

JOÃO LUCAS DE PINHO CARVALHO

**THE EFFECTS OF TAX EVASION ON ECONOMIC
GROWTH: A STOCHASTIC GROWTH MODEL
APPROACH**

Dissertação apresentada à Universidade Federal de Viçosa, como parte das exigências do Programa de Pós-Graduação em Economia Aplicada, para obtenção do título de *Magister Scientiae*.

VIÇOSA
MINAS GERAIS - BRAZIL
2019

**Ficha catalográfica preparada pela Biblioteca Central da Universidade
Federal de Viçosa - Câmpus Viçosa**

T

C331e
2019
Carvalho, João Lucas de Pinho, 1991-
The effects of tax evasion on economic growth : a
stochastic growth model approach / João Lucas de Pinho
Carvalho. – Viçosa, MG, 2019.
iv, 35 f. : il. ; 29 cm.

Texto em inglês.

Inclui apêndices.

Orientador: Ian Michael Trotter.

Dissertação (mestrado) - Universidade Federal de Viçosa.

Referências bibliográficas: f. 27-29.

1. Macroeconomia. 2. Sonegação fiscal.
3. Desenvolvimento econômico. 4. Programação dinâmica.
I. Universidade Federal de Viçosa. Departamento de Economia
Rural. Programa de Pós-Graduação em Economia Aplicada.
II. Título.


CDD 22. ed. 339

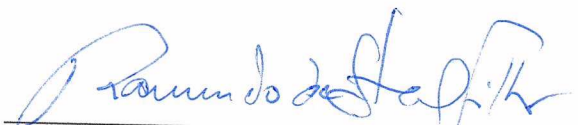
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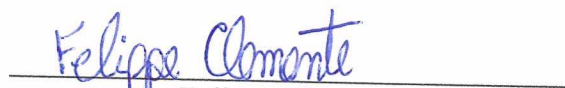
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APROVADA: 26 de março de 2019.


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Abstract

CARVALHO, João Lucas de Pinho, M.Sc., Universidade Federal de Viçosa, March, 2019. **The Effects of Tax Evasion on Economic Growth: a stochastic growth model approach.** Advisor: Ian Michael Trotter. Co-advisor: Felipe Clemente.

The present work aims to analyze the macroeconomic relations between tax evasion and public and private investment and their implications for economic growth through a stochastic growth model in discrete-time. Taxation is important for many aspects for growth. Tax evasion creates negative impacts on the economy such as the fall in government revenues, that fund public infrastructure, education and other services. When private capital and public spending are substitutes in the productive sector, tax evaded can be used by private agents to raise funds to finance private domestic investment, in order to mitigate the negative externalities of tax evasion on productive public spending. We use the stochastic dynamic programming approach to derive the optimal plans for consumption and tax evasion rate. Therefore the effect of tax evasion on economic growth is ambiguous because the trade-off between the gain from tax evasion to disposable income and the loss because of the lower productivity due to lower public input. Also, we did the comparative dynamics on optimal values of consumption and tax evasion rate. Changes in the tax rate and penalty have ambiguous effects on optimal consumption and depend on the enforcement parameters of the tax authority, while the effects on optimal tax evasion rate are mostly positive and consistent with theory.

Key-words: Macroeconomics, Tax Evasion, Development Economics, Dynamic Programming.

Resumo

CARVALHO, João Lucas de Pinho, M.Sc., Universidade Federal de Viçosa, março de 2019. **The effects of tax evasion on economic growth: a stochastic growth model approach.** Orientador: Ian Michael Trotter. Coorientador: Felipe Clemente.

O presente trabalho busca analisar as relações macroeconômicas entre evasão fiscal e investimento público e privado e suas implicações para o crescimento econômico, utilizando um modelo de crescimento estocástico em tempo discreto. A tributação é importante para muitos aspectos do crescimento. A evasão fiscal cria impactos negativos na economia como a queda nas receitas do governo, que financiam a infraestrutura pública, a educação e outros serviços. Quando o capital privado e os gastos públicos são substitutos no setor produtivo, a evasão fiscal pode ser usada por agentes privados para levantar fundos para financiar investimentos privados, a fim de mitigar as externalidades negativas da evasão fiscal sobre os gastos públicos produtivos. Usamos a abordagem da programação dinâmica estocástica para obter os planos de consumo e taxa de evasão fiscal ótimos. Portanto, o efeito da evasão fiscal sobre o crescimento econômico é ambíguo, devido ao *trade-off* entre o ganho da evasão fiscal para a renda disponível e a perda por causa da menor produtividade devido à menor participação do insumo público. Além disso, fizemos a dinâmica comparativa sobre os valores ótimos de consumo e taxa de evasão fiscal. Mudanças na taxa de imposto e na penalidade têm efeitos ambíguos sobre o consumo ótimo e dependem dos parâmetros de fiscalização da autoridade tributária, enquanto os efeitos sobre a taxa de evasão fiscal ótima são, na maior parte, positivos e consistentes com a teoria.

Palavras-chaves: Macroeconomia, Sonegação Fiscal, Desenvolvimento Econômico, Programação Dinâmica.

1 Introduction

The public sector is an essential contributor to the economic growth by making investments in a variety of ways. Such investments can only be made through the collection of taxes, which is the main resource available to the State. However, economic performance may be compromised when agents fail to pay taxes, thus increasing their disposable income. The literature calls it a income tax evasion, which it constitutes a sizable share of underground economy, a chronic problem in virtually every country. The problem is more critical in developing countries, where tax evasion is directly linked to cultural aspects, weak institutions and low quality of governance. Therefore, tax authorities should be concerned with controlling the level of tax evasion.

Tax evasion, internationally known as the tax gap, is defined as a specific collection deficiency and it consists in the difference between actual payments and the legally required obligation. It is important to emphasize that the tax gap is not exactly equal to the amount of additional revenue that would be collected by a stricter taxation, since a perfect tax would significantly affect the economic scenario, so that the tax base would certainly be changed. As a consequence, at least in theory, net income could even be lower. Thus, the standard tax gap measures should be interpreted cautiously. They are only an approximation of the likely immediate effects of marginal improvements in the imposition. (FRANZONI, 1998).

Another concept related to the loss of revenue is tax avoidance, in which individuals reduce their own tax in a way that was not desired by legislators but was not expressly foreseen and prohibited by law. Tax avoidance is typically carried out through structured transactions in order to minimize tax liability. The evasion differs from the avoidance of being illegal, and hence subject to punishment. However, in concern to the economic function, evasion and avoidance obviously has very strong similarities and for the most part, can hardly be distinguished. Therefore, the work focuses only on tax evasion, due to the difficulty in identifying and measuring tax avoidance, since it is not illegal. (SIQUEIRA; RAMOS, 2005; FRANZONI, 1998).

Tax evasion creates negative impacts on the economy and the welfare of society. One of the impacts is the fall in government revenue which can lead to higher tax rates, negatively affecting firms and households. Measurement is another problem with tax evasion, since it's a criminal activity, it's difficult to measure the amount of tax effectively evaded from an economy.

The problem of tax evasion is common in both developed and developing countries. Crivelli, Mooij and Keen (2016) estimated global revenue losses at about \$650 billion per

year, of which about 1/3 are from developing countries. The table below shows the lowest rates of tax evasion among the Organization for Economic Co-operation and Development (OECD) countries.

Table 1 – Tax evasion rate among developed countries in 2010

Countries	Tax Evasion Rate (% GDP)
Spain	0.4
Portugal	0.5
Belgium	0.7
Canada	0.8
Japan	0.9
Germany	1.1
French	1.1
Netherlands	1.3
Finland	1.5
Denmark	1.7
Norway	1.7
Sweden	1.7

Source: [Clemente and Lirio \(2017\)](#), [Cobham and Janský \(2018\)](#).

Despite the great efforts expended by the Brazilian tax authority, tax evasion in Brazil is still quite pronounced. In Brazil, there is no precise estimate of its level. [Wasilewski \(2001\)](#) places the level of tax evasion in amounts ranging from 15% to 40% of potential collection, which is already high. The data show a very marked rate of tax evasion among Latin American countries. The table below shows the rates of tax evasion among developing countries.

