they calculate returns of up to 17.5% for Brazilian men. Additionally, the findings of Psacharopoulos & Patrinos (2004) indicate that the average returns to higher education investments in Brazil is approximately 20%. In comparison to advanced economies, investments in human capital through schooling in Brazil offer higher returns (PSACHAROPOULOS; PATRINOS, 2004). Admitting that returns to investments in education are usually high in developing economies (BARRO, 2000), why is it the case that the current level and distribution of educational attainment in developing countries is only comparable to what developed countries exhibited in the 1960s (BARRO; LEE, 2013)?

Before attending to that question, it is prudent to acknowledge the considerable improvements in human capital formation made by developing economies over the last few decades. The empirical work of Barro & Lee (2013) showed that the average years of schooling in developing economies rose from 2.0 years to 7.2 years since the 1950s⁹. Furthermore, they reveal that most of the progress relates to increased enrollment and completion ratios of higher primary and secondary school, which differs from the scenario in advanced countries, where the increase in years of schooling is mostly due to completion and enrollment ratios of higher secondary and tertiary school (BARRO;

⁸ Mincer (1974) was the first to formalize a function of returns to education.

⁹ Results for individuals aged 15 and above. For further details, see Barro & Lee (2013).

LEE, 2013). This comparison suggests that the proportion of “more” skilled to “less” skilled labor (not to say skilled to unskilled labor) is higher in advanced countries (as we already know) and has increased in the last decades.

The discrepancies in human capital formation between countries with different levels of development can be attributed to a number of factors. Caselli & Ciccone (2013) argue that the differences could be related to difficult access to schooling in some developing countries, among other things. Barro (2000) suggests that the existence of setup costs for certain types of human capital investment could contribute to the gap between developed and developing countries. For instance, investments in secondary school – which requires a higher setup cost – could present higher returns than investments in primary school (BARRO, 2000). This mechanism guarantees that countries where families attain formal education more equally be more successful in accumulating human capital. Hanushek & Woessman (2008) attribute deficient accumulation of human capital to investments in schooling that do not result in skill gains, suggesting that quality of schooling should be at the center of attention.

Leaving comparisons aside, now we can turn back to the matter of human capital attainment per se, which, for the purposes of this work, requires reviewing some of the literature on growth and development accounting, along with some more empirical results of earlier works. Cavalcanti, Guimarães & Sampaio (2010) demonstrated empirically that inequality of opportunity¹⁰ in Brazil – on what concerns human capital formation – prevents people who cannot afford early private education from being admitted in a public university, and thus, impairs the ability of these individuals to better their skills. Their results testify to the perverse effects of the combination between inequality and low social mobility between generations (BOURGUIGNON; FERREIRA; MENÉNDEZ, 2003). In the same spirit, Galor (2011) asserts the following:

While heterogeneity of ability may permit upward mobility of high ability, low income individuals and downward mobility of low ability, high income individuals, income inequality nevertheless operates towards the segmentation of society into two clubs; a club of poor, uneducated individuals and a club of rich, educated individuals. This outcome, which may raise important social and economic concerns, has significant direct

¹⁰ Inequality of opportunity is borne by the fact that wealth and social status affect how an individual does in life (ALEXOPOULOS; CAVALCANTI, 2010).

effects on aggregate economic activity and economic growth (GALOR, 2011, p. 17).

One of the channels through which inequality may negatively affect economic development¹¹ is credit market imperfections (GALOR; ZEIRA, 1993). When lack of resources, and complementarily, difficult access to credit prevent individuals from investing in education or training, the long term result is deficient human capital accumulation, which hinders growth. Galor & Moav (2004) developed a general equilibrium model that highlights the role of human capital in this interaction, and demonstrates that underinvestment in education and training is associated with credit constraints and diminishing marginal returns in human capital formation. The mere fact that investments in human capital are subject to people's physiological limitations is an indication of diminishing returns (GALOR; MOAV, 2004). It is also worth noting that "human capital accumulation is a social activity, involving groups of people in a way that has no counterpart in the accumulation of physical capital" (LUCAS, 1988). This emphasizes the unique and complex nature of this process, which involves family background, social status, innate abilities, wealth, and so on.

Notwithstanding the fact that human capital attainment can assume various forms, such as on-the-job training and learning-by-doing – which are just as important as schooling for the process of accumulation (LUCAS, 1988) – we emphasize the role of the latter in the workings of social mobility mechanisms. Education provides individuals with more than just tools to achieve a successful career accompanied by a sizeable income wage. Education spills over to society and contribute to development in many fronts, by inducing individuals to innovate, stimulate critical thinking and raise political awareness. Through these channels education enables poverty-trapped

¹¹ Classical economists considered inequality to be conducive to economic development, largely because marginal propensity to save is higher when income is higher, which favors accumulation of savings. Marrero & Rodríguez (2013) include unobservable effort and investment project size as routes through which inequality may affect development, according to the classical approach. Nevertheless, at the time these assertions were made, the stage of development of the world was vastly different than what we experience nowadays, and physical capital (relative to human capital) had a larger impact on development. Currently, the rise of skill-biased technologies, among other factors, make human capital relatively more important to economic growth (GALOR, 2011). Note that a technology is skill-biased when its creation and application require a specific set of abilities. They are often associated with computers and telecommunication technologies (MORTENSEN; PISSARIDES, 1999).

individuals to promote the structural changes necessary for them to actually move upward in the social ladder. The challenges are numerous, and their political nature undeniable. Bourguignon & Verdier (2000), Kochar (2008), Alexopoulos & Cavalcanti (2010) and others have demonstrated both theoretically and empirically how local elites benefit from (and contribute to) underinvestment in public schools in order to maintain their economic powers and social status.

Goñi, López & Servén (2011) also draw attention to politics in their analysis of income inequality and fiscal policy in Latin America. For the case of Brazil, specifically, they report that poor targeting of redistributive programs is predominantly to blame for overall inequality in the country. Stiglitz (2014) adds that many of the policy changes needed to reduce inequality and simultaneously stimulate growth have already been identified, but inequality of political power – which favors the wealthy – is keeping these policies from being implemented. That is particularly alarming for less developed countries. When the institutional scenario of a developing economy, such as Brazil, is so that credit market imperfections are not dealt with (or are poorly addressed), the effects of inequality on economic development can be quite extensive (BARRO, 2000).

