

**UNIVERSIDADE FEDERAL DE VIÇOSA**

**ÂNGELO SALTON**

**IMPACTS OF MICROECONOMIC SHOCKS ON THE AGGREGATE ECONOMY: AN  
ANALYSIS FOR BRAZIL**

**VIÇOSA - MINAS GERAIS  
2019**

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Tese apresentada à Universidade Federal de Viçosa, como parte das exigências do Programa de Pós-Graduação em Economia Aplicada para a obtenção do título de *Doctor Scientiae*.

Orientador: Wilson da Cruz Vieira

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Assentimento:



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Angelo Salton  
Autor



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Wilson da Cruz Vieira  
Orientador

*“O otimismo é altamente valorizado no mercado. Pessoas e empresas remuneram mais aqueles que trazem informações controversas do que aquelas que trazem a verdade. Uma das lições trazidas da Grande Depressão é de que existem períodos onde a competição entre especialistas e entre organizações cria forças que favorecem uma cegueira coletiva ao risco e à incerteza.” (Daniel Kahnemann)*

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## ABSTRACT

SALTON, Ângelo, D.Sc., Universidade Federal de Viçosa, November, 2019. **Impacts of microeconomic shocks on the aggregate economy: an analysis for Brazil.** Adviser: Wilson da Cruz Vieira.

This work aims to evaluate the impact of microeconomic uncertainty shocks in the Brazilian economy through a dynamic stochastic general equilibrium model (DSGE) with financial frictions. The microeconomic shocks emerge from the time-varying output dispersion of entrepreneur agents and their interaction with the financial sector. Calibrating the model using Brazilian macroeconomic quarterly data from 2003 to 2018, we find that agency costs in the entrepreneurial and financial sector impact economic cycles, capital accumulation and household's welfare.

**Keywords:** Uncertainty. Volatility. Growth. Development.

## RESUMO

SALTON, Ângelo, D.Sc., Universidade Federal de Viçosa, novembro de 2019. **Impacts of microeconomic shocks on the aggregate economy: an analysis for Brazil.** Orientador: Wilson da Cruz Vieira.

O objetivo deste trabalho é avaliar o impacto de choques de incerteza de ordem microeconômica no Brasil através de um modelo de equilíbrio geral dinâmico estocástico (DSGE) com fricções financeiras. Estes choques se caracterizam pela dispersão das possibilidades de produção de um grande número de agentes empreendedores, que por sua vez interagem com o setor bancário para satisfazer suas necessidades de financiamento. Calibrando o modelo com dados trimestrais da economia brasileira de 2003 à 2018, os resultados apontam que custos de agência nessa relação impactam o crescimento econômico, a formação de capital e o bem-estar das famílias.

**Palavras-chave:** Incerteza. Volatilidade. Crescimento. Desenvolvimento.

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## 1 INTRODUCTION

This is a research in the field of macroeconomics that aims to provide a deeper understanding of economic uncertainty in Brazil. This work addresses an inquiry on the importance of *microeconomic uncertainty shocks* in the Brazilian economy. In the last decade, macroeconomists turned their attention to the role of firms and individuals in the performance of economies, and nowadays microeconomic elements are ubiquitous in structural and dynamic macroeconomic models. In example, microeconomic elements can be inserted in such models in form of a large number of firms or agents, (instead of a single, representative agent) that need not be homogeneous, interacting in competitive or monopolistic markets. The result of the aggregate economy will emerge from market clearing and other equilibrium conditions. By using this strategy, it is possible to answer more intricate questions in economic policy without leaving the general equilibrium framework.

More than investigating the impacts of microeconomic shocks itself, we turn our attention to microeconomic uncertainty shocks. In this work, following the existing literature, we interpret such shocks as perturbations to the firms' expectations about the return of their investments. In other terms, think of a statistical distribution of firm outputs: rather than shocks to the mean, this work focuses on shocks to the standard deviation of the distribution. In fact, the production decision of firms can be affected by a multitude of factors: economic, administrative, political. In our model economy, firms respond to fluctuations in their own idiosyncratic shocks, macroeconomic policy shocks, input prices and loan interest rates.

Recent work in the real business cycle theory (JUSTINIANO; PRIMICERI, 2008; BORN; PFEIFER, 2014) focuses on time-varying dispersion effects in aggregate variables, and findings from Christiano, Motto and Rostagno (2014) suggest that microeconomic uncertainty shocks are as important as the traditional technology and policy shocks as a source of business cycle fluctuations, meaning that such elements must be incorporated in dynamic stochastic general equilibrium (DSGE) models used by policymakers. In this work, we will focus on microeconomic shocks as perturbations to economic cycles. This research departs from the view that evaluates how the stan-

standard economic policy shocks affect business cycles. In the context of DSGE models, microeconomic uncertainty shocks are introduced by adding agents or firms that face uncertainty about their output, and such uncertainty need not show a strong correlation with the usual aggregate total factor productivity (TFP) shocks.

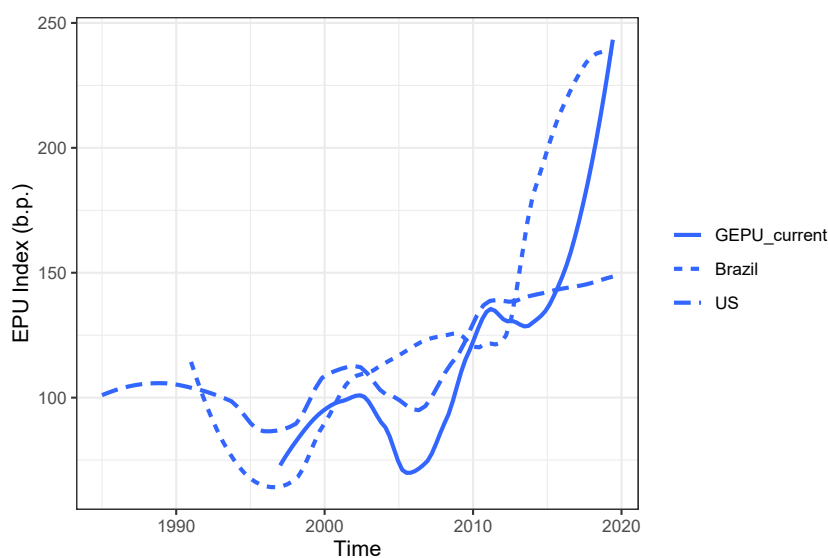
Empirical evidence on the subject is largely devoted to the United States economy. The matter of economic uncertainty received greater attention since the work of Lucas (1988). Beginning in the mid-1980s, the U.S. economy began to experience a phenomenon called the Great Moderation, which was an abrupt decline in the volatility of gross domestic product growth. The causes of the Great Moderation were at the center of a long debate, and Stock and Watson (2003) argues that while efficient monetary policy helped diminish output volatility, about half of the reduction in volatility remained unexplained. In their conclusions, the authors suggest that some of the moderation could be attributed to greater predictability of the economy, and rule out other factors such as productivity shocks and better inventory management of firms. One of the purposes of this work is to bring this knowledge to discuss symptoms and causes of economic uncertainty in Brazil.