Table 2 – Tax evasion rate among developing countries in 2010

Countries	Tax Evasion Rate (% GDP)
Chile	11
Mexico	20
Argentina	21.2
Colombia	23.5
Uruguai	26.3
Brazil	27.6
Bolivia	29
Peru	37.3

Source: [Clemente and Lirio \(2017\)](#), [Cobham and Janský \(2018\)](#).

Taxation is important for many growth aspects: tax revenues fund public infrastructure, education, legal systems, and others. When there is misallocation of resources, corruption and the government provides poor public services, agents cannot see why they should pay taxes properly. In the case of firms, the unstable economic environment favors tax evasion. ([AGHION et al., 2016](#); [LITINA](#); [PALIVOS, 2016](#)).

In economic terms, the problems of tax evasion arise from the fact that the variables that define the tax base such as sales, income, wealth, and others, are not often observable.

In other words, an external entity can not observe the real value of an individual's tax base, and hence can not know its true tax liability. Thus, taxpayers can take advantage of the imperfect information that the tax administration has on their responsibility and evade taxes. However, sometimes this knowledge can be obtained through audits, and in this case it is said that the tax base is verifiable at a certain cost. (SIQUEIRA; RAMOS, 2005).

Although the consumer-producer does not derive utility from public spending, he knows that tax evasion affects the volume of per capita spending in the economy and therefore the amount of income he will receive from production. This knowledge could encourage tax evasion if the return on the equities generated by tax evasion was higher enough that the positive impact on the production of a larger share of private capital outweighed the negative impact of the externality of public spending. This is likely to happen if the agent faces a favorable gamble, for example with a low probability of being caught and convicted and if the probability of paying a bribe when detected is high. (CÉLIMÈNE et al., 2014).

Let's assume a developing country, where the government faces fiscal noncompliance, but there are taxpayers who want to buy stocks (assuming they have a low risk aversion). Also, let's suppose that the productivity of public spending is low, that people have incentives to pay bribes to government tax collectors, and that tax evasion is widespread. Generally, developing countries have difficulties in implementing an effective fight against corruption and tax evasion. Given these features, this country will experience volatile production fluctuations, as well as possible negative effects on the average growth rate due to the diversion of public resources. Therefore, to reduce the size of production fluctuations, the government could increase the productivity of public spending. In this case, since bureaucrats cannot combat tax evasion, such a policy will only reduce the volatility of public spending, but private capital will still be volatile. However, the situation would be better than the initial situation where both components of the per capita output growth rate are volatile. (CÉLIMÈNE et al., 2014).

There are several ways in which a government can smooth the cyclical fluctuations in the economy from tax evasion. The government could increase the efficiency of public spending to reduce the degree of externality of public spending in the presence of tax evasion. Another possibility would be to reduce the incentive to fraud by employing efficient technology to detect tax evasion or fight against corruption. The government can also limit the negative effects of tax evasion on average growth by allowing people to invest their illegal benefits in the stock markets. How can the government make investing in stock markets an attractive activity for taxpayers? First, by reducing the tax rate and secondly by improving the productivity of public spending. In this case, private equity markets act as substitutes for anti-corruption policies to combat tax evasion. However, if agents are highly risk averse, the effects of wealth on consumption will be important,

thus implying a decrease in their holdings of private capital. (CÉLIMÈNE et al., 2014; PANTEGHINI, 2000; BARONE; MOCETTI, 2011).

A firm's activity levels are affected by the decision to evade taxes. The intuition behind this assertion is that the level of activity in a given period depends on the level of investment. The level of investment, in turn, depends on the firm's level of tax evasion. The latter relation is due, first, to the fact that tax evasion increases future payoffs, arguing for higher investment to increase the survival probability, and second, the fact that a statement deviation in an earlier period may be detected later, arguing for a decrease in investment. (BAUMANN; FRIEHE, 2010).

From the point of view of public investment, the efficient behavior of the public sector in the provision of public goods can stimulate a cooperative reaction of the taxpayers in the form of a better attitude towards the fiscal obligations. For example, in the taxpayer's cost-benefit calculation, the inefficiency of public spending amounts to a waste of resources and implies a less favorable relationship between the supply of public goods and the taxes used to finance them. Consequently, the taxpayer may react with a lower propensity to pay taxes because of fiscal injustice. (BARONE; MOCETTI, 2011).

Over the past few years, the fight against corruption and tax evasion, particularly in developing countries, has become high on the agenda of various international organizations, such as the World Bank and the International Monetary Fund (IMF). This has been motivated by a deepening belief that good quality governance is essential for sustained economic development. (ROSE-ACKERMAN, 1999). Thus, tax authorities have been struggling to develop mechanisms that discourage tax evaders and reduce tax evasion.

To solve problems such as high budget deficits that can cause macroeconomic instability, public agencies have used tools to increase the detection and punishment of evaders. This procedure aims to avoid the loss of revenue of the State in order to provide quality goods and services for the population. (CLEMENTE, 2016).

The proposed model considers tax evasion, public and private investment as endogenous variables and creates a link between them. The representative household will choose consumption, fraction of the evaded income and private capital in order to maximize its discounted lifetime utility. This paper contributes to the literature since it analyzes the macroeconomic implications of tax evasion considering the government's optimizing behavior, which will choose an optimal tax rate in order to maximize the household's utility. There is no work in the Brazilian literature that discusses tax evasion from the macroeconomic point of view, using the stochastic growth models approach. Therefore, unlike Eichhorn (2006), we begin the analysis of how tax evasion affects the optimal tax rate.

Based on this, the following question arises: what are the effects of tax evasion

on the level of public and private investment, since both are directly linked to economic growth?

When private capital and public spending are substitutes in the productive sector, tax evaded is used by private agents to raise funds to finance private investment, in order to mitigate the negative externalities of tax evasion on productive public spending.

To confirm this hypothesis, the main objective was to analyze the macroeconomic relations between tax evasion and the level of private and public investment, that are both factors of economic growth. To achieve the expected results, we incorporate tax evasion within the context of stochastic growth models in discrete time for a general economy. Also, we analyze how tax evasion affects the economic growth and derive the comparative dynamics between the variables of interest.

The present work is divided as follows: in chapter 2 we present a brief literature review of the main studies about tax evasion from the macroeconomic point of view; in chapter 3 we present the model, its assumptions and the representative agent's problem; in chapter 4 the main results of the analytical solution and the comparative dynamics were discussed; and, finally, the main conclusions of the work.

2 Literature Review

In the economic literature, tax evasion as a topic for theoretical investigation was suggested by [Mirrlees \(1971\)](#). The literature highlights¹ that positive or negative effect depends on the quality of governance, the size of the public sector and the level of economic development, and how these factors play a significant role in the results of corruption and tax evasion. ([DZHUMASHEV, 2014](#)). [Allingham and Sandmo \(1972\)](#) in their seminal work, developed a model of tax evasion using the crime-economics approach² and the optimal portfolio analysis in the economics of uncertainty³. There are also empirical works⁴ that strive to understand the behavior of taxpayers and tax authorities. Recently, in order to verify the relation between tax evasion and corruption and its effects on macroeconomic variables (inflation and investment, for example), the authors have used dynamic stochastic general equilibrium models (DSGE) and stochastic growth models⁵, the latter being the focus of this work.

First, we present the main results of the seminal work of [Allingham and Sandmo \(1972\)](#) and its first extension of [Yitzhaki \(1974\)](#). Finally, we present a brief review of the main works developed in the literature until recently.

The relation between taxation and risk-taking has been concerned with the effect of taxes on consumer portfolio decisions. However, there are some problems which are not naturally classified under this rubric and one of these problems is tax evasion. The model presented here is related to the studies of economics of crime, as in [Becker \(1968\)](#), and also to the optimal portfolio analysis of the economy of uncertainty by [Arrow \(1974\)](#). A simple static model is presented in the decision to evade taxation is the one that the individual is concerned about, ignoring the interrelationships that probably exist with other types of economic choices. ([ALLINGHAM; SANDMO, 1972](#)).