It is important to bear the idiosyncrasies of human capital accumulation in mind, along with the political aspects that permeate the interactions of economic agents. But also, it is fundamental to be aware of the methodological limitations we are faced with. For now, keeping analysis simple through a model that captures individual choices of investment in human capital through higher education is a good way to start, seeing that human capital is viewed to play a major role in equalization (ACEMOGLU, 2012). In face of the assumption that technology is skill-biased, widening the supply of skilled professionals – through upgrade of school quantity and quality, as instance – is the best way to counter inequality. Furthermore, human capital increases labor productivity and is one of the driving forces of innovation and adoption of new technologies (ACEMOGLU, 2012; BARRO; LEE, 2013), which makes it vital for economic progress.

3 THE MODEL

3.1 The Basic Model

The economy is defined through an overlapping generations model, which was introduced by Samuelson (1958). In this model, agents type i live for two periods, defined as youth and adulthood. Population growth rate is 0 – each adult has only one parent and gives birth to only one child, with measure equal to 1. In this economy, agents' initial wealth is a_0^{2i} , and it varies across individuals. They optimize their discounted lifetime utility, where the discount rate is given by β . Agents' preferences are homogeneous, and defined only over consumption, c_t^{1i} , if the agent hasn't reached adulthood. The instantaneous utility for a young individual can be defined as $u(c_t^{1i})$.

If the agent is an adult, lifetime utility is defined over consumption, c_t^{2i} , and future utility of offspring, $u(c_{t+1}^{1i})$. The mechanism through which parents contribute to their children's welfare, and therefore, to their own, is by saving and making intergenerational transfers, a_{t+1}^{2i} . Instantaneous utility for an adult can be written as:

$$(1) \quad u(c_t^{2i}, a_{t+1}^{2i}) = (c_t^{2i})^\gamma (a_{t+1}^{2i})^{1-\gamma}$$

Where $0 < \gamma < 1$. This equation presents a basic configuration of instantaneous utility functions, and can be found in several papers, such as Antunes, Cavalcanti & Villamil (2008) and Alexopoulos (2012).

3.2 The Production Sector

In the production sector, firms produce the single good of the economy, Y_t , using capital, K_t , and labor, L_t^s and L_t^u , so that:

$$(2) \quad Y_t = K_t^\alpha (AL_t^s + L_t^u)^{1-\alpha}$$

Where $A > 1$ and $0 < \alpha < 1$. Each adult supplies one unit of skilled or unskilled labor, defined respectively as l_t^s and l_t^u , where l_t^s is more productive than l_t^u . Each unit of labor is rewarded with an income wage, which corresponds to w_t^s for skilled labor, and w_t^u for unskilled labor. Firms do not differentiate between skilled and unskilled labor, therefore both L_t^s and L_t^u are larger than zero in a competitive equilibrium.

Proposition 1: In any competitive equilibrium, $L_t^{u*} > 0$.

Proof: If it is the case that $L_t^{u*} = 0$, then $w_t^{s*} \leq Aw_t^{u*}$, which cannot be a competitive equilibrium. It is not possible that every worker in this economy be skilled, because the probability of becoming a skilled worker after having graduated from university is smaller than one. So it has to be the case that $L_t^{u*} > 0$.

Assumption 1: $L_t^{s*} > 0$

Given the goal of this paper, we will assume that there is always someone getting educated. The model will become too trivial if there aren't any skilled people graduating and entering the job market. Therefore, $L_t^{s*} > 0$. Provided that $L_t^{s*} > 0$ and $L_t^{u*} > 0$, we have:

$$(3) \quad \frac{w_t^{s*}}{w_t^{u*}} = A$$

Agents own all capital in the economy, k_t , and make loans to firms at rate r_t .

From the production function we have that:

$$(4) \quad (1+r_t^*) = \alpha K_t^{\alpha-1} (AL_t^s + L_t^u)^{1-\alpha}$$

Equation (4) is true only when the rate at which capital depreciates is zero.

3.3 An Overview of the Agents' Problems

The agents' problems will be solved recursively. To make notation simpler, we will not use a time subscript on variables in this section, neither will we use a superscript to differentiate individuals. In this model, young agents choose whether they will further their education or not. If they choose not to go to a university, then they will join the workforce by offering one unit of unskilled labor now and in the adulthood, earning w^u as income wage. If they choose to go to a university, they subsequently have to choose between a public or a private institution. Let $e \in \{0,1\}$ be the discrete variable that indicates the agent's choice – its value is 1 when she goes to a university, and 0 when she chooses to work.

There exists a probability Q^n of becoming a skilled professional associated with graduation in a university, where $n \in \{0,1\}$ describes the type of institution – public or private. If the agent attends a public university, with $n=0$ (given $e=1$), the probability of becoming a skilled worker is Q^0 . Similarly, if the agent goes to a private institution, the probability of becoming a skilled professional is Q^1 . In this economy, public institutions graduate more skilled workers than private ones, so $Q^0 > Q^1$. Furthermore, private universities require payment of a tuition, δ . Public universities do not, but they have an entrance cost, ϕ . Both δ and ϕ are larger than zero and given exogenously, in terms of the final good in the economy.

Moreover, there exists a temporal gap which makes the payment of ϕ relatively more costly than δ . In contrast to private institutions, the entrance in a public university is conditional to the attainment of a good grade in the entrance exam which, in turn, depends on the quality of basic schooling and preparatory courses. Both these elements require that the family wealth is sufficiently large when the individual is at a young age, so public universities are often inaccessible to poor agents.

The aggregate state variables for all agents are the aggregate capital in the economy, K , and the total amount of skilled and unskilled labor, L^s and L^u . Let Ω be the set of aggregate state variables.

3.4 The Adult's Problem

For the adults in this economy, the individual state variables are wealth, a^1 , past education, $e \in \{0,1\}$, past choice of institution, $n \in \{0,1\}$, and characterization of labor, $h \in \{u,s\}$ (unskilled or skilled). The control variables for adults are consumption, c^2 , and intergenerational transfers, a^2 . The resulting problem can be written as follows:

$$\begin{aligned}
 (5) \quad & V_{hen}(a^1; \Omega) = \max_{c^2, a^2} \{u(c^2, a^2)\} \\
 & s.t.: \\
 & c^2 + a^2 = w^h + (1+r)a^1 - en\delta[(1+r)(1-\tau)] \\
 & c^2 \geq 0; a^2 \geq 0
 \end{aligned}$$

Where τ is a governmental subsidy which may or may not be given to agents to encourage them to attend university.