Figure 1 presents the Economic Policy Uncertainty <sup>1</sup> (EPU) monthly index, proposed by Baker, Bloom and Davis (2013), from January 1990 to December 2018. Moving averages were calculated to smooth out the series, using a 12-month window. Data shows the effects of the 2008 financial crisis, political turmoils and a recent comparison between Brazil, U.S. and European countries. The index is based on political news coverage, stock market fluctuations and disagreements among forecasters about economic perspectives. While the index is not a direct proxy for microeconomic uncertainty, it gives an overview of instabilities faced by economies. Data shows the effects of the 2008 financial crisis and recent political turmoils, in all sets of countries examined (impeachment and economic recession in Brazil, U.S. elections, Brexit in Europe). The solid line represents the Global EPU index, while the dotted and dashed line represents, respectively, Brazil and the United States.

The matter of economic uncertainty gained momentum on the macroeconomic research agenda, in the last ten years, due to the 2008 subprime financial crisis. As noted by Castro (2016, p. 4), in the aftermath of the crisis, Brazil “adopted a new

<sup>1</sup> Available at: <<http://www.policyuncertainty.com/>>. Accessed in Oct. 20, 2019.

Figure 1 – Moving averages of EPU Index, 1990 - 2018.



Source: The authors, based on methodology from Baker, Bloom and Davis (2013).

regime, with public banks stepping in strongly in the credit markets to offset the retraction in private banks' credit origination". Some of the findings in the literature and lessons learned from recent financial crises materialized in the form of macroprudential policies and regulations, such as capital controls, corporate governance in firms and restrictions on banking instruments such as credit default swaps (i.e., the Dodd-Frank Act in the United States)<sup>2</sup>. Recently, the literature on economic uncertainty advanced when studies turned their attention to two facts. First, beyond many analyses on aggregate GDP growth volatility, researchers started looking at the microeconomic level. This includes sector and firm-level data. Another rich source of data were surveys with banking sector personnel (CHRISTIANO; MOTTO; ROSTAGNO, 2014; BACHMANN; ELSTNER; SIMS, 2012). Simultaneously, researchers pursued different sources and methods of estimating economic uncertainty (BAKER; BLOOM; DAVIS, 2013). A challenge is to determine how well we can consider theoretical implications from studies based on developed economies. Throughout this research, one of the strategies will be to compare the existing results found in the literature with our findings. We should also be careful when applying existing models, especially because of economic policy. The options for monetary policy in the Brazilian economy are limited in comparison to developed economies, not only because of the exchange risk, but also in terms

<sup>2</sup> Available at: <<https://www.sec.gov/about/laws/wallstreetreform-cpa.pdf>>. Accessed in Apr. 15, 2018.

of dependence and lack of coordination with fiscal policy (GADELHA; DIVINO, 2008; MENDONÇA; MOREIRA; SACHSIDA, 2017).

It is important to stress how this work deals with terms such as uncertainty, risk and volatility. We interpret uncertainty as a disagreement about economic forecasts, because of a multitude of factors, such as fluctuations in domestic and foreign markets, political and institutional instabilities, expectations about future inflation and interest rates. In this work, the microeconomic uncertainty shocks are essentially uncertainty about the return of investments for entrepreneurs. This closely follows the recent literature in economic uncertainty that will be explored in Section 3, which states that fluctuations in business cycles are composed of a combination concerning factors stated above.

## 1.1 The problem and its importance

The matter of microeconomic uncertainty shocks poses challenges for policy-makers. Governments attempt to alleviate economic uncertainty with counter-cyclical measures, using the standard fiscal and monetary policy tools. However, when making investment decisions, agents consider not only current government actions but the sentiment of other investors and potential customers. The importance of the subject cannot be understated, because it is of the interest of policymakers to know how fiscal and monetary policy work under agents uncertainty, and how credit markets amplify these effects. Moreover, uncertainty rapidly propagates among markets and economies due to financial globalization, as shown by the slow recovery of developed countries, following the 2008 financial crisis.

The most common topic in macroeconomic uncertainty is the volatility-growth correlation (RAMEY; RAMEY, 1995; BLACKBURN; PELLONI, 2004; IMBS, 2007). Initially, researchers aimed to fill this gap in classical growth theories. Despite the long-existing discussion, empirical evidence found by these authors is mixed. Nevertheless, key factors were found to be relevant, such as financial development (AGHION et al., 2010; YAVAS; DEDI, 2016), because (i) economic agents can diversify their portfolios or engage in precautionary savings in times of greater uncertainty; (ii) long-term investments and R&D projects can be sustained because of the greater access to credit.

Studies such as Acemoglu et al. (2003) and Jetter (2014) also stress the importance of institutional factors, in the sense that foreign investors have limited information about emerging economies and expected returns are harder to calculate with greater risk.

Many theories offer explanations for the dynamics of aggregate volatility and economic growth. The precautionary savings channel predicts that, in times of greater uncertainty, agents save to smooth consumption, generating future economic growth. Hence, uncertainty shocks could be pro-cyclical over time (AIYAGARI, 1994). On the other hand, the irreversible investments channel says that, in the face of uncertainty shocks, agents postpone long-term investment projects, especially in face of capital constraints (PINDYCK, 1991). Moreover, in the presence of capital constraints, mixed evidence appears, depending on whether economies are emerging or developed (SALTON; ELY, 2017).