The decision to declare a tax is subject to uncertainty. The reason for this is that failure to declare a person's entire income to the tax authorities does not automatically result in a penalty. The taxpayer chooses between two main strategies: i) he can declare his actual income; ii) he can declare less than your actual income. If he chooses the latter strategy, his reward will depend on the investigation by the tax authorities. If it is not investigated, it will be clearly better than in strategy i). If he is investigated, he will be worse off. The choice of a strategy is therefore non-trivial.

¹ See [Leff \(1964\)](#), [Lui \(1985\)](#), [Mauro \(1995\)](#), [Barreto \(2000\)](#), [Blackburn, Bose and Emranul Haque \(2006\)](#).

² See also [Yitzhaki \(1974\)](#), [Cowell \(1985\)](#), [Sandmo \(2005\)](#), [Lee \(2016\)](#).

³ See also [Lin and Yang \(2001\)](#), [Litina and Palivos \(2016\)](#), [Levaggi and Menoncin \(2015\)](#).

⁴ See [Fisman and Svensson \(2007\)](#), [Alm, Martinez-Vazquez and McClellan \(2015\)](#), [Slemrod \(2007\)](#).

⁵ See [Dzhumashev and Gahramanov \(2009\)](#), [Blackburn and Powell \(2011\)](#), [Cerqueti and Coppier \(2011\)](#), [Berliant and Fujishima \(2017\)](#).

The model assumes that the taxpayer follows the von Neumann-Morgenstern axioms for the behavior under uncertainty. Its function of cardinal utility has income as its sole argument, that is, it must be understood as the function of indirect utility with constant prices. Marginal utility will be considered positive and strictly decreasing, so that the individual is risk averse.

Actual income, W , is given exogenously and is known by the taxpayer, but not by the government tax collector. The tax is levied at a constant rate, θ , on the declared income, X , which is the taxpayer's decision variable. However, with some probability p , the taxpayer will be subjected to investigation by the tax authorities, who will then know the exact amount of their actual income. It turns out that the taxpayer will have to pay tax on the undeclared value, $W - X$, at a penalty rate π that is greater than θ . This formal representation of the taxpayer's choice situation is a simplification of his situation in the real world. In particular, the present formulation ignores some of the elements of uncertainty. (ALLINGHAM; SANDMO, 1972).

The taxpayer will choose X in order to maximize:

$$E[U] = (1 - p)U(Y) + pU(Z) \quad (2.1)$$

where

$$Y = W - \theta X$$

$$Z = W - \theta X - \pi(W - X)$$

The first and second-order condition for an interior maximum of (2.1) can be written as:

$$\frac{\partial E[U]}{\partial X} = -\theta(1 - p)U'(Y) - (\theta - \pi)pU'(Z) = 0 \quad (2.2)$$

$$\frac{\partial^2 E[U]}{\partial X^2} = \theta^2(1 - p)U''(Y) + (\theta - \pi)pU''(Z) < 0 \quad (2.3)$$

being the latter satisfying the concavity assumption of the utility function.

In this analysis, the conditions for the existence of an interior maximum are of particular importance. It is not assumed that $0 < X < W$ because it must depend on the values of the parameters. To see what conditions of the parameter values are required for

an interior solution, we evaluate the expected utility at $X = 0$ and $X = W$. Since the expected marginal utility is decreasing with X , it follows that:

$$\left. \frac{\partial E[U]}{\partial X} \right|_{X=0} = -\theta(1-p)U'(W) - (\theta - \pi)pU'[W(1-\pi)] > 0 \quad (2.4)$$

$$\left. \frac{\partial E[U]}{\partial X} \right|_{X=W} = -\theta(1-p)U'[W(1-\theta)] - (\theta - \pi)pU'[W(1-\theta)] < 0 \quad (2.5)$$

Equations (2.4) and (2.5) can be rewritten as:

$$p\pi > \theta \left[p + (1-p) \frac{U'(W)}{U'[W(1-\theta)]} \right] \quad (2.6)$$

$$p\pi < \theta \quad (2.7)$$

Equation (2.7) implies that the taxpayer will evade tax if the expected payment of the tax on the undeclared income is less than the standard rate. In other words, an individual engages in tax evasion if, and only if, his expected return is positive, that is, if the tax rate is greater than the expected fine. This result stems from the theory of investment under uncertainty. Since the factor in brackets (2.6) is positive and less than one, both conditions provide a set of positive parameter values that will guarantee an interior solution. (ALLINGHAM; SANDMO, 1972). The authors conclude that when the tax rate increases, there will be two opposing effects, an income effect and a substitution effect.

Starting from the above model, Yitzhaki (1974) shows that if the fine is imposed on the tax evaded, there are no contradictory effects. Assuming that the taxpayer has an absolute risk aversion which decreases with income, it is concluded that, as the tax rate increases, the evaded income decreases. In this case, there is no substitution effect.

Since $F > 1$ is the fine, now the taxpayer will choose X so as to maximize:

$$E[U] = (1-p)U(Y) + pU(Z) \quad (2.8)$$

where

$$Y = W - \theta X \quad (2.9)$$

$$Z = W - \theta X - F\theta(W - X) \quad (2.10)$$

The first-order condition is:

$$\theta[-(1-p)U'(Y) + p(F-1)U'(Z)] = 0 \quad (2.11)$$

The conditions for interior solution are:

$$\frac{U'(W)}{U'[W(1-F\theta)]} < \frac{p(F-1)}{1-p} \quad (2.12)$$

$$pF < 1 \quad (2.13)$$

Differentiating (2.11) according to θ yields:

$$\frac{\partial X}{\partial \theta} = -\frac{\theta}{D}(1-p)U'(Y)X[R_A(Z) - R_A(Y)] + F(W-X)R_A(Z) \quad (2.14)$$

where $D \equiv \theta^2[(1-p)U''(Y) + P(F-1)^2U''(Z)]$ and $R_A(Y) < R_A(Z)$ leads to $\partial X/\partial \theta > 0$. The intuition behind this result is the absence of the substitution effect: a change in the tax rate does not alter the relative price of income across the two states of the world. Changing the tax rate encourages evasion by the same rate as it prevents it (tax evasion savings as well as penalty increase proportionally with increasing tax rate).

Although growth theory is a very active field of economic research, models that explicitly take into account the possibility of tax evasion are rare. Noteworthy exceptions will be discussed in more detail. It is surprising that the literature does not provide a clear prediction of the relationship between tax evasion and economic growth. However, two different approaches describe the influence of tax evasion from a macroeconomic point of view - a short-term approach that focuses on the effects of aggregate demand for tax evasion on static Keynesian models and a long-term approach that emphasizes the implications of tax evasion for private investment and economic growth.

Tax evasion can affect the allocation of productive factors in various ways. On the one hand, every effort to evade taxpayers and, on the other hand, every effort of the tax authority to detect evaders. Tax evasion can also affect incentives to invest and therefore long-term economic growth. This is also true for redistribution associated with tax evasion. Tax evasion redistributes income from the honest to the dishonest and from the detected evaders to the undiscovered. If the marginal propensity to save and invest is different for both groups (perhaps because both are determined by an underlying difference in risk-taking propensity), such redistribution is likely to affect the total amount of the investment.

Wrede (1995) formulates a model of endogenous growth of overlapping generations (discrete time) with a completely rival productive public good, where saving and tax

evasion (interest income) decisions are endogenous. With individuals having logarithmic preferences and a production function with increasing returns to scale, it shows that tax evasion has a negative impact on growth because the loss in tax revenue leads to lower levels of public good and lower income (and savings). Regarding the parameters of tax execution, its results are ambiguous and depend on the intertemporal elasticity of substitution.