3.5 The Young Agent's Problem

The individual state variable for young individuals is the bequest they receive from their parents, a^2 . The control variables are consumption, c^1 , savings, a^1 , education, $e \in \{0,1\}$, and choice of institution, $n \in \{0,1\}$, so that:

$$(6) \quad W_{en}(a^2; \Omega) = \max_e \{W_{00}(a^2; \Omega); W_{1n}(a^2; \Omega)\}$$

In order to make this problem simpler, we will solve two different problems – one for the agents who attend university and other for the ones who do not. The agent whose choice is $e = 0$, that is, work instead of studying, knows that she will be offering one unit of unskilled labor now and in adulthood, so she knows exactly what her income wage will be in the future. The resulting problem is:

$$\begin{aligned}
W_{00}(a^2; \Omega) &= \max_{c^1, a^1} \{u(c^1) + \beta[V_{u00}(a^1; \Omega)]\} \\
(7) \quad \text{s.t.} & \\
c^1 + a^1 &= w^u + (1+r)a^2 \\
c^1 \geq 0, a^1 &\geq 0
\end{aligned}$$

Young agents who decide to attend university, $e = 1$, solve a problem to choose whether they will go to a public or a private institution:

$$(8) \quad W_{1n}(a^2; \Omega) = \max_n \{W_{10}(a^2; \Omega); W_{11}(a^2; \Omega)\}$$

Again, we will solve a different problem for type of institution. The agent who chooses to go to a public university ($n = 0$) solves:

$$\begin{aligned}
W_{10}(a^2; \Omega) &= \max_{c^1, a^1} \{u(c^1) + \beta[Q^0 V_{s10}(a^1; \Omega) + (1-Q^0)V_{u10}(a^1; \Omega)]\} \\
(9) \quad \text{s.t.} & \\
c^1 + a^1 + \phi &= (1+r)a^2 \\
c^1 \geq 0, a^1 &\geq 0
\end{aligned}$$

Likewise, the young agent who decides to attend a private university ($n = 1$) solves:

$$\begin{aligned}
W_{11}(a^2; \Omega) &= \max_{c^1, a^1} \{u(c^1) + \beta[Q^1 V_{s11}(a^1; \Omega) + (1-Q^1)V_{u11}(a^1; \Omega)]\} \\
(10) \quad \text{s.t.} & \\
c^1 + a^1 &= (1+r)a^2 \\
c^1 \geq 0, a^1 &\geq 0
\end{aligned}$$

3.6 The Government's Problem

In this economy, the government finances the agents' studies at private universities, provided that they commit to paying back the government when they graduate and become workers – qualified or not. That means that the existence of this loan, $\delta(1-\tau)$, allows all individuals, poor and rich, to invest in their education.

Let λ be the distribution of wealth¹², and let $P(a^j; A) = \text{Prob}\{a' \in A | a^j\}$, so that:

$$(11) \quad \lambda' = \sum_{j=1}^2 \int_{a^j} P(a^j; A) \lambda(d a^j)$$

Additionally, let $I_{\{e_{-1}=1\}}(a^2, \Omega)$ be an indicator function that assumes the value 1 when $n = 1$ (agent goes to a private university) and 0 when $n = 0$ (agent goes to public university). And let $S(a^1, \Omega)$ be another indicator function that is equal to 1 when $h = s$ (agent is a skilled worker) and is equal to 0 when $h = u$ (agent is an unskilled worker). The government's revenue, T , can be defined as:

$$(12) \quad T = \tau \delta \int_{a^2} I_{\{e_{-1}=1\}}(a^2, \Omega) \lambda(da^2) + \theta \int_{a^2} [1 - I_{\{e_{-1}=1\}}(a^2, \Omega)] \lambda(da^2)$$

Where θ is the cost of public university per student.

3.7 Competitive Equilibrium

A dynamic competitive equilibrium is a set of functions $c^{2*}(a^1; \Omega)$, $c^{1*}(a^2; \Omega)$, $a^{1*}(a^2; \Omega)$, $a^{2*}(a^1; \Omega)$, $V_{hen}^*(a^1, \Omega)$, $W_{en}^*(a^2, \Omega)$, $T^*(a^1, a^2; \Omega)$ and τ^* such that:

- i. Given τ^* , $V_{hen}^*(a^1, \Omega)$ and $W_{en}^*(a^2, \Omega)$ solve the agents' problem. The optimal policy functions are $c^{2*}(a^1; \Omega)$, $c^{1*}(a^2; \Omega)$, $a^{1*}(a^2; \Omega)$ and $a^{2*}(a^1; \Omega)$.
- ii. Given τ^* , $T^*(a^1, a^2; \Omega)$ balances government's budget.
- iii. Given τ^* , the market clearing conditions are:

$$(13) \quad \int_{a^1} c^2(a^1, \Omega) \lambda da^1 + \int_{a^1} \lambda da^1 + \int_{a^2} c^1(a^2, \Omega) \lambda da^2 + \int_{a^2} \lambda da^2 + T = Y^*$$

$$(14) \quad L^s = \int_{a^1} S(a^2, \Omega) da^1$$

¹² See Aiyagari (1994) and Ljungqvist & Sargent (2004) for more on wealth distributions.

$$(15) \quad L^u = \int_{a^1} [1 - S(a^1, \Omega)] da^1 + \int_{a^2} I(a^2, \Omega) da^2$$

$$(16) \quad K = \sum_{j=1}^2 \int_{a^j} \lambda da^j$$

4 RESULTS

We will start by solving the problems of each type of agent in the economy – adults, young individuals who go to public university, young individuals who attend private university, and young individuals who don't further their studies and go straight to the work market. Those problems will be solved recursively, therefore we will start with the adult's problem and replace the optimal value function into each young agent's problem, so that we can find the optimal value functions for these individuals as well.

Next, we will present some hypotheses and derive their implications for the model economy we have constructed. Subsequently, we will conduct a partial equilibrium analysis and present the results graphically. The analytical results from this section, as well as the partial equilibrium results, should link directly to the two latter specific objectives of this research.