Christiano, Motto and Rostagno (2014) build a model based on the *financial accelerator* specification in Bernanke, Gertler and Gilchrist (1999), which adds financial frictions in an otherwise standard DSGE model, studying the U.S. economy in the last 25 years. They introduce agents called entrepreneurs that can produce effective capital that depends on a stochastic process, and calls the time-varying standard deviation of this process as “risk”. This stochastic process should emulate the fact that some businesses succeed and others fail because of many possible circumstances assumed to be exogenous to the model. In other terms, capital returns are idiosyncratic. Besides, there is an agency problem in the financial sector because loan suppliers have monitoring costs, so they can be able to retain assets from entrepreneurs in case of default. Finally, they argue that microeconomic uncertainty shocks are the main forces that drive the business cycles. Whether this is true in Brazil remains an open question, and this is the research problem that will be investigated. As the Brazilian economy is more volatile, if this mechanism is found to be relevant, we also expect to find prominent effects.

Where does our contribution fit among the studies about economic uncertainty for Brazil? There are a series of works that develop dynamic general equilibrium models with financial frictions adapted to the Brazilian economy (CASTRO et al., 2011; CAVALCANTI; VEREDA, 2011; ARANHA, 2012; KANCZUK, 2013; AREOSA; COELHO,

2015; DIVINO; KORNELIUS, 2015), but these authors do not analyze productivity shocks other than aggregate, total factor productivity shocks, using this framework. Recently, a paper from Melo and Silva (2019) studied the impact of uncertainty shocks in Brazil using simulations from a dynamic general equilibrium model. However, the model built by the authors used a different strategy: domestic and foreign production volatility parameters are introduced in the total factor productivity process. This skips the financial frictions mechanism and is, in essence, a macroeconomic risk shock. Precisely, our contribution lies in the evaluation of the impact of economic uncertainty, emerging from the agent level, on economic growth and welfare, with a focus on the financial frictions mechanism and an extensive study of recent Brazilian business cycles. To our knowledge, these are open questions in the scientific economic literature in Brazil.

## 1.2 Hypothesis

As briefly discussed, and presented in greater detail in the theoretical reference, recent evidence from the literature proposes uncertainty shocks as a generator of business cycles, and specifically the relevance of microeconomic uncertainty shocks, compared to productivity and policy shocks. In these terms, present evidence from the literature gives us support to define our hypothesis: similar to policy and total factor productivity shocks, microeconomic uncertainty shocks significantly affect business cycles in Brazil.

## 1.3 Objectives

The broad objective of this work is to evaluate the importance of uncertainty shocks at the microeconomic level in Brazil, from 2003 to 2018. This work aims to provide empirical evidence for the Brazilian economy in a literature that is dedicated mostly to developed countries. There are key dissimilarities as Brazil is an emerging economy, such as credit constraints and institutional factors (which are reflected in economic policy uncertainty).

### 1.3.1 Specific objectives

- Calibrate a dynamic stochastic general equilibrium model with financial frictions, sticky prices, policy and uncertainty shocks to simulate the business cycles of the Brazilian economy.
- Evaluate if microeconomic uncertainty shocks impact business cycles in Brazil. The null hypothesis is that uncertainty shocks do not generate deviations from the simulation steady state for macroeconomic aggregates. If the null hypothesis is rejected, we should compare the effect of microeconomic uncertainty shocks to policy and total factor productivity shocks, using the impulse response functions as a metric of contribution to the business cycles,
- Perform a welfare analysis of microeconomic uncertainty shocks by aggregation of households utility functions.

## 2 ANALYSIS OF BRAZILIAN ECONOMIC CYCLES

This section aims to analyze Brazil's economic cyclical behavior and uncertainty measures for our empirical period of study. In our research, we found studies that approach business cycles up to 2012 (CHAUVET, 2002; ELLERY; GOMES; SACHSIDA, 2002; ELLERY; GOMES, 2005; LOPES, 2014; VIEIRA; PEREIRA, 2014). In his work, Lopes (2014) studies the Brazilian economy from 1947 to 2012, and presents some stylized facts: (i) business cycles are asymmetric, with expansions greater than recessions; (ii) convergence to lower GDP growth rates, combined with a reduction in volatility since the launch of *Plano Real*. From the latter fact, the author argues that Brazil also experienced a Great Moderation. Vieira and Pereira (2014) study the business cycles from 1900-2012, focusing on the identification of expansion and recession dates. However, they also point out for periods of different growth volatility.

Krznar and Matheson (2017) present a few stylized facts about Brazil's financial cycles in the 2000s. First, the economic boom favored an increase in both public and private loans. Second, credit supply follows GDP growth more than the other way around. Third, private credit responses to GDP fluctuations are quicker than public credit. Araujo, Sant'anna and Junior (2009) discusses the development of credit markets in Brazil, from 2004 to 2008. The country experienced a continuous increase in credit to GDP ratio, and a movement towards financial development, and a decline in interest rates, until the 2008 financial crisis. The relationship between private and public credit remained steady in the period, but in 2008 there was an expansion in credit provided by public banks, a counter-cyclical movement in response to the financial crisis.

First, we go on a brief overview of recent economic growth in Brazil, which can be divided into three epochs. First, the period from 2003 to 2008, marked for fast economic growth, a result from a combination of factors: (i) stable economic growth macroeconomic conditions in the whole world, unlike the financial crises of the 1990's; (ii) increased government spending and economic development; (iii) successive surpluses in the balance of trade. The real GDP growth was 4.2%, and the *per capita*

growth was around 2.9%. The year of higher performance was 2007, with a GDP growth of around 6%.