Caballé and Panadés (1997) study in particular how the tax compliance policy in the form of audit and the fine rate affect the rate of economic growth in a model of overlapping generations (discrete time) with identical individuals and logarithmic preferences, where the public goods are productive and financed by taxes. The authors indicate that this effect is generally ambiguous and depends on the importance of public inputs in the production process because (if compliance is not perfect) more rigorous enforcement increases compliance, taking the two effects in opposite directions. On the one hand, private saving decreases with expected disposable income. On the other hand, the increase in public inputs leads to greater investment due to the increase in the productivity of private capital.

Lin and Yang (2001) adapted part of the growth model of Barro (1990) into an endogenous continuous-time growth model with tax evasion. If public goods are only consumer goods, Barro (1990) finds that the growth rate is strictly decreasing on tax rate. Lin and Yang (2001) show that for individuals with logarithmic preferences, economic growth is increasing on tax evasion because resources are diverted from the unproductive government sector to the productive private sector.

Chen (2003) investigates an endogenous continuous-time growth model with a Cobb-Douglas production function with public capital financed by a tax that can be evaded. It investigates the optimal saving and evasion decision in an environment without uncertainty, assuming that individuals hold a sufficient number of firm assets such that auditing for a fraction of the income is guaranteed by law or by large numbers. The government optimizes the tax rate, audit probability and penalty rate given the consumer evasion decision. In general, these policies have ambiguous effects, but for a set of realistic parameters, the author finds that growth decreases with tax evasion.

Célimène et al. (2014) study the impact of tax evasion and tax corruption on private investment and government spending, two main determinants of growth rate and per capita GDP volatility. For this, the authors use the standard portfolio arguments adopting a continuous-time stochastic growth model for an open economy. The results indicate that a decrease in the probability of being caught, or a lower penalty rate or a greater probability of detecting a corrupt bureaucrat, increases the risk-adjusted return on undeclared income. Hidden income is used to buy foreign stock (or equivalently to hold a fraction of the foreign country's physical capital). The gains from this investment

are consumed (effect-wealth on consumption). This wealth-effect reduces savings (and therefore negatively affects the growth rate of private capital) and its magnitude depends on the curvature of the utility function.

The greater the risk aversion of the domestic agent, the stronger the negative impact on the growth rate of savings. In addition, public expenditure financing is decreasing as tax evasion increases. This, in turn, reduces the gross domestic return of a unit of undeclared income, and thus decreases the share of domestic capital in the total wealth held by the family. The authors emphasize the role of the stock market, showing that the outcome of evasion to the private sector is not necessarily seen as negative, but as an opportunism and optimal response of individual agents to a governance failure of the tax administration. The stock market plays the same role as a tax exemption policy. In societies where the share of private investment in relation to GDP is increasing, where evaders generally choose to protect revenues from their illegal activities from official financial institutions, and where the productivity of public spending is generally low, tax evasion and tax corruption can contribute to the development of private capital if people find an opportunity to invest the proceeds of their illegal activities in the stock markets.

Levaggi and Menoncin (2015) shows that the level of tax evasion depends on the investor's preference parameters, but is not affected by uncertainty: the evaders adjust their consumption path according to uncertainty, not tax evasion. This result, which is in line with earlier findings on dynamic tax evasion⁶ shows that government should not increase uncertainty in fiscal parameters, as traditional literature⁷ seems to suggest. This policy will simply reduce consumption with potentially perverse effects on economic growth. Without tax evasion, a high investment in risk-free assets implies high risk aversion. When evasion is appropriate, a riskless portfolio is a byproduct of evasion, and optimal asset allocation cannot be used to measure investor risk aversion. In this framework, high liquidity can be used to direct audits, as long as it is coupled with other indicators that prevent audits of highly risk-averse individuals who most likely will not be able to escape the tax. Tax evasion has an interesting compensating effect on the distortion created by a symmetrical fiscal system. Through tax evasion, the government shares the expected losses with investors only for the assets that have been declared. This increases the risk taken by the investor and leads to a reallocation between financial assets.

A preliminary result of the literature review is that the results are sensitive to the particular specification of the problem as an individual or firm tax evasion problem. These differences may arise because of the different evasion technologies used - sometimes related to the objective function in question. Even if tax evasion is discussed from the perspective of firms, the particular specification of evasion technology plays an important role. Assuming a particular evasion technology implies that government can influence the

⁶ See Bernasconi, Levaggi and Menoncin (2015)

⁷ See Alm (1988) and Slemrod and Yitzhaki (2002)

production decision through the possibility of evading taxes.

3 The Model

The model developed here was adapted and simplified from [Eichhorn \(2006\)](#). I present a discrete-time stochastic growth model, which describes the choice of a representative agent and presents the household preferences, production possibilities and the role of government. The contribution of the present model lies in the fact of incorporate the enforcement costs and analyze the effects on economic growth through closed-form solutions.

The structure of this chapter is as follows: first, the assumptions of the model will be presented; in the next section, the representative agent problem will be described and the solutions presented in the next chapter.

3.1 Assumptions

Assumption 3.1.1 (Utility) *There is a continuum of identical individuals of mass 1 without population growth. Each individual has a utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$, $(c, g) \mapsto u(c, g)$, defined on private consumption c and a public consumption good g .*

The utility function u is increasing (reflecting a desire for more consumption) and concave (reflecting the decreasing marginal value of additional consumption), additive separable, state and time independents, exhibits a constant coefficient of relative risk aversion and is separable in c and g . The utility function has the form:

$$u(c_t, g_t) := \begin{cases} (1 - \theta) \frac{c_t^{1-\gamma} - 1}{1-\gamma} + \theta v(g_t) & \text{for } \gamma \neq 1 \\ (1 - \theta) \ln(c_t) + \theta v(g_t) & \text{for } \gamma = 1 \end{cases} \quad (3.1)$$

where

- γ is the Arrow-Pratt Coefficient of relative risk aversion;
- θ is the weight attributed to consumption of the public good;
- $v : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $g \mapsto v(g)$ is a strictly increasing, strictly concave and continuously differentiable function.

An important feature of these preferences in growth theory is not that the relative risk aversion coefficient is constant, but that the intertemporal elasticity of substitution

is constant (because most growth models do not present uncertainty). The intertemporal elasticity of substitution regulates how individuals are willing to replace consumption over time, thus determining their savings and consumption behavior.

The time-separability assumption just means that the intertemporal utility function is additive, i.e., $U(c_0, c_1, \dots, c_{T-1}) = u^{(0)}(c_0) + u^{(1)}(c_1) + \dots + u^{(T-1)}(c_{T-1})$, where $u^{(t)}(c_t)$ is the utility contribution from period- t consumption, $t = 0, 1, \dots, T - 1$.

Moreover, we assume that individuals maximize their expected utility (von Neumann-Morgenstern axioms). Individuals discount future utility at a constant rate $\beta \in (0, 1)$, i.e., they value current consumption more than consumption in the future time.

Note that γ also denotes the inverse of the substitution elasticity of consumption between two periods, that is, for high values of γ , an individual is less willing to move away from a smooth consumption path.

The representative agent optimizes its stream of consumption and evasion over an infinite planning horizon $\mathcal{T} := \{0, 1, 2, \dots\}$, where time is discrete, taking into account that the restrictions imposed by the production possibility set and by the taxation and penalty system are exogenous.

Assumption 3.1.2 (Production) *Unlike Eichhorn (2006), a stochastic growth model is used assuming that the production function $F : \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$ is C^2 in K and L , and satisfies the Inada conditions. Moreover, F exhibits constant returns to scale in K and L .*

The aggregate production technology is Cobb-Douglas but now also includes an aggregate stochastic shock z_t , which z_t follows a Markov chain with values in the set $\mathcal{Z} \equiv \{z_t, \dots, z_N\}$. The production function is of the form:

$$Y_t = z_t K_t^\alpha L_t^{1-\alpha} \quad (3.2)$$

Expressing this in per capita terms yields:

$$\frac{Y_t}{L_t} = \frac{z_t K_t^\alpha L_t^{1-\alpha}}{L_t} \quad \Rightarrow \quad y_t = z_t k_t^\alpha \quad (3.3)$$

Taxes are levied on income and tax revenues are used to finance public goods. For simplicity, the functions considered for tax and penalties are linear.