4.1 Solution to the Adult's Problem

Given that R^A denotes the resources of agents in adulthood, that is, $R^A = w^h + (1+r)a^1 - en\delta(1+r)(1-\tau)$, the adult's problem can be written as:

$$\begin{aligned}
 V_{hen}(a^1; \Omega) &= \max_{c^2, a^2} (c^2)^\gamma (a^2)^{1-\gamma} \\
 (17) \quad s.t.: \\
 c^2 + a^2 &= R^A \\
 c^2 \geq 0, a^2 &\geq 0
 \end{aligned}$$

$$(18) \quad L = (c^2)^\gamma (a^2)^{1-\gamma} + \lambda(R^A - c^2 - a^2)$$

The first order conditions are:

$$(19) \quad \{c^2\}: \gamma(c^2)^{\gamma-1}(a^2)^{1-\gamma} - \lambda = 0$$

$$(20) \quad \{a^2\}: (1-\gamma)(c^2)^\gamma (a^2)^{-\gamma} - \lambda = 0$$

$$(21) \quad \{\lambda\}: c^2 + a^2 = R^A$$

From equations (19) and (20) we have that:

$$(22) \quad c^2 = \frac{\gamma a^2}{(1-\gamma)}$$

Using equations (22) and (21) we get to the adult's optimal savings, as well as to the optimal consumption:

$$(23) \quad a^{2*} = (1-\gamma)R^A$$

$$(24) \quad c^{2*} = \gamma R^A$$

Note that both savings and consumption depend only on R^A (available resources). They do not depend on past decisions of consumption, which makes them history independent (LJUNGQVIST; SARGENT, 2004). From equations (17), (23) and (24) we have that:

$$(25) \quad V_{hen}^*(a^1; \Omega) = \gamma^\gamma (1-\gamma)^{1-\gamma} R^A$$

This means that the solution for the adult's problem is linear in R^A , with slope $\gamma^\gamma (1-\gamma)^{1-\gamma}$.

4.2 Solution to the Young Agents' Problems

4.2.1 Agent who chooses a public university ($e = 1$ and $n = 0$)

The problem of the agent who chooses to attend a public university is:

$$(26) \quad W_{10}(a^2; \Omega) = \max_{c^1, a^1} \{u(c^1) + \beta[Q^0 V_{s10}(a^1; \Omega) + (1 - Q^0) V_{u10}(a^1; \Omega)]\}$$

s. t. :

$$c^1 + a^1 + \phi = (1 + r)a^2$$

$$c^1 \geq 0, a^1 \geq 0$$

$$(27) \quad L = u(c^1) + \beta[Q^0 V_{s10}(a^1; \Omega) + (1 - Q^0) V_{u10}(a^1; \Omega)] + \lambda[(1 + r)a^2 - c^1 - a^1 - \phi]$$

The first order conditions are:

$$(28) \quad \{c^1\}: u'(c^1) - \lambda = 0$$

$$(29) \quad \{a^1\}: \beta[Q^0 V'_{s10}(a^1; \Omega) + (1 - Q^0) V'_{u10}(a^1; \Omega)] - \lambda = 0$$

$$(30) \quad \{\lambda\}: c^1 + a^1 + \phi = (1 + r)a^2$$

In equation (29), the values of $V'_{s10}(a^1; \Omega)$ and $V'_{u10}(a^1; \Omega)$ can be derived from equation (25), which corresponds to the solution to the adult's problem. Therefore, from equation (25) we have that:

$$(31) \quad V'_{hen}(a^1; \Omega) = \frac{dV_{hen}^*(a^1; \Omega)}{da^1} = \gamma^\gamma (1 - \gamma)^{1-\gamma} (1 + r)$$

From equations (28), (29) and (31) we have that:

$$(32) \quad u'(c^1) = \beta \gamma^\gamma (1 - \gamma)^{1-\gamma} (1 + r)$$

The agent's optimal level of consumption, c^{1*} , is given implicitly by (32). Note that consumption is independent of bequest a^2 . Moreover, the optimal level of savings can be derived from equation (30):

$$(33) \quad a^{1*} = (1 + r)a^2 - \phi - c^{1*}$$

Note that the consumption of these young individuals do not depend on the bequest they received, which derives from the solution to the adult's problem – equation (25), or the value function of adults, is linear in R^A .

4.2.2 Agent who chooses a private university ($e = 1$ and $n = 1$)

The problem of the agent who chooses to attend a private university, and resulting Lagrange equation, are:

$$(34) \quad \begin{aligned} W_{11}(a^2; \Omega) &= \max_{c^1, a^1} \{u(c^1) + \beta[Q^1 V_{s11}(a^1; \Omega) + (1-Q^1) V_{u11}(a^1; \Omega)]\} \\ \text{s.t.:} \\ c^1 + a^1 &= (1+r)a^2 \\ c^1 \geq 0, a^1 &\geq 0 \end{aligned}$$

$$(35) \quad L = u(c^1) + \beta[Q^1 V_{s11}(a^1; \Omega) + (1-Q^1) V_{u11}(a^1; \Omega)] + \lambda[(1+r)a^2 - c^1 - a^1]$$

From (35), the first order conditions are:

$$(36) \quad \{c^1\}: u'(c^1) - \lambda = 0$$

$$(37) \quad \{a^1\}: \beta[Q^1 V'_{s11}(a^1; \Omega) + (1-Q^1) V'_{u11}(a^1; \Omega)] - \lambda = 0$$

$$(38) \quad \{\lambda\}: c^1 + a^1 = (1+r)a^2$$

As done in the previous problem, equation (31) will be used to find the values of $V'_{s11}(a^1; \Omega)$ and $V'_{u11}(a^1; \Omega)$. Therefore, using (31), (36) and (37) we will find that the optimal level of consumption of the agent who attends a private university is the same of the agent who attends a public university. In other words, it is given implicitly by equation (32). Conversely, the optimal level of savings varies. From equation (38) we have that:

$$(39) \quad a^{1*} = (1+r)a^2 - c^{1*}$$

4.2.3 Agent who chooses not to attend a university ($e = 0$ and $n = 0$)

$$\begin{aligned}
 W_{00}(a^2; \Omega) &= \max_{c^1, a^1} \{u(c^1) + \beta[V_{u00}(a^1; \Omega)]\} \\
 (40) \quad \text{s.t. :} \\
 c^1 + a^1 &= w^u + (1+r)a^2 \\
 c^1 \geq 0, a^1 &\geq 0
 \end{aligned}$$

$$(41) \quad L = u(c^1) + \beta[V_{u00}(a^1; \Omega)] + \lambda[w^u + (1+r)a^2 - c^1 - a^1]$$

From (41), the first order conditions are:

$$(42) \quad \{c^1\} : u'(c^1) - \lambda = 0$$

$$(43) \quad \{a^1\} : \beta V'_{u00}(a^1; \Omega) - \lambda = 0$$

$$(44) \quad \{\lambda\} : c^1 + a^1 = w^u + (1+r)a^2$$

We will use equation (31) to find the value of $V'_{u00}(a^1; \Omega)$. Combining that with equations (42) and (43) we will find that the optimal level of consumption of the agent who does not attend a university is the same of the agent who attends a public university, which, in turn, is the same of the agent who goes to a private institution. Therefore, optimal consumption is given implicitly by equation (32). Equation (44) gives us the optimal level of savings:

$$(45) \quad a^{1*} = w^u + (1+r)a^2 - c^{1*}$$

4.3 Implications

Proposition 2: If $a^2 \geq \frac{\phi + c^{1*}}{(1+r)}$, then:

1. The consumption is optimal, i.e., c^{1*} is given by equation (32).
2. $a^{1*} \geq 0$

Proof: According to Proposition 2, bequest does not constrain the agent's decision as long as a^2 is large enough to pay for both the present value of university

entrance cost, ϕ , and present consumption, c^{1*} . With regard to a^{1*} : by contradiction, if $a^{1*} < 0$, then $a^2 < (\phi + c^{1*})/(1+r)$, which cannot be true, since the agent's received bequest needs to be large enough to cover her expenses. Therefore, $a^{1*} \geq 0$. This condition holds if and only if $a^2 \geq (\phi + c^{1*})/(1+r)$.