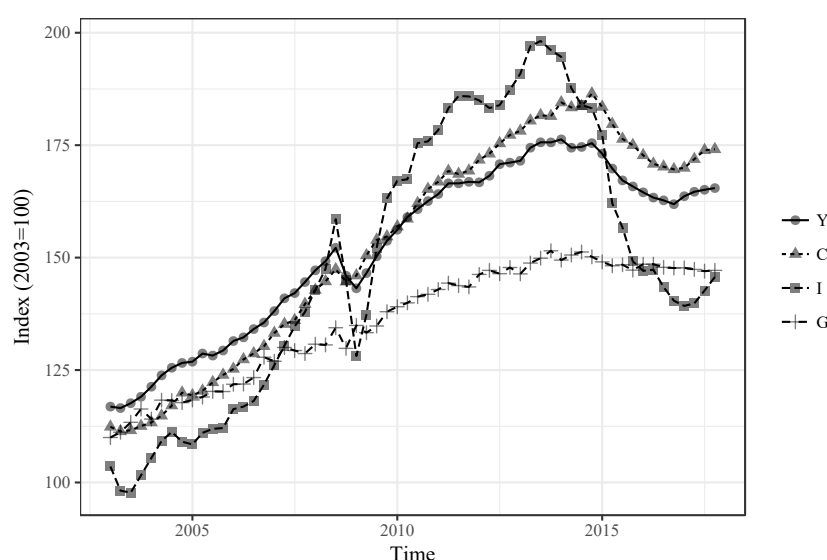
The next epoch goes from 2009 to 2014. In 2009, the impact of the 2008 financial crisis appeared on the numbers, with GDP shrinking (-0.1%). The average GDP growth in the 2009-2014 period was 2.8%, and *per capita* growth was 1.8%. In 2010, Brazilian GDP grew 7.5%, the highest rate in 24 years, led by internal demand and poor performance in the previous year. Later, in 2011, output growth was 2.7%. Despite a significantly lower performance in comparison to 2010, Brazilian growth beat the world average, still suffering from the 2008 financial crisis. In 2012 and 2013, Brazil experienced slow but steady growth (1.4% on average) and continuous *per capita* GDP growth since 2010.

Last, the period spanning from 2014 to 2018. In 2014, the recession came after political instabilities and misconducts of economic policy (price controls, mixed signals sent from monetary policy). Calculated output growth was near zero, the worst since the 2009, in the crisis. For the first time since 2010, there was a fall in *per capita* GDP growth. The Brazilian economy spiraled down in 2015, with GDP shrinking (-3.8%). This is the greatest contraction in GDP since the present method of calculation started, in 1996, and considering earlier years, the worst result since 1990, when Brazil struggled with the aftermath of *Plano Verão*, when the access to savings accounts were restricted. Output did not improve in 2016, when GDP growth was -3.5%. With respect to 2015, *per capita* output fell more than 4%. Finally, in 2018 GDP growth was 1%, signaling a recovery of the Brazilian economy. In absolute terms, Brazil reached the same output level from 2011, adjusted for inflation.

In order to provide benchmarks and stylized facts for our empirical model, we attempt to characterize the Brazilian business cycles for the period 2003-2018, using techniques similar to those cited above. First, as in Ellery, Gomes and Sachsida (2002), we gather quarterly data from national accounts published by IBGE, the Brazilian national statistics bureau. Data spans from the first quarter of 2003 to the fourth quarter of 2018. Throughout this work – and unless stated otherwise – all time se-

ries are seasonally adjusted using the X13-ARIMA-SEATS model<sup>1</sup>. Initially, we present data for gross domestic product  $Y_t$ , household consumption  $C_t$ , investment (gross fixed capital formation)  $I_t$ , government consumption  $G_t$ , labor force  $N_t$  and wage levels  $W_t$ . The choice of variables for the model aggregates follows Castro et al. (2011). All data is presented as indexes, with a fixed base equal to 100 for the year 2002, as provided by IBGE. This transformation is suitable for comparisons, as we can obtain quarterly percent changes by taking first differences of the time series.

Figure 2 – Brazilian macroeconomic aggregates, 2003-2018



Source: The author, based on IBGE (2019).

According to Figure 2, by the end of 2018 investment was at the same level as in the 2008 financial crisis. Next, we examine the correlation between the gross domestic product and macroeconomic aggregates in Table 1:

Table 1 shows that despite household consumption representing the biggest share of domestic income, investment exhibits the greatest correlation with GDP, with both contemporaneous and lagged GDP time series. As expected, government consumption is the least correlated aggregate, mainly because of counter-cyclical fiscal policy measures. Also, investment is three times as volatile as GDP. Surely, simple correlations are economically difficult to interpret, because causation can become from both directions. However, it is enough to give us a sense of co-movement between vari-

<sup>1</sup> This is a methodology developed by the United States Census Bureau, in conjunction with the Bank of Spain. The X13-ARIMA-SEATS model contains methods for seasonal decomposition and extraction of seasonal factors, taking into account outliers, holidays, trading days and level changes.

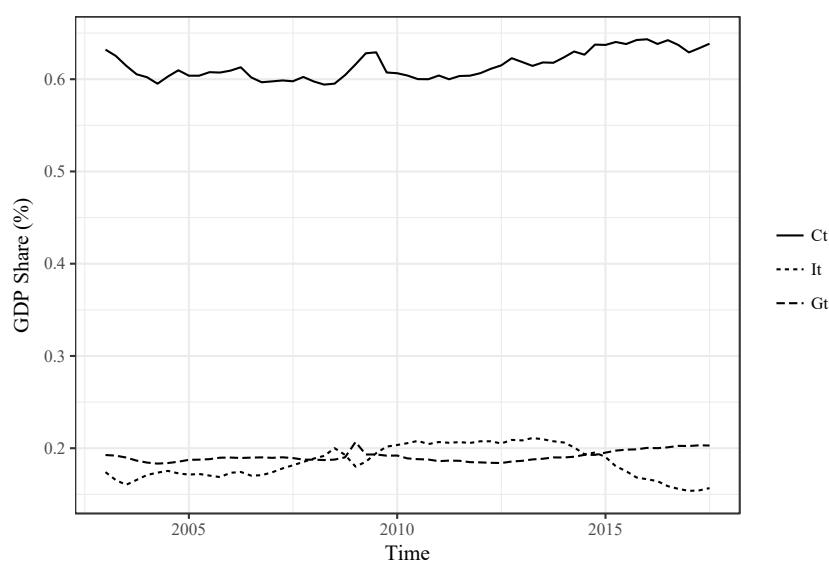
Table 1 – Correlation between Brazil GDP and its macroeconomic aggregates, 2003-2018