Assumption 3.1.3 (Tax System) *The tax system is fully specified by a constant income tax rate $\tau \in (0, 1)$ for declared positive income. The tax schedule is given by the function $T : \mathbb{R} \rightarrow \mathbb{R}_+$, $y_d \mapsto T(y_d)$:*

$$T(y_d) := \begin{cases} 0 & \text{for } y_d < 0 \\ \tau y_d & \text{for } y_d \geq 0 \end{cases} \quad (3.4)$$

where y_d is the declared income. The values defined for the T function are very intuitive: if the declared income is negative, taxation will be null; and for non-negative amounts of declared income, a tax rate τ on the declared income is charged.

Assumption 3.1.4 (Penalty System) *The penalty system is described by a constant fine rate $\phi > 0$, to be paid on the amount of evaded tax. The fine schedule is given by the random function $\tilde{\phi} : \mathbb{R}_{++} \times \Omega \rightarrow \mathbb{R}_+$, $(y_e, \omega) \mapsto \tilde{\phi}(y_e, \omega)$:*

$$\tilde{\phi}(y_e, \omega) := \begin{cases} \phi \tau y_e & \text{for } \omega = y_e \\ 0 & \text{for } \omega = 0 \end{cases} \quad (3.5)$$

where

- $(\Omega, \mathcal{F}, \mu)$ is the probability space with event set $\Omega := \{y_e, 0\}$;
- $\mathcal{F} := \mathcal{P}(\Omega)$ is the σ – algebra;
- a function $\mu : \mathcal{F} \rightarrow \mathbb{R}_+$ called probability measure, where $\mu(y_e) := p$ and $\mu(0) := 1 - p$, for some detection probability $p \in (0, 1)$.

The expressions for the function $\tilde{\phi}$ tell us that if event y_e occurs, that is, with probability p the agent is discovered by evading taxes, so a fine will be charged to the evader; if the agent is not discovered, with a probability $1 - p$, the evader will be free to pay a fine.

As there is no tax repayment (in case of tax overpayment), the above specifications of the tax and penalty structure ensures that a risk-averse individual always chooses $y_d \in [0, y]$. With identical individuals, this configuration leads to an economy of representative consumers and producers. Per capita output accrues to the consumer as revenue. At each point in time t , an individual with a given income y_t has to decide simultaneously how much income to declare y_d and, respectively, how much to evade, y_e , where $y_d + y_e = y_t$ always holds. Tax evasion is therefore possible underreporting income. This approach is common in the literature on tax evasion since the seminal paper by [Allingham and Sandmo \(1972\)](#). ([EICHHORN, 2006](#)).

The government doesn't know the initial level of capital stock per capita k_0 (hence, it cannot infer the real income stream), investigates a fraction p of all individuals with a

enforcement cost $\varphi(p)$, that is defined by the function $\varphi : (0, 1) \rightarrow \mathbb{R}_+$, $p \mapsto \varphi(p)$. This function starts at 0 ($\varphi(0) = 0$), is strictly increasing ($\varphi' > 0$), strictly convex ($\varphi'' > 0$) and is infinite when p approaches 1 ($\lim_{p \rightarrow 1} \varphi(p) = \infty$). The government detects evasion if, and only if, the evader is subject to a random audit. Government actions are exogenous, that is, the parameters τ , p and ϕ are given.

It is assumed that individuals are fully informed about the penalty and the audit rate. Also, individuals chooses a fraction $e_t \in (0, 1)$ of income to hide from the government. Thus, from an individual's point of view, auditing is random and disposable income after taxes and fines is a binary random variable:

$$\tilde{y}_d(t, \omega) := \begin{cases} y_d(t, y_e) = (1 - \tau)y_t + \phi\tau y_e & \text{w.p. } p \\ y_d(t, 0) = (1 - \tau)y_t + \tau y_e & \text{w.p. } 1 - p \end{cases} \quad (3.6)$$

At each point in time t , the individual is faced with the decision to pay less taxes by evading income with the risk of a fine in case of an audit. Tax evasion is similar to portfolio decision with a safe and risky asset (evaded income). Denote by $\bar{r} := 1 - p\phi$ the expected return of one unit of evaded tax. In a static setup, a risk-averse individual takes risk (evades taxes) if and only if the expected return on the first unit is positive, that is, $\bar{r} > 0$. (ALLINGHAM; SANDMO, 1972; YITZHAKI, 1974; CÉLIMÈNE et al., 2014; EICHHORN, 2006).

Assumption 3.1.5 (Government Budget) *Tax revenues and all tax collection with fines are used to finance public goods. Then,*

$$g_t = \tau_t^e y_t - \varphi(p) \quad , \forall t \in \mathcal{T}. \quad (3.7)$$

where τ_t^e is the expected tax rate and $\varphi(p)$ is the enforcement cost.

Lemma 3.1.1 (Expected Tax Rate) *For given income y_t , the statutory tax rate τ and enforcement parameters p and ϕ , and share of evaded income $e_t := \frac{y_e}{y_t}$, the expected tax rate is:*

$$\tau^e = (1 - \bar{r}e_t)\tau \quad (3.8)$$

Proof 3.1.1 *Fix the following parameters: t , y , τ , p , ϕ , e and remember $\bar{r} = 1 - p\phi$. So the expected tax and penalty payment is:*

$$\begin{aligned}
E[T(y_d) + \tilde{\phi}(y_e, \omega)] &= \tau y(1-p)(1-e) + p[(1-e)\tau y + \phi\tau e y] \\
&= \tau y[(1-e)(1-p) + p(1-e) + p\phi e] \\
&= \tau y(1-e + p\phi e) \\
&= \tau y(1-e \underbrace{(1-p\phi)}_{\bar{r}}) \\
&= \tau y(1-e\bar{r})
\end{aligned}$$

Therefore, income is taxed at the expected rate $\tau^e := \tau(1-e\bar{r})$.

As a continuum of individuals has been assumed, the law of large numbers ensures that for the whole economy the expected rate is equal to the effective tax rate. Also it is assumed that all revenue is spent for a public good.

3.2 Household Optimization

The first step of the analysis is the optimal allocation of capital over time in an environment of uncertainty from the point of view of the representative agent. As disposable income is random, consumption and savings depend on whether the individual has been audited or not. Savings, s , is equal to the investment in this closed economy and increases the capital stock. The sources of economic growth stem from them. Considering that the audit is uncertain, the stock of capital per capita in discrete-time follows a random difference equation:

$$k_{t+1} = (1-\delta)k_t + \tilde{s}(\tilde{y}_d, \omega) \quad (3.9)$$

where $\delta \geq 0$ is a constant depreciation rate and

$$\tilde{s}(\tilde{y}_d, \omega) := \begin{cases} ((1-\tau) + \tau e_t)y_t - c_t & \text{for } \omega = 0 \\ ((1-\tau) - \phi\tau e_t)y_t - c_t & \text{for } \omega = y_e \end{cases} \quad (3.10)$$

The saving in period t is described by \tilde{s}_t and is random because it can be paid after an audit that can identify tax evasion. Therefore, we have two possible values for k_{t+1} :

$$\begin{aligned}
k_{1,t+1} &= (1-\delta)k_t + ((1-\tau) + \tau e_t)y_t - c_t & \text{w.p. } 1-p \\
k_{2,t+1} &= (1-\delta)k_t + ((1-\tau) - \phi\tau e_t)y_t - c_t & \text{w.p. } p
\end{aligned}$$

The utility function derived from the public good, $v(g_t)$, can be discarded from the optimization problem because we're assuming that the level of the public good is given

(exogenous). Since the taxpayer has to decide how much to consume and save respectively and how much to evade, the individual's problem is:

$$\begin{aligned} & \max_{\{c_t, e_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] & (3.11) \\ \text{s. t. } & k_{t+1} = (1 - \delta)k_t + \tilde{s}(t, \omega) \\ & 0 \leq c_t \leq y_t \\ & 0 < e_t < 1 \\ & k_t \geq 0 \quad , \quad k_0 > 0 \quad , \quad \forall t \in \mathcal{T}. \end{aligned}$$

4 Results

The problem of the representative agent will be done through stochastic dynamic programming. Essentially, the idea behind the Optimality Principle is an optimal plan can be broken into two parts, what is optimal to do today, and the optimal continuation path. Stochastic Dynamic Programming exploits this principle and provides us with a set of powerful tools to analyze optimization in discrete-time infinite-horizon problems.