From now on, we will distinguish between savings made by agents who attend public institutions, a_{10}^1 , and agents who go to public ones, a_{11}^1 . It is already known, from (34) and (41), that $a_{10}^{1*} \neq a_{11}^{1*}$, given that $\phi \neq 0$.

$$\text{Assumption 2: } \frac{E(w^h | n = 0) - E(w^h | n = 1)}{(1+r)} \geq \phi - \delta(1-\tau)$$

The assumption above guarantees that, if an agent is rich enough, i.e., $a^2 \geq (\phi + c^1)/(1+r)$, with c^{1*} given implicitly by (32), then the agent who decides to further her education will choose a public university instead of a private one. Therefore, the choice of university depends on the agent not being constrained by her wealth. Moreover, the higher the cost of entrance of public universities, the higher the likelihood of being constrained by wealth, and the higher the share of young agents who choose private institutions (if it is the case that they can afford both consumption and school debt in the next period of their lives).

Putting such information in context with the governmental incentive for public university in Brazil that we chose to work with, that is, REUNI, an increase in the number of openings in the public system of higher education is associated with a decline in relative competition. In the model economy, this is equivalent to a decrease in the cost of entrance, ϕ . Consequently, expansions in public universities increase the share of young agents who attend public institutions.

$$\text{Proposition 3: } W_{10}(a^2; \Omega) \geq W_{11}(a^2; \Omega)$$

Proof: Since optimal consumption is the same for all agents who choose to further their education, then agent with wealth a^2 will choose a public university if and only if $W_{10}(a^2; \Omega) \geq W_{11}(a^2; \Omega)$. Using the proposed relation and equations (26), (32) and (34) we have that:

$$(46) \quad Q^0 w^s + (1 - Q^0) w^u - [Q^1 w^s + (1 - Q^1) w^u] \geq (1 + r)(a_{11}^{1*} - a_{10}^{1*}) - \delta(1 + r)(1 - \tau)$$

I will call the expression $Q^0 w^s + (1 - Q^0) w^u$ the expected income wage for a public university attendant, or $E(w^h | n = 0)$. Similarly, the expression $Q^1 w^s + (1 - Q^1) w^u$ will be called the expected income wage for a private university attendant, or $E(w^h | n = 1)$. Also, from equations (33) and (39) we have that $a_{11}^{1*} - a_{10}^{1*} = \phi$. Replacing these expressions into equation (46) and rearranging the terms we get Assumption 2:

$$(47) \quad \frac{E(w^h | n = 0) - E(w^h | n = 1)}{(1 + r)} \geq \phi - \delta(1 - \tau)$$

Moreover, the government may choose to give a subsidy, τ , to young agents who attend private universities. The government may choose any value between 0 and 1 as a subsidy rate. If $\tau = 1$, then the agent's debt will be equal to zero, which means that the government is paying in full for the agent's education. Using equation (47):

$$(48) \quad \frac{E(w^h | n = 0) - E(w^h | n = 1)}{(1 + r)} \geq \phi$$

The existence of a full subsidy makes it less likely that an agent will choose a public university, because the subsidy makes it relatively less costly to attend a private institution. Accordingly, the share of *financially constrained* young agents who choose to attend private university in detriment of public university is higher. Note that an agent who is not constrained by wealth will choose a public university (see Assumption 2). In contrast, when $\tau = 0$ the agent will have to pay δ to the government after her education is complete and she gets a job. Once more, we will use equation (47) to show that:

$$(49) \quad \frac{E(w^h | n = 0) - E(w^h | n = 1)}{(1 + r)} \geq \phi - \delta$$

Firstly, if the value of ϕ does not constrain the agent, then she will choose a public university regardless of whether the government is paying the costs of private

education or not (again, see Assumption 2). On the other hand, for the case of a financially constrained individual (for whom public university is desirable but not necessarily accessible), the absence of a subsidy – which means that the agent will own δ to the FIES fund in the future – makes attendance to a private university relatively more costly. If a poor agent goes through with university with $\tau = 0$, when she becomes an adult with budget constraint $c^2 + a^2 = w^h + (1+r)a^1 - \delta(1+r)$, it may be the case that she will not be able to both pay for her FIES loan and save. This means that in the next generation her offspring will likely be facing the same situation. This result is consistent with Galor & Zeira (1993), who have found that skilled individuals who pay student loans do not leave as many resources for the next family generation as individuals who have come from a wealthy family.

Therefore, equations (47) to (49) assure that the higher the subsidy τ from the government, the higher the share of agents who choose to attend a private university. For those individuals who cannot access public university due to wealth constraints (where $a^2 < (\phi + c^1)/(1+r)$), and conversely, for those who do not want to (or cannot) risk acquiring educational debt (FIES) without guarantees of future income, subsidy in the form of a program such as ProUni may be the only path for escaping poverty.

As done previously for other agents, we will call the savings made by agents who do not further their education a_{00}^1 . From (33), (39) and (45) we know that $a_{00}^1 \neq a_{10}^1 \neq a_{11}^1$, given that $\phi \neq 0$.

$$\text{Assumption 3: } \frac{E(w^h | n=0) - w^u}{(1+r)} \geq w^u + \phi$$

This assumption makes it so that the higher the share of young agents who are constrained by their wealth, the higher the share of agents who choose to work instead of going to a university. Furthermore, the higher the unskilled wage, the higher the proportion of young agents who choose to work. Additionally, Assumption 3 guarantees that, if agent is rich enough, she will choose to further her education.

Proposition 4: If initial wealth, a^2 , is so that $W_{10}(a^2; \Omega) \geq W_{11}(a^2; \Omega)$, then $W_{00}(a^2; \Omega) < W_{10}(a^2; \Omega)$.