	$\sigma_x$	$\frac{\sigma_x}{\sigma_Y}$	$\text{corr}(x, Y_{t-1})$	$\text{corr}(x, Y_t)$	$\text{corr}(x, Y_{t+1})$
$Y_t$	1.867	1	0.402	1	0.402
$C_t$	1.729	0.926	0.390	0.747	0.394
$I_t$	5.630	3.015	0.492	0.865	0.351
$G_t$	1.729	0.926	0.097	0.296	0.209
$N_t$	1.988	1.065	0.474	0.582	0.303
$W_t$	20.306	10.874	0.086	-0.013	-0.141

Source: The author, based on IBGE (2019) and Ellery, Gomes and Sach-sida (2002).

ables, in terms of business cycles. Our empirical model should be consistent with the results above, and stylized facts can be drawn accordingly. We also plot the evolution of aggregates as a share of GDP in Figure 2:

Figure 3 – Plots of  $\frac{C_t}{Y_t}$ ,  $\frac{I_t}{Y_t}$  and  $\frac{G_t}{Y_t}$ , 2003-2018



Source: IBGE (2019).

As Figure 2 shows, household consumption is responsible for the largest share of GDP, ranging from 60 to 70% of the gross domestic product (61.4% on average). In the same period, investment to GDP was 18.5% on average, and the average government consumption to GDP was 19%. The Figure also summarizes the key facts of the recent economic history of Brazil. First, the reaction to the 2008 financial crisis: in the period, investment depressed and active counter-cyclical government spending kicked in. An intertemporal reallocation of resources happens as present consumption is favored in opposition to investment. After the correction, investment to GDP rose again

to a peak of 21,5% in the third quarter of 2010. From 2010 to around 2013, the monetary authority aggressively pursued a reduction in interest rates, through multiple cuts in Selic rate targets. This allowed for a sustainment of investment, combined with low exchange rates, caused in part for successive rounds of quantitative easing made by the U.S. Federal Reserve (U.S. dollars reached a minimum of R\$ 1.66 in 2010). Finally, from 2014 onwards, a trend shift occurs with a sharp fall in the share of investments to GDP, reflecting a sub-par economic performance in the period 2014-2016.

Next, we plot quarterly percent changes in gross domestic product in Figure 4, according to the three main sectors of the economy: agriculture and husbandry, industries and services. An important insight emerges from data: agriculture is the most volatile of the main sectors, while activities related to services exhibit smaller variability. Two main reasons are plausible: First, the agriculture and husbandry sector is more volatile for its very nature, because agricultural production depends on crop yields, which in turn depends on environmental conditions and other exogenous factors. Second, the service sector is responsible for the largest share of GDP. Data also shows the recent instabilities in the Brazilian economy, such as the impacts of the 2008 financial crisis and the aftermath of the presidential impeachment, among other misconducts in economic policy.

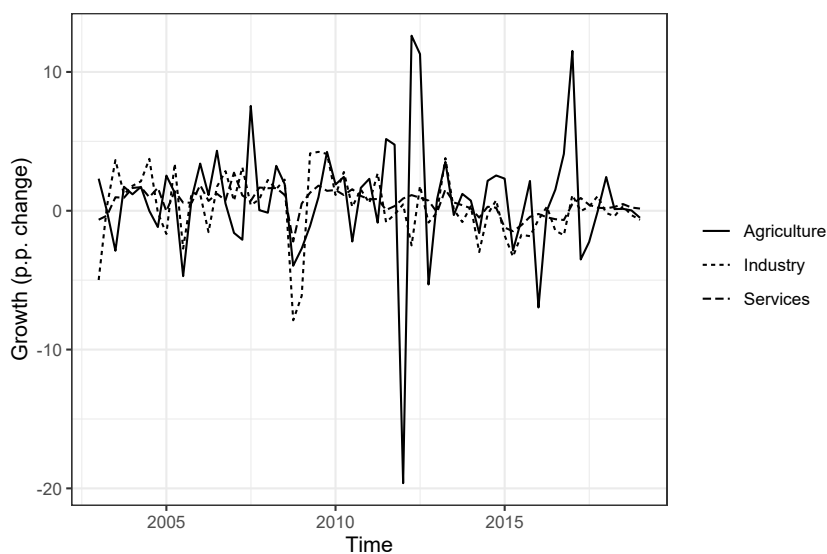
In both recent crises, industrial production suffered the most. The latest recession was led by industry, and by 2016 investment levels were 30% lower than 2013 (TINOCO; GIAMBIAGI, 2018). In addition, government consumption and debt grew in the same period, in what appears to be a crowding-out<sup>2</sup> effect. Data from IBGE shows that gross fixed capital formation declined 23% in the 2014-2016 triennium.

Now, we use the IGP/DI index to construct the real quarterly GDP, and then take logarithms. Next, we apply the Hodrick-Prescott filter, with smoothing parameter adjusted to quarterly data ( $\lambda=1600$ , a standard value in the literature), to remove the trend component and extract the economic cycles. Figure 5 represent shows both the GDP data for Brazil and the trend decomposition generated by the filter.