The decision to evade tax resembles the portfolio decision, i.e., investing in a risky asset is associated with the decision to evade tax, and investing in a risk-free asset is associated with the decision not to evade tax. Moreover, the agent's decision to evade tax will depend on the tax authority's enforcement parameter, i.e., the probability of the agent being discovered or not.

4.1 Optimal Plans

Let us consider the logarithmic utility ($\gamma = 1$). First, the Bellman equation for problem (3.11) is:

$$\begin{aligned} V(k_t) &= \max_{c_t, e_t} \left\{ (1 - \theta) \ln(c_t) + \beta \mathbb{E}_0[V(k_{t+1})] \right\} \\ &= \max_{c_t, e_t} \left\{ (1 - \theta) \ln(c_t) + \beta \left\{ (1 - p)V(k_{1,t+1}) + pV(k_{2,t+1}) \right\} \right\} \end{aligned} \quad (4.1)$$

Taking the first-order conditions of the right-hand side, we find the following expressions:

$$\frac{\partial V(k_t)}{\partial c_t} = 0 \Rightarrow \frac{(1 - \theta)}{c_t} = \beta \left[(1 - p)V'(k_{1,t+1}) + pV'(k_{2,t+1}) \right] \quad (4.2)$$

$$\frac{\partial V(k_t)}{\partial e_t} = 0 \Rightarrow \frac{p}{1 - p} \frac{V'(k_{2,t+1})}{V'(k_{1,t+1})} = \frac{1}{\phi} \quad (4.3)$$

Result (4.3) gives us an interesting result. The marginal rate of substitution between income in the state of the economy where evasion is detected and the state where evasion isn't detected is equal to the inverse of penalty rate, i.e., the taxpayer will evade less taxes when the penalty rate increases. An obvious result written mathematically. Result (4.2) is straightforward: the marginal utility of present consumption is equal to the expected value of discounted capital at a rate β .

In order to find the optimal plans for consumption and evasion we use the Method of Undetermined Coefficients (guess-and-verify)¹. The idea is to guess a particular functional form of a solution and then verify that the solution has in fact this form.

Let us guess that the value function has the following shape:

$$V(k_t) = A \ln(k_t) + B \quad (4.4)$$

where A and B are constants to be determined.

If we substitute our guess into (4.1), the right-hand side of the equation is then:

$$V(k_t) = \max_{c_t, e_t} \left\{ (1 - \theta) \ln(c_t) + \beta [A \ln(k_{t+1}) + B] \right\} \quad (4.5)$$

Taking first-order conditions with respect to c_t and e_t respectively yields:

$$\frac{1 - \theta}{c_t} = \beta A \left[\frac{1 - p}{h_t + \tau e_t y_t - c_t} + \frac{p}{h_t - \phi \tau e_t y_t - c_t} \right] \quad (4.6)$$

$$\frac{(1 - p)\tau}{h_t + \tau e_t y_t - c_t} = \frac{p\phi\tau}{h_t - \phi \tau e_t y_t - c_t} \quad (4.7)$$

where $h_t := (1 - \delta)k_t + (1 - \tau)y_t$. Solving the system above, yields the solutions² for c and e :

$$c = \frac{(h_t - \phi \tau e_t y_t)(-\phi(1 - p) + p)}{(1 - \theta)(\beta A + 1)} \quad (4.8)$$

$$e = \frac{h_t \beta A \bar{r}}{(1 - \theta)(\beta A + 1)\phi\tau} \quad (4.9)$$

Substituting (4.8) and (4.9) to rewrite (4.5) in terms of the optimal values of c and e we get:

$$V(k_t) = (1 - \theta) \ln(c) + \beta \left[(1 - p)A \ln(k_{1,t+1}) + pA \ln(k_{2,t+1}) \right] + \beta [A \ln(k_t) + B] \quad (4.10)$$

Given the expression above and our guess $V(k_t) = A \ln(k_t) + B$, we can rewrite (4.10):

$$A \ln(k_t) + B = (1 - \theta) \ln(c) + \beta \left[(1 - p)A \ln(k_{1,t+1}) + pA \ln(k_{2,t+1}) \right] + \beta A \ln(k_t) + \beta B \quad (4.11)$$

¹ For more details, see [Sargent and Ljungqvist \(2004\)](#).

² We provide the step-by-step on Appendix B.

Rearranging terms yields³:

$$A \left[\ln(k_t)(1 - \beta) - \beta \left[(1 - p)A \ln(k_{1,t+1}) + pA \ln(k_{2,t+1}) \right] \right] + B(1 - \beta) = (1 - \theta) \ln(c) \quad (4.12)$$

If the first term on the left side equals zero, yields:

$$\frac{\beta}{1 - \beta} = \frac{\ln(k_t)}{(1 - p) \ln(k_{1,t+1}) + p \ln(k_{2,t+1})} \quad (4.13)$$

Then we have the values of the coefficients A and B :

$$A = 0 \quad (4.14)$$

$$B(1 - \beta) = (1 - \theta) \ln(c) \Rightarrow B = \frac{1 - \theta}{1 - \beta} \ln(c) \quad (4.15)$$

Thus, the optimal plans for consumption and evasion are given by the following expressions:

$$c_t^* = \frac{(h_t - \phi\tau e_t y_t)(-\phi(1 - p) + p)}{1 - \theta} \quad (4.16)$$

$$e_t^* = \frac{h_t \beta A \bar{r}}{(1 - \theta) \phi \tau} \quad (4.17)$$

The optimal consumption is ambiguous and depend on parameters such as the probability of the evader be discovered and the penalty rate. The risk-averse taxpayer maximizes its expected utility by evading the optimal share of its income when $\bar{r} > 0$, i.e., when the expected return of one unit of evaded tax is positive.

4.2 Comparative Dynamics

In order to visualize the effects of some parameters on the optimal values of the consumption and the share of evaded income, we will make the comparative dynamics. First we analyze the comparative dynamics for the optimal consumption plan.

Proposition 4.2.1 *Given a change in the tax rate, ceteris paribus, its effect on optimal consumption will be ambiguous.*

³ In Appendix C we provide the step-by-step.

$$\frac{\partial c_t^*}{\partial \tau} = \frac{e_t y_t \phi (\phi(1-p) - p)}{1-\theta} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (4.18)$$

If $p = 0$ (taxpayer isn't discovered), an increase in the tax rate implies a positive effect on consumption. On the other hand if $p = 1$ (taxpayer is discovered), an increase in the tax rate implies a negative effect on consumption.

Proposition 4.2.2 *Given a change in the penalty rate, ceteris paribus, its effect on optimal consumption will also be ambiguous.*

$$\frac{\partial c_t^*}{\partial \phi} = \frac{\tau e_t y_t (2\phi(1-p) - p) - h_t(1-p)}{1-\theta} \begin{matrix} \geq \\ \leq \end{matrix} 0 \quad (4.19)$$

The behavior of the penalty rate is similar to that found above. If $p = 0$, an increase in the penalty rate implies a positive effect on consumption. If $p = 1$, an increase in the penalty rate implies a negative effect on consumption.

Now we analyze the comparative dynamics for the optimal share of evaded income plan.

Proposition 4.2.3 *Given a change in the tax rate, ceteris paribus, its effect on optimal share of evaded income will be positive.*

$$\frac{\partial e_t^*}{\partial \tau} = \frac{\beta h_t' \bar{r}}{(1-\theta)\phi\tau^2} > 0 \quad (4.20)$$

An increase in the tax rate causes an increment in the rate of tax evasion. This result is consistent with the literature.