Proof: If studying in a public university is at least just as good as studying in a private one, then studying in a public university has to be better than not studying at all. Using equations (26), (32) e (40) on $W_{00}(a^2; \Omega) < W_{10}(a^2; \Omega)$ we can show that:

$$(50) \quad w^u + (1+r)(a_{00}^{1*} - a_{10}^{1*}) < Q^0 w^s + (1-Q^0)w^u$$

As we did previously, we will call the expression $Q^0 w^s + (1-Q^0)w^u$ the expected income wage for a public university attendant, or $E(w^h | n = 0)$. In addition to that, from equations (33) and (49) we have that $a_{00}^{1*} - a_{10}^{1*} = w^u + \phi$. Using that information on equation (50) we get that:

$$(51) \quad \frac{E(w^h | n = 0) - w^u}{(1+r)} \geq w^u + \phi$$

It is important to notice that the term $w^u/(1+r)$ represents the discounted income wage of a young agent who chooses to work instead of studying. We don't call it *expected income wage* because we know for sure that an agent who does not attend university will offer one unit of unskilled labor, and therefore will receive w^u as payment for her work. Moreover, from equations (49) and (51) we have that:

$$(52) \quad \frac{E(w^h | n = 1) - w^u}{(1+r)} \leq w^u + \delta$$

The analysis of equation (52) is similar to the previous ones. Higher values of unskilled wage w^u and private university's costs δ are associated with a smaller share of agents attending private university and consequently a higher share of young agents joining the labor force. When we consider individuals with no family wealth, faced with the *possibility* of a position as a skilled professional *in the future* (alongside with a student loan to pay) and, on the other hand, *the certainty of wage right now* (which will guarantee consumption and maybe saving), foregoing university attendance may be the best option for the bulk of young agents in this position. The temporal gap between unskilled and (possibly) skilled wage may be too costly for some, so they choose consumption and saving over investments in human capital which, in turn, may result in families being poverty trapped for generations to come.

4.4 Partial equilibrium analysis

In order to evaluate the implications of higher education incentives to the model economy, we will now conduct a partial equilibrium analysis, which takes all prices as given. Through the relationship between the young agents' diverging bequests and their lifetime utilities (where W_{00} is the value function of the young individual who chooses work instead of university, W_{11} is that of the agent who goes to private university, and W_{10} is the function of the agent who goes to public university), we can show how they interact with the agents' choices of higher education in Figure 1, considering a subsidy equivalent to $\tau = 0.5$:

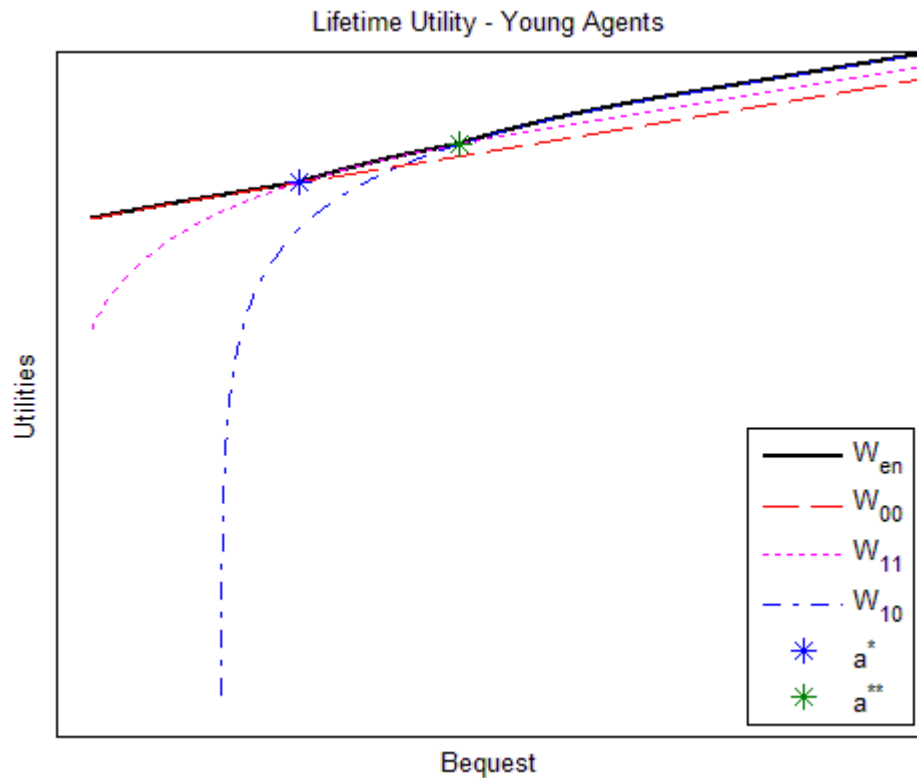


Figure 1: Lifetime utility of young agents with $\tau = 0.5$
Source: Research results

The graph above shows that individuals born with bequest inferior to a^* , corresponding to the blue star above, choose not to attend university. Those born with bequest between a^* and a^{**} choose to study in a private institution, and pay back the FIES fund when they are through with their studies. Note that the government is paying a fraction of the costs through ProUni. And last, anyone born with bequest superior to a^{**} will go to a public university. This corroborates Assumption 2 in the previous section: as long as the agent is not constrained by her wealth, that is,

$a^2 \geq (\phi + c^1)/(1+r)$, she will choose to attend a public university, where her chances to become a skilled professional in the future are higher.

Wealth inequality makes it so that only individuals with a minimal amount of wealth are able to make unconstrained decisions of higher education, in detriment of poor agents. As Banerjee & Newman (1993) have argued, differences in wealth explain, in part, different choices of occupation (or comparatively, education, as it is the case here). A change in the subsidy parameter to $\tau = 0$, which means that the government gives no financial aid to poor individuals, results in the situation depicted in Figure 2:

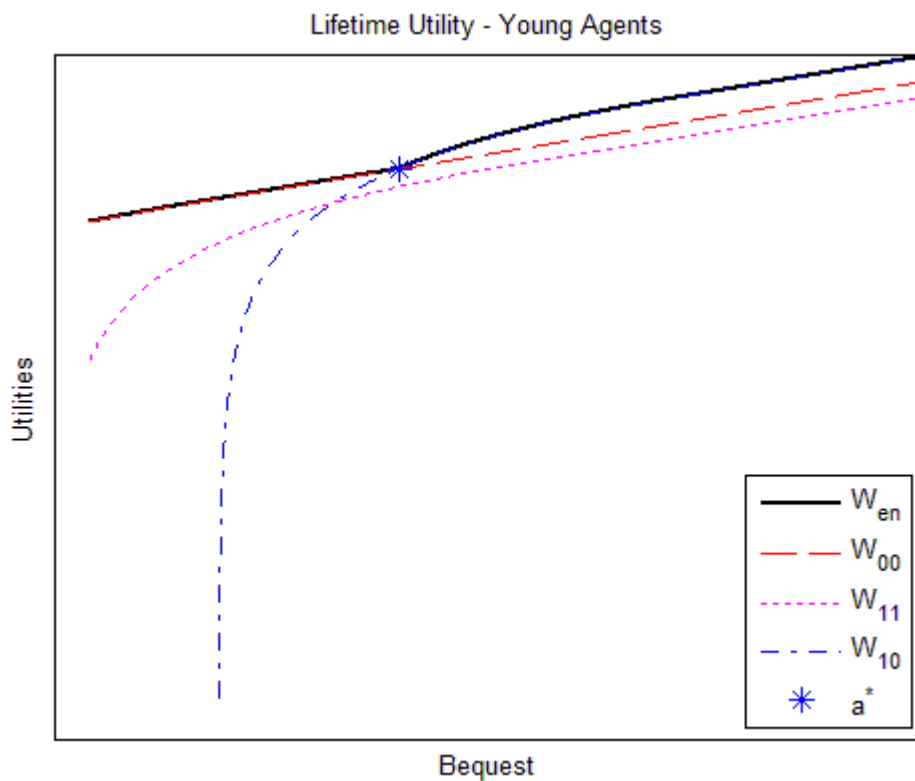


Figure 2: Lifetime utility of young agents with $\tau = 0$
Source: Research results

According to Figure 2, when subsidies for private university (ProUni) do not exist, attending university is not an option for poor individuals (see the trajectory of function value W_{11}). The blue star in the graph is the level of wealth associated with attendance of public university. Actually, the fact that an individual has the means to attend a public institution of higher education is indicative of the existence of enough resources to go through with private university. However, as we already know, an agent who is not financially constrained will prefer to continue her studies in the public system.

The outcome we get when the government gives full subsidy to poor individuals is illustrated in Figure 3, the case of ProUni full scholarships ($\tau = 1$):

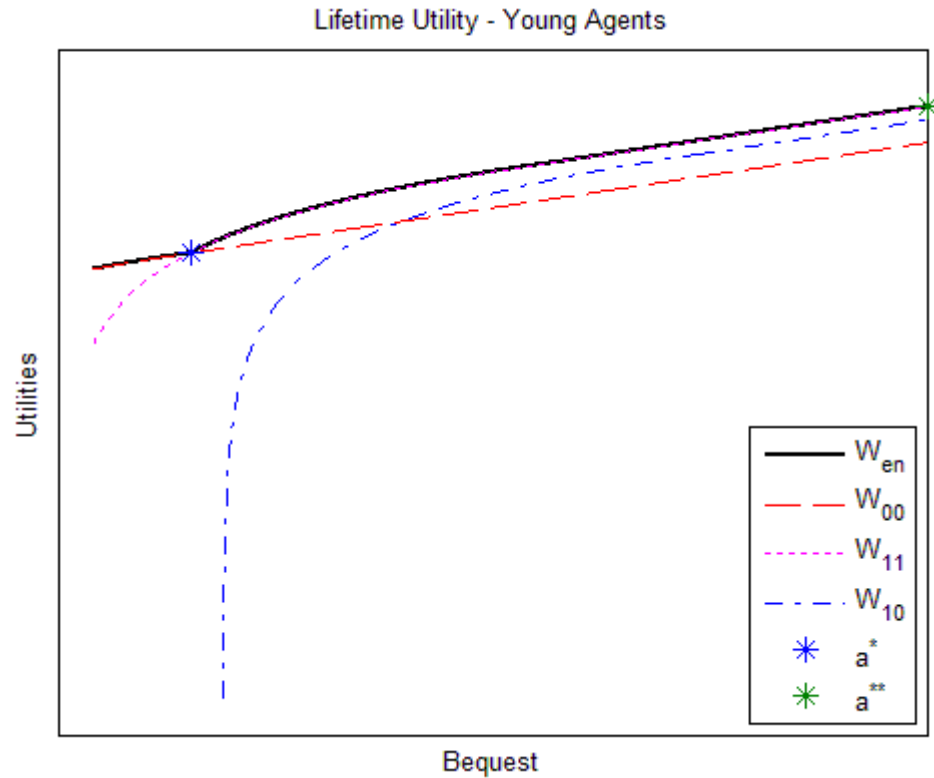


Figure 3: Lifetime utility of young agents with $\tau = 1$
Source: Research results

The graph above shows that the existence of full subsidy allows individuals born with wealth between a^* and a^{**} access to higher education in private institutions, which accounts for a much wider range of assistance than what we observed in Figure 1. The wealth threshold for public university, in contrast, becomes extremely high. Note that the trajectory of W_{10} does not reach W_{en} in the visible area of the graph. This suggests on the one hand, that for this model economy public university is accessible to a very small share of young individuals in the presence of $\tau = 1$. On the other hand – and more intuitively – it may simply be a case of ill-directed incentive. The outcome of a different kind of higher education incentive – the case of REUNI – is illustrated in Figure 4:

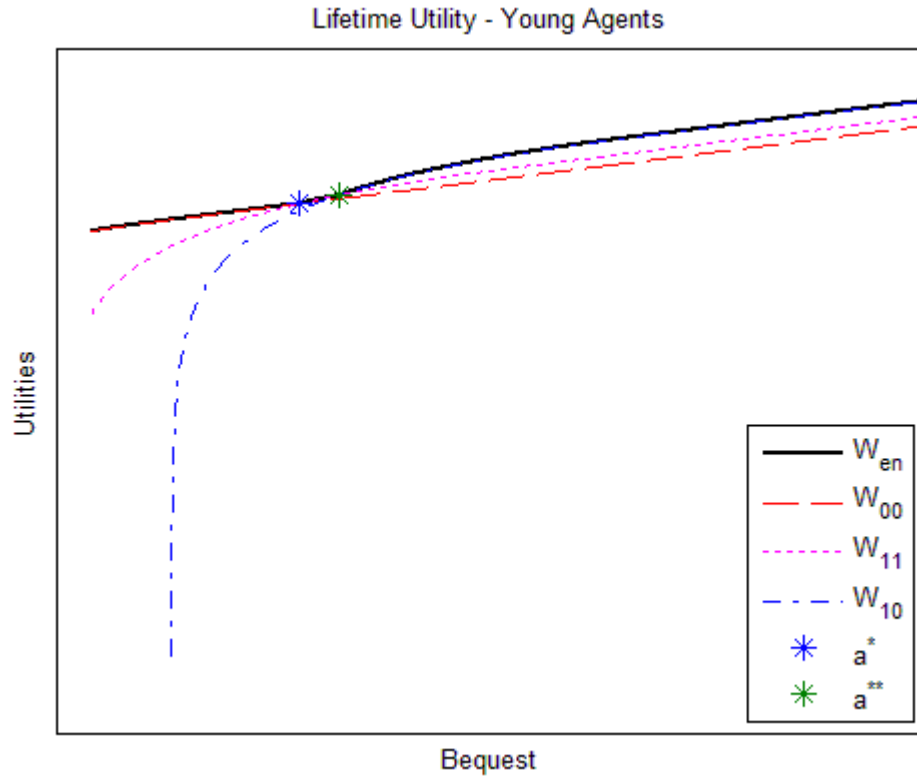


Figure 4: Lifetime utility of young agents with $\tau = 0.5$ and $\phi = 0.4$ (initially, $\phi = 0.5$)
Source: Research results