In respect of business cycles identification for the Brazilian economy, there are the expansion and recession dates determined by CODACE (*Comitê de Datação de*

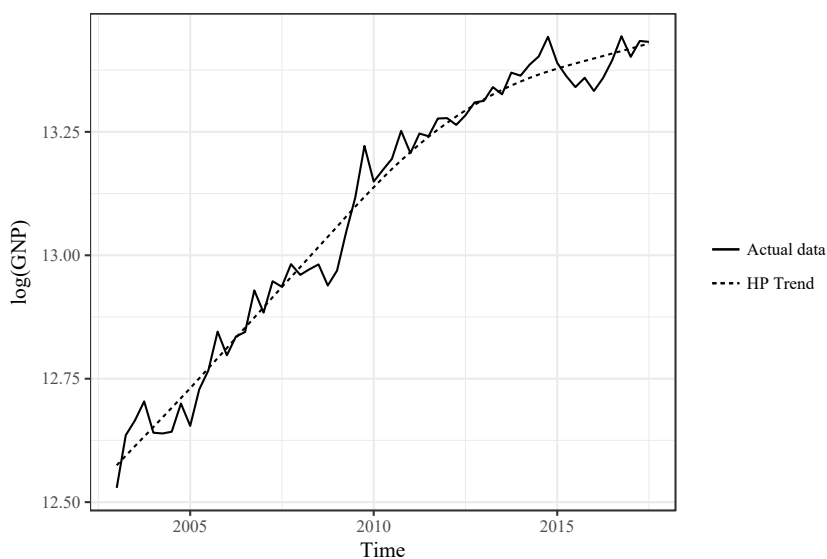
<sup>2</sup> This effect describes a phenomenon where increasing government intervention affects private investment. The main hypothesis is that the financing needs of governments raise the cost of capital.

Figure 4 – Percent changes in Brazilian GDP, by sectors



Source: IBGE (2019).

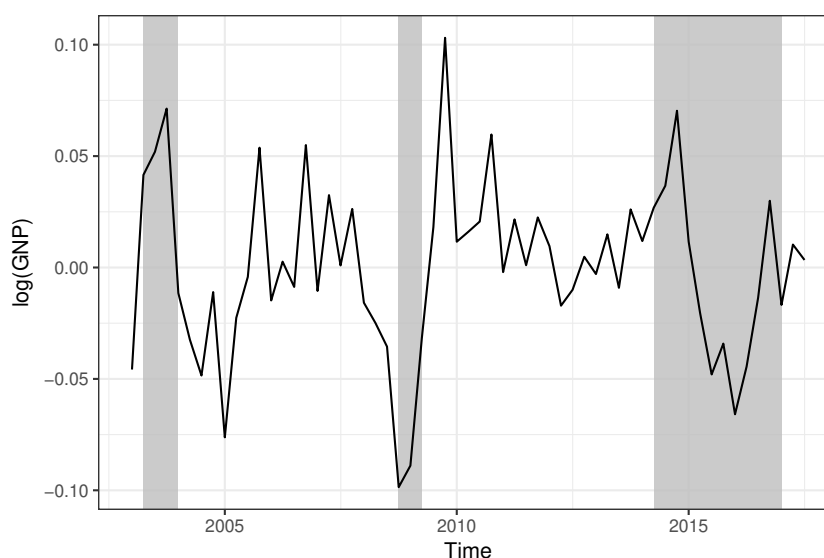
Figure 5 – Trend component of HP-filtered log Brazilian GDP



Source: The author.

*Ciclos Econômicos*), a committee from IBRE/FGV, inspired by the NBER Business Cycle Dating Committee in the United States. The committee specialists characterize cycles by observing economic indicators such as real GDP, unemployment, industrial production, household consumption, wholesale and retail sales. Figure 6 represents the cycle decomposition of the Brazilian log GDP. Areas shaded in gray are recession date ranges determined by CODACE.

Figure 6 – Cycle component of HP-filtered log Brazilian GDP



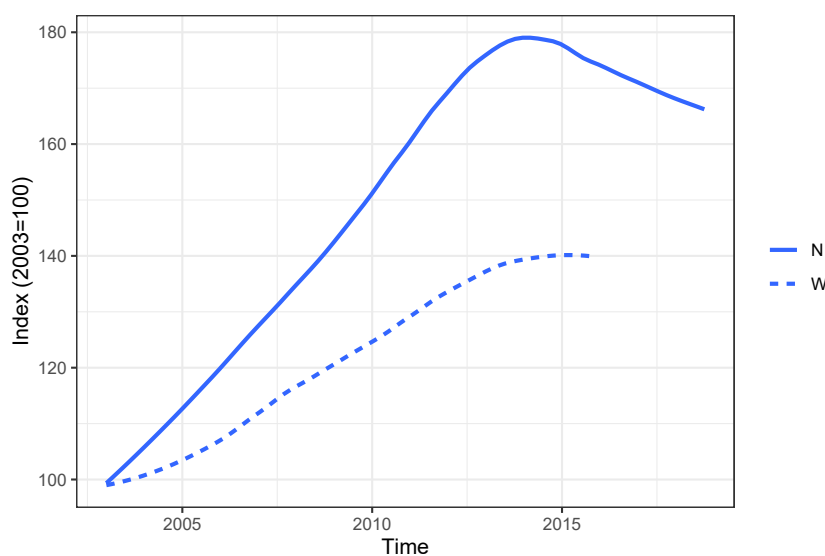
Source: The author.

Both figures above show that the cycle component of the series does not precisely identify the late 2003 recession, caused by uncertainty about the aftermath of the 2002 presidential elections, and the 2016 recession, following the impeachment of President Dilma Rousseff. However, the 2008 financial crisis valley approximates the CODACE dates. We believe the mismatches could be corrected with additional data points on both extremes, as the recovery of the Brazilian economy in 2018 resulted in another trend shift.

## 2.1 Employment

Data about employment went through methodological changes, with respect to data collection. IBGE published employment data through PME (*Pesquisa Mensal de Emprego*) up to 2016, when the survey was discontinued. Data is now available in *Pesquisa Nacional por Amostra de Domicílios Contínua* (PNADC), the national household sample survey conducted by IBGE. We present the level of employment (in number of occupied persons) combining both data sources. We also present data for the evolution of real average wages, calculated until 2016 with PME data. Figure 7 shows the evolution of data, where the solid line denotes the employment and the dashed line denotes real wages. Both time series were smoothed out for better readability, using a two-sided, 4-quarter moving average filter.