Proposition 4.2.4 *Given a change in the penalty rate, ceteris paribus, its effect on optimal share of evaded income will be negative.*

$$\frac{\partial e_t^*}{\partial \phi} = -\frac{p\beta h_t}{(1-\theta)\phi^2\tau} > 0 \quad (4.21)$$

An increase in the penalty rate leads to a lower rate of tax evasion, which confirms the result (4.3).

Proposition 4.2.5 *Given a change on expected return of one unit of evaded tax, ceteris paribus, its effect on optimal share of evaded income will also be positive.*

$$\frac{\partial e_t^*}{\partial \bar{r}} = \frac{\beta h_t}{(1-\theta)\phi\tau} > 0 \quad (4.22)$$

An increase in the expected return of one unit of evaded tax has a positive effect on tax evasion, i.e., making it more attractive.

Proposition 4.2.6 *Given a change in the discount rate, ceteris paribus, its effect on optimal share of evaded income will also be positive.*

$$\frac{\partial e_t^*}{\partial \beta} = \frac{h_t \bar{r}}{(1-\theta)\phi\tau} > 0 \quad (4.23)$$

An increase in the instantaneous rate of time preference has a positive effect on tax evasion, that is the individual becomes less patient and evades more taxes in the present.

Proposition 4.2.7 *Given a change in the probability of detection, ceteris paribus, its effect on optimal share of evaded income will be negative.*

$$\frac{\partial e_t^*}{\partial p} = -\frac{\beta h_t}{(1-\theta)\tau} < 0 \quad (4.24)$$

An increase in the probability of the evader being discovered, the effect on the tax evasion is negative. The result is straightforward.

4.3 Growth Implications

In order to analyze the effects of tax evasion on economic growth, we first derive the expected growth rate of capital per capita from (3.9). Substituting the optimal plans c_t^* and e_t^* into k_{t+1} , we obtain the capital evolution:

$$\tilde{s}(\tilde{y}_d, \omega) := \begin{cases} ((1-\tau) + \tau e_t^*)y_t - c_t^* & \text{for } \omega = 0 \\ ((1-\tau) - \phi\tau e_t^*)y_t - c_t^* & \text{for } \omega = y_e \end{cases} \quad (4.25)$$

Dividing k_{t+1} by k_t we get:

$$\frac{k_{t+1}}{k_t} = (1-\delta) + \frac{\tilde{s}(y_d, \omega)}{k_t} \quad (4.26)$$

Let $\bar{c}_t := c_t^*/k_t$ and $\bar{e}_t := e_t^*/k_t$. Thus the expected growth rate of capital per capita equals to⁴:

⁴ In Appendix D we provide the complete derivation.

$$\begin{aligned}
\pi_k = \mathbb{E} \left[\frac{k_{t+1}}{k_t} \right] &= (1 - \delta) + (1 - p) \left[(1 - \delta) + \left((1 - \tau) + \tau \bar{e}_t y_t - \bar{c}_t \right) \right] \\
&\quad + p \left[(1 - \delta) + \left((1 - \tau) - \phi \tau \bar{e}_t y_t - \bar{c}_t \right) \right] \\
&= (1 - \delta) + (1 - p) \left[n_t - \bar{c}_t + \frac{m_t}{(1 - \theta)\phi} \right] + p \left[n_t - \bar{c}_t - \frac{m_t}{1 - \theta} \right] \quad (4.27)
\end{aligned}$$

where $m_t := z_t k_t^{\alpha-1} n_t \beta \bar{r}$ and $n_t := (1 - \delta) + z_t k_t^{\alpha-1} (1 - \tau)$.

Proposition 4.3.1 *The impact of changes in the rate of tax evasion and the tax rate on economic growth are ambiguous.*

$$\frac{\partial \pi_k}{\partial e_t} \geq 0 \quad ; \quad \frac{\partial \pi_k}{\partial \tau} \leq 0 \quad (4.28)$$

There is a trade-off between the gain from tax evasion to disposable income and the loss because of the lower productivity due to lower public input. Therefore, the overall effect of tax evasion on growth is ambiguous.

By analogy, we can show that in the absence of tax evasion the growth rate would be given by:

$$\pi_0 = (1 - \delta) + 2n_t \quad (4.29)$$

Proposition 4.3.2 *The growth rate with evasion is higher than it would be without evasion.*

$$\pi_k > \pi_0 \quad (4.30)$$

This higher rate of growth is obviously due to the existence of tax evasion. An economy with tax evasion grows at a higher rate because the increase in expected income is partially saved and contributes to the accumulation of productive capital, while it is used as a pure consumption good in government spending. For this, it is important to consider that the government does not change its policy.

5 Conclusion

We propose a theoretical model in which we incorporate tax evasion under an approach to stochastic growth models and analyze their effects on economic growth. Note that a crucial assumption for the growth effect is, that the possibility that the public good is productive has been excluded.

Our first interesting result concerns the marginal rates of substitution between rents in the two states of the economy, which equals the inverse of the penalty rate. This means that if the tax authority increases the penalty for evasion, the tax evasion rate will be lower. Result consistent with Proposition 4.2.4. Through the comparative dynamics it was possible to analyze, *ceteris paribus*, the effects of changes in some parameters on optimal consumption and evasion. Changes in the tax rate and penalty have ambiguous effects on optimal consumption and depend on the enforcement parameters of the tax authority, while the effects on optimal tax evasion rate was consistent with the literature (see [Yitzhaki \(1974\)](#), [Wrede \(1995\)](#), [Chen \(2003\)](#) and [Célimène et al. \(2014\)](#)).

Finally, tax evasion can be conducive for economic growth if government revenues are used for a good that is unproductive, but the overall effect is ambiguous and also depends on the enforcement parameters. The assumption that policy is not changed if taxes are evaded is crucial for this result. There is a trade-off between the gain from tax evasion to disposable income and the loss because of the lower productivity due to lower public input.

It is important to take into account the limitations of this research. The realization that tax evasion can be conducive to economic growth must be interpreted with caution. One failure in the portfolio theory approach to tax evasion is that it ignores the factors that emerge from social interactions, such as tax evasion can be caused by perceived inequities in the tax system, or it may simply be a problem of corruption. Some results obtained in this work may need to be qualified after taking into account the social context. Another reason to interpret our results with caution is that transaction costs resulting from taxpayer compliance and/or evasion activities, as in most models, are ignored in this work.

To suggest future research it would be ideal to separate the effects of risk and time preferences into expected utility models. [Chatterjee, Giuliano and Turnovsky \(2004\)](#) showed that the effects of tax changes on the equilibrium growth rate, its volatility, and welfare are sensitive to independent variations of the rate of time preference and the coefficient of risk aversion using a numerical analysis with recursive preferences. It is therefore of interest to investigate how results change when such preferences are used.

Another relevant suggestion would be to incorporate social interaction parameters (for example, bureaucratic corruption, taxpayers compliance, cultural issues, among others) in theoretical models, getting closer to reality.

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APPENDIX A – First-order condition with respect to e_t

We show in Appendix A how to get the first-order condition given in (4.3).

$$\frac{\partial V(k_t)}{\partial e_t} = 0 \Rightarrow \beta(1-p)V'(k_{1,t+1})\tau y_t - \beta p V'(k_{2,t+1})\phi \tau y_t = 0$$

$$\Rightarrow (1-p)V'(k_{1,t+1})\tau = p V'(k_{2,t+1})\phi \tau$$

$$\Rightarrow \frac{p}{1-p} \frac{V'(k_{2,t+1})}{V'(k_{1,t+1})} = \frac{1}{\phi}$$

APPENDIX B – How to obtain c and e

In Appendix B we will only show how to obtain the expressions for c and e given in (4.8) and (4.9), respectively. We start taking the first-order conditions from (4.5):

$$\frac{1-\theta}{c_t} = \beta A \left[\frac{1-p}{h_t + \tau e_t y_t - c_t} + \frac{p}{h_t - \phi \tau e_t y_t - c_t} \right]$$

$$\frac{(1-p)\tau}{h_t + \tau e_t y_t - c_t} = \frac{p\phi\tau}{h_t - \phi \tau e_t y_t - c_t}$$