When the government chooses to direct part of the budget for higher education incentive to the expansion of public universities (REUNI, which is associated with a decrease in the cost of entrance ϕ , as discussed in Assumption 2 of the previous section) we get that the wealth gap between individuals who choose private university and those who choose public university diminishes substantially – note the distance between a^* and a^{**} in the graph. Although individuals who are not as rich as a^* will not be able to further their education, among those who will do it there will be more equality of opportunity in the competition for spots in public universities. Consequently, poor and rich individuals will have, if not equal, at least similar chances of becoming skilled professionals when they graduate.

In order to assess how the model's output changes when model's parameters change, we perform a sensitivity analysis. For that, one of the most common approaches is the one-factor-at-a-time procedure. This methodology can be summarized by two simple steps: (i) changing one input parameter while keeping others at their baseline values and (ii) repeating first step for all interesting parameters while returning the previous parameter to its baseline value. The one-factor-a-time procedure provides then, important comparability results since the experiment's results are computed with

reference to the same baseline model and minimizes computer programming crashes, in comparison when several input parameters are changed simultaneously. However, it is important to highlight that this analysis is concerned to the partial equilibrium model, since no calibration was performed, therefore, the baseline model is chosen subjectively¹³. Moreover, prices are given, hence, the skill premium is constant what enhances the results in comparison with a general equilibrium model.

The sensitivity analysis focuses on the educational and government policies parameters: Q^0 , Q^1 , φ , τ and δ ¹⁴. Moreover, we will analyze how these parameters will change the threshold bequests: a^* and a^{**} , since these bequest levels summarize all three possible education strategies for the optimal solution of young agent's problem. Table 1 describes the sensitivity analysis results:

Model	a^*	a^{**}
Baseline (Q^0, Q^1, φ, τ e δ)	100.00%	100.00%
1.01Q^0	100.00%	98.47%
1.05Q^0	100.00%	93.22%
1.01Q^1	98.00%	101.05%
1.05Q^1	90.62%	105.44%
1.01 φ	100.00%	100.96%
1.05 φ	100.00%	104.97%
1.01 τ	99.80%	100.10%
1.05 τ	98.60%	100.67%
1.01 δ	100.20%	99.81%
1.05 δ	101.40%	99.24%

Table 1: Sensitivity analysis for educational and government policies parameters.

Table 1 above shows how the baseline model results change when the parameters of interest increase in 1% and 5% while keeping all the other parameters at the baseline nominal values. For instance, an increase of 1% (5%) of the quality of public universities (Q^0), decreases the threshold value of a^{**} in 1.53% (6.78%) while keeps a^* constant. In other words, since a^{**} decreased, more young (richer) agents will choose public universities, furthermore, since a^* did not change, the increase in the

¹³ Although the baseline parameters were chosen subjectively, the qualitative results of sensitivity analysis will not change with respect to the change of the nominal baseline parameters value, as long as, the theoretical assumptions of the model hold.

¹⁴ The baseline parameters values are: 0.9, 0.6, 0.6, 0.5, 0.5, respectively.

quality of public universities will not have any effect for constrained young agents. The interpretation of changes in the quality of private universities (Q^1) is similar (although the effect is different): an increase of 1% (5%) in Q^1 decreases in 2% (0.68%) the value of a^* , so that a smaller proportion of poor young agents will have access to private university (or to university, generally). At the same time, it increases in 1.05% (5.44%) the threshold value of a^{**} , which means that a smaller share of rich high school graduates will choose public university. The same logic can be applied to the parameters φ , τ and δ , but as explicated in Table 1, the model is robust to them.

All the parameters have the expected results. Parameters Q^0 and Q^1 results indicate that they have higher effect in the model. This can be explained for the fact that this is a partial equilibrium, as noted before. Note that by increasing the quality of universities, the supply of skilled workers increases, therefore, in a general equilibrium setup, it is expected that the skill premium (difference between skilled and unskilled wage) decreases, offsetting the incentives to become a skilled worker. Therefore, there is an overestimation of the effects of these parameters in table 1. Despite that, it is an interesting result the fact that decreasing the gap of quality between private and public universities has the highest effect on the agent's optimal strategy.

5 CONCLUDING REMARKS

This research aimed to contribute to the debate on human capital investment, and to develop a theoretical model that is fit to explain how individuals make educational choices in Brazil, and how the government can affect these choices through educational policies. This is the first effort of a broader investigation – the model developed in this paper will be calibrated for the case of Brazil in order to analyze the effects of governmental incentives on macroeconomic variables such as economic growth and wealth inequality. Moreover, simulation will provide quantitative results for variables such as skill premium.

The analytical results are consistent with the observed reality of higher education in Brazil – they reveal that public university is more desirable than private university, and that agents who have enough resources to succeed in the selection process will further their studies in the public system. Furthermore, the results reveal that individuals who are constrained by their wealth may choose not to further their education. Finally, the results indicate that the existence of governmental subsidies make it more likely that a financially constrained individual attend university.

The partial equilibrium analysis give us some insight on the impacts of governmental incentives on human capital formation through education. While the existence of the FIES fund allows that some financially constrained individuals be given a chance of becoming qualified in the future, the fact that they will be giving up the

opportunity of working and saving right now (and at the same time acquire a significant debt) deters many of them from attending private university. On the other hand, a ProUni scholarship is, obviously, more effective in increasing the share of poor individuals in the higher education system. However, the analysis suggests that this form of incentive tends to amplify the gap between poor and rich individuals on what concerns public university access. Given that the share of public university attendees that become skilled in the future is higher than that of private university graduates, it is debatable whether ProUni does much for reducing inequality of opportunity. Accordingly, the partial equilibrium analysis indicates that the expansion of public universities is the most effective educational incentive in the sense that it prevents wealth bias and promotes equality of opportunity in the higher education selection system.

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