Figure 7 – Brazilian employment and real wages, 2003-2018



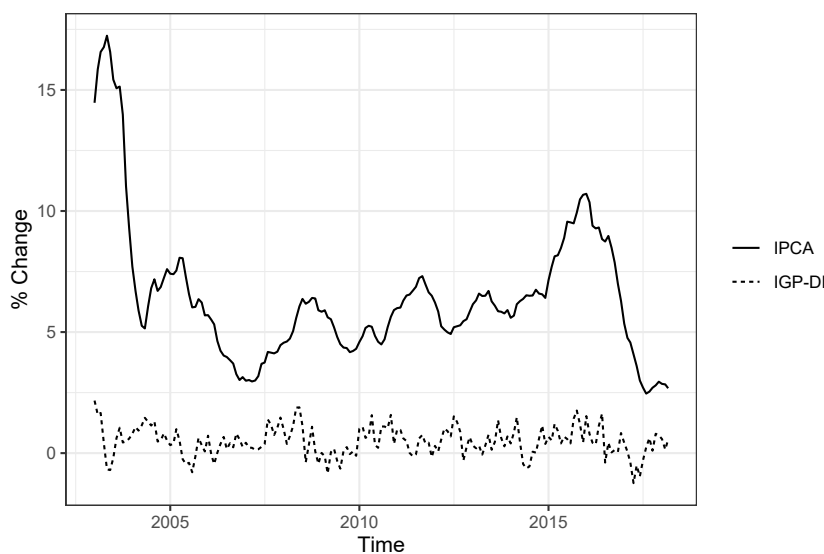
Source: IBGE (2019).

According to Figure 7, labor force employed in production experienced a growth of approximately 40% from 2003 to 2015, while real wages accumulated a 25% growth in the same period. In fact, data shows the formation of a gap between the use of labor and real wages paid. Most of it could be explained by the rise in inflation rates that affected the purchasing power of wages, and also by the shift in activities from industry to services in the period.

## 2.2 Inflation

Figure 8 shows the evolution of IPCA, the national consumer price index, calculated by IBGE and available in bi-weekly and monthly observations, as percent changes from the last period. The IPCA index aims to measure price changes in retail goods and services. To present data in quarterly format, we sum observations within each quarter. We also present the IGP-DI index, used as the GDP deflator. Unlike IPCA, the IGP-DI index have a wider scope: it is a weighted mean of IPA (the national wholesale price index), IPC (a consumer price index measured in São Paulo and Rio de Janeiro) and INCC (the national construction price index, that considers inputs and goods used in construction and labor). Data is seasonally adjusted:

Figure 8 – IPCA and IGP-DI indices, 2003-2018



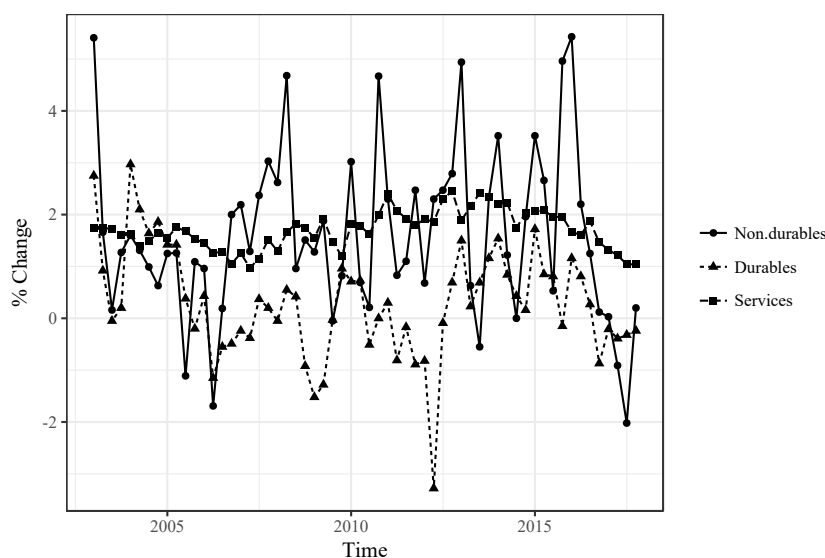
Source: IBGE (2019).

Because IGP-DI is a weighted mean of multiple indices, it is less volatile than IPCA, but the two series appear to be positively correlated. The average quarterly price changes for IGP-DI and IPCA are approximately the same: 1.47%. As expected and discussed in the literature, an inflation-uncertainty nexus emerges: in the short term, firms adjust prices in response to economic conditions and expectations. Corrections can go up or down, with a few examples that can be characterized with help of Figure 8. First, in the first quarter of 2003, when the first term of President Luiz Inácio Lula da Silva's started. Uncertainty about his economic policy, combined with high exchange rates (also a price, in this case of foreign currency) helped the prices go up, both in wholesale and retail. Second, in the 2008 financial crisis, prices rose with the expectations of the repercussions of recession in Brazil. Then, inflation reduced, as firms started realizing their first losses.

Figure 9 decomposes the IPCA index into price changes in durable goods, non-durable goods and services. Durable goods (i.e. automobiles, electrical and home appliances) account for the smallest contribution for the price index (average quarterly change of 0.29%), many times registering negative quarterly percent changes. Non-durables (i.e. foods, beverages, other perishable products), on the other hand, registered an average change of 1.58% every quarter. Finally, the price of services registered a steady evolution, increasing around 1.7% every quarter. Empirical data

on inflation gives us a hint of how prices should behave in our general equilibrium model. Durable goods could be compared to unfinished capital, meaning that the price of capital should be less elastic than the price of final goods.

Figure 9 – Inflation, breakdown by types of goods, 2003-2018



Source: IBGE (2019).