Rearranging terms in (4.6) to obtain c , we get:

$$\frac{1-\theta}{c_t} = \beta A \left[\frac{-\phi(1-p) + p}{h_t - \phi \tau e_t y_t - c_t} \right]$$

$$= \frac{-\phi(1-p) + p}{h_t - \phi \tau e_t y_t - c_t} \beta A$$

With a little algebra we get the following equality:

$$\frac{h_t - \phi \tau e_t y_t}{c_t} - 1 = \frac{-\phi(1-p) + p}{1-\theta} \beta A$$

$$\Rightarrow \frac{h_t - \phi \tau e_t y_t}{c_t} = \frac{-\phi(1-p) + p}{1-\theta} \beta A + 1$$

Isolating c_t from the above equality, we get (4.8):

$$c = \frac{(h_t - \phi \tau e_t y_t)(-\phi(1-p) + p)}{(1-\theta)(\beta A + 1)}$$

To obtain (4.9), we substitute c in (4.7):

$$\frac{(1-p)\tau}{h_t + \tau e_t y_t - c} = \frac{p\phi\tau}{h_t - \phi \tau e_t y_t - c}$$

Rearranging terms yields:

$$\begin{aligned} e &= \frac{h_t \beta A \overbrace{(1 - p\phi)}^{\bar{r}}}{(1 - \theta)(\beta A + 1)\phi\tau} \\ &= \frac{h_t \beta A \bar{r}}{(1 - \theta)(\beta A + 1)\phi\tau} \end{aligned}$$

APPENDIX C – Coefficients A and B

We provide in Appendix C the step-by-step to obtain the values of the coefficients A and B . Starting from (4.11) and applying some algebra yields:

$$A \ln(k_t) + B = (1 - \theta) \ln(c) + \beta \left[(1 - p) A \ln(k_{1,t+1}) + p A \ln(k_{2,t+1}) \right] + \beta A \ln(k_t) + \beta B$$

$$\Rightarrow A \ln(k_t) - \beta A \ln(k_t) + B - \beta B = (1 - \theta) \ln(c) + \beta \left[(1 - p) A \ln(k_{1,t+1}) + p A \ln(k_{2,t+1}) \right]$$

$$\Rightarrow A \ln(k_t)(1 - \beta) + B - \beta B = (1 - \theta) \ln(c) + \beta \left[(1 - p) A \ln(k_{1,t+1}) + p A \ln(k_{2,t+1}) \right]$$

$$\Rightarrow A \ln(k_t)(1 - \beta) - \beta A \left[(1 - p) \ln(k_{1,t+1}) + p \ln(k_{2,t+1}) \right] + B(1 - \beta) = (1 - \theta) \ln(c)$$

$$\Rightarrow A \left[\ln(k_t)(1 - \beta) - \beta \left[(1 - p) \ln(k_{1,t+1}) + p \ln(k_{2,t+1}) \right] \right] + B(1 - \beta) = (1 - \theta) \ln(c)$$

If the first term on the left side equals zero, we get:

$$A \left[\ln(k_t)(1 - \beta) - \beta \left[(1 - p) \ln(k_{1,t+1}) + p \ln(k_{2,t+1}) \right] \right] = 0$$

$$\Rightarrow \ln(k_t)(1 - \beta) = \beta \left[(1 - p) \ln(k_{1,t+1}) + p \ln(k_{2,t+1}) \right]$$

$$\Rightarrow \frac{\beta}{1 - \beta} = \frac{\ln(k_t)}{(1 - p) \ln(k_{1,t+1}) + p \ln(k_{2,t+1})}$$

Since $A = 0$, we obtain the value of B :

$$B(1 - \beta) = (1 - \theta) \ln(c) \Rightarrow B = \frac{1 - \theta}{1 - \beta} \ln(c)$$

APPENDIX D – Complete derivation of the growth implications

First we show how to obtain the values for \bar{c}_t and \bar{e}_t , respectively. Remark that $h_t := (1 - \delta)k_t + (1 - \tau)y_t$ and $y_t = z_t k_t^\alpha$.

$$\begin{aligned}
c_t^* &= \left[p h_t - \phi h_t (1 - p) - p \phi \tau e_t y_t + \phi^2 \tau e_t y_t (1 - p) \right] \left[1 - \theta \right]^{-1} \\
&= p k_t (1 - \theta) + p y_t (1 - \tau) - \phi k_t (1 - \delta) (1 - p) - \phi y_t (1 - \tau) (1 - p) \\
&\quad - p \phi \tau e_t y_t + \phi^2 \tau e_t y_t (1 - p) \\
&= p k_t (1 - \delta) + p z_t k_t^\alpha (1 - \tau) - \phi k_t (1 - \delta) (1 - p) - \phi z_t k_t^\alpha (1 - \tau) (1 - p) \\
&\quad - p \phi \tau e_t z_t k_t^\alpha + \phi^2 \tau e_t z_t k_t^\alpha (1 - p) \quad (\div k_t) \\
&= p (1 - \delta) + p z_t k_t^{\alpha-1} (1 - \tau) - \phi k_t (1 - \delta) (1 - p) - \phi z_t k_t^{\alpha-1} (1 - \tau) (1 - p) \\
&\quad - p \phi \tau e_t z_t k_t^{\alpha-1} + \phi^2 \tau e_t z_t k_t^{\alpha-1} (1 - p) \quad (\text{Collecting terms}) \\
&= p \left[\underbrace{(1 - \delta) + z_t k_t^{\alpha-1} (1 - \tau)}_{n_t} \right] - \phi (1 - p) \left[\underbrace{(1 - \delta) + z_t k_t^{\alpha-1} (1 - \tau)}_{n_t} \right] - \phi \tau e_t z_t k_t^{\alpha-1} \left[\underbrace{p - \phi (1 - p)}_P \right] \\
&= p n_t - \phi (1 - p) n_t - \phi \tau e_t z_t k_t^{\alpha-1} [p - \phi (1 - p)] \\
&= P n_t - \phi \tau e_t z_t k_t^{\alpha-1} P \\
&= \frac{P (n_t - \phi \tau e_t z_t k_t^{\alpha-1})}{(1 - \theta) k_t} = \frac{c_t^*}{k_t} = \bar{c}_t
\end{aligned}$$

$$\begin{aligned}
e_t^* &= \frac{\beta h_t \bar{r} [(1 - \theta) \phi \tau]^{-1}}{k_t} \\
&= \frac{n_t \beta \bar{r}}{(1 - \theta) \phi \tau} = \frac{e_t^*}{k_t} = \bar{e}_t
\end{aligned}$$

Now we show how to obtain (4.27). Substituting the optimal plans into (3.9), we get:

$$\begin{aligned}
k_{t+1} &= (1-p) \left[(1-\delta)k_t + \left((1-\tau) + \tau e_t^* \right) y_t - c_t^* \right] \\
&\quad + p \left[(1-\delta)k_t + \left((1-\tau) - \phi \tau e_t^* \right) y_t - c_t^* \right]
\end{aligned}$$

Taking the expected value of k_{t+1}/k_t yields the expected growth rate of capital per capita:

$$\begin{aligned}
\mathbb{E} \left[\frac{k_{t+1}}{k_t} \right] &= (1-p) [h_t + y_t \tau \bar{e}_t - \bar{c}_t] + p [h_t - y_t \phi \tau \bar{e}_t - \bar{c}_t] \\
&= (1-p) \left[n_t + \frac{z_t k_t^{\alpha-1} n_t \beta \bar{r}}{(1-\theta)\phi} - \bar{c}_t \right] + p \left[n_t - \frac{z_t k_t^{\alpha-1} n_t \beta \bar{r}}{(1-\theta)} - \bar{c}_t \right]
\end{aligned}$$

Since $m_t := z_t k_t^{\alpha-1} n_t \beta \bar{r}$, we get:

$$\pi_k = (1-p) \left[n_t - \bar{c}_t + \frac{m_t}{(1-\theta)\phi} \right] + p \left[n_t - \bar{c}_t - \frac{m_t}{(1-\theta)} \right]$$