## 2.3 Financial variables

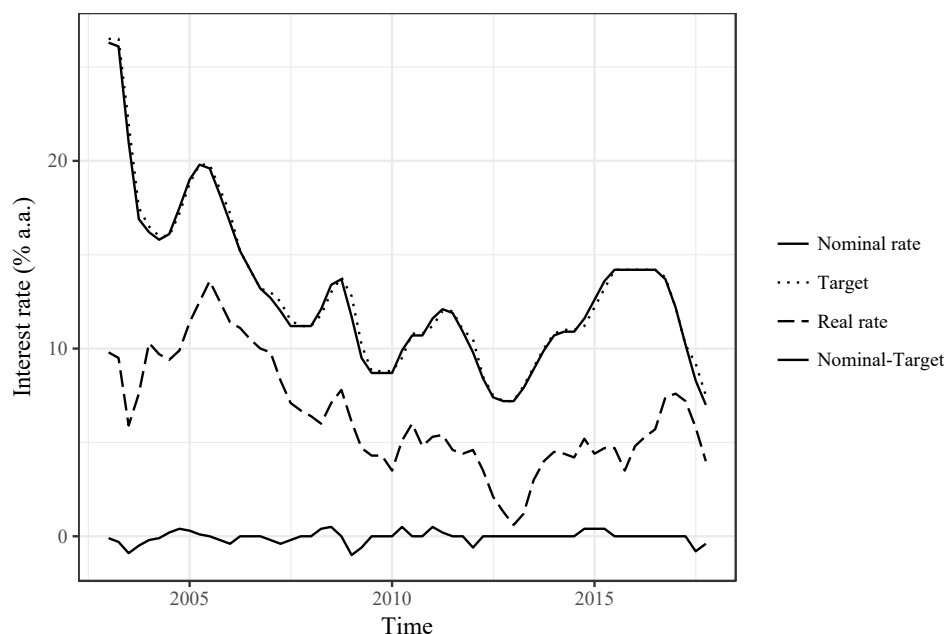
Now, we turn our attention to the financial variables and parameters present in the empirical model, starting with interest rates, more specifically the Selic rate. Named after the acronym Selic (*Sistema Especial de Liquidação e Custódia*), it is the national interbank settlement system. BACEN, the Brazilian central bank is an active player in the interbank market, using its power to steer the market interest rate into the target. Data on nominal and target interest rates are provided daily through BACEN. We annualize both rates (to base 252)<sup>3</sup> and evaluate them at the end of the quarter, to harmonize with our database.

Figure 10 summarizes the historical data. The solid and dotted lines on top are the nominal and target Selic rates. The dashed line in the middle is the real rate, calculated as the nominal Selic rate minus the annualized IPCA inflation rate (according to

<sup>3</sup> In this process, we annualize daily interest rates for 252 (the number of working days in Brazil each year), using the formula:  $(1 + \text{daily rate})^{252} - 1$ .

the Fisher equation). Finally, the solid line on the bottom is the gap between the nominal and target rates (the first two curves), presented on its own for better visualization.

Figure 10 – End-of-quarter Brazilian interest rates, 2003-2018



Source: BACEN (2018).

Interest rate swings become clear from the Figure above. Also, smaller gaps between nominal and real rates mean smaller inflation rates. Some remarks can be made about the 2003-2018 period. First, the series begins with nominal interest rates over 20% a year, a heritage from the high interest rate policy that marked the 1990's stabilization. From this time on, the 2000's combined fast economic growth and fiscal surpluses that reduced the burden of debt<sup>4</sup>, giving the monetary authority room to pursue lower real interest rates. Another pattern comes from data: since 2013, when the conditions of the Brazilian economy started to deteriorate, the gap between the market and target Selic rates became much tighter, meaning that the monetary authority increased its efforts in the open market to control interest rates.

Lower real interest rates emerged in the first quarter of 2013, marking the end of the aggressive interest rate reduction policy conducted by then minister Guido Mantega. At the time, the reduction of interest rates was opposite to the prescription of mainstream economic theory, as inflation rates were rising. According to Barbosa Filho (2017), this shift in the stance of the monetary authority affected its credibility.

<sup>4</sup> Earlier in the text, we remarked the reduction of the debt-to-GDP ratio in the 2000's.

Thus, interest targets had to go up again. In 2015, the new minister of the economy, Joaquim Levy, promised rigorous control of inflation rates, and the interest target stayed at 14.25% from the second quarter of 2015 to the first quarter of 2016. Now, Brazil is entering in a time of financial stabilization, with nominal and real interest rates entering historical lows, under the supervision of minister Paulo Guedes and central bank president Roberto Campos Neto.

There are a few stylized facts about the external finance premium, since the works of Bernanke and Gertler (1995), but the most remarkable is that firm size seems to be a proxy variable to credit access. Since small firms cannot finance their project themselves, they resort to external credit, and given the state of economic development in Brazil, we expect to find estimates for the premium higher than, say, United States. Recent works analyzed the external finance premium in Brazil (OLIVEIRA, 2012; OLIVEIRA; Ronchi Neto, 2012). In particular, Oliveira (2012) studied the external finance premium, using publicly available and confidential balance sheet data from more than 5,000 public and private firms, with data spanning from 1994 to 2010. From the author's remarks, two factors appear to greatly affect the premium:

- *Firm size*. Oliveira (2012) shows that the external finance premium is not only higher, but also more volatile for small firms than for large firms. Firm size is determined by comparison among all firms in the sample and balance sheet metrics. Greater firms experience a lower premium due to a reduced probability of default.
- *BNDES*. The Brazilian development bank offers lower interest rates and longer maturities for loans. In the recent economic policy, interest rates were subsidized, therefore not responding immediately to monetary policy.

The main problem is that the external finance premium is unobservable. Estimation of the premium often come from DSGE models. De Graeve (2008) builds a model based on Smets and Wouters (2007) with financial frictions similar to those found in our model to provide estimates of the premium. Often, authors start with steady-state estimates of the premium. For the U.S. economy, some authors used 200 basis points (equivalent to 0.2%) (BERNANKE; GERTLER; GILCHRIST, 1999; De Graeve, 2008).

For Brazil, the spread between the annualized CDI<sup>5</sup> rate and the Selic rate yields similar results.

## 2.4 Economic uncertainty

To better understand the perception of agents about Brazil's recent economic performance, we now present data from the Focus report. The report is an online publication from the Brazilian central bank, with daily frequency, that registers the forecasts from financial institutions about economic indicators, representing an important measure of agents' expectations. Arithmetic means of institutional forecasts are calculated, and the standard deviation of all predictions is also informed. Thus, every day a statistical distribution of forecasts for each economic indicator is characterized. Increases in the dispersion of predictions represent greater disagreement among forecasters, a proxy for economic uncertainty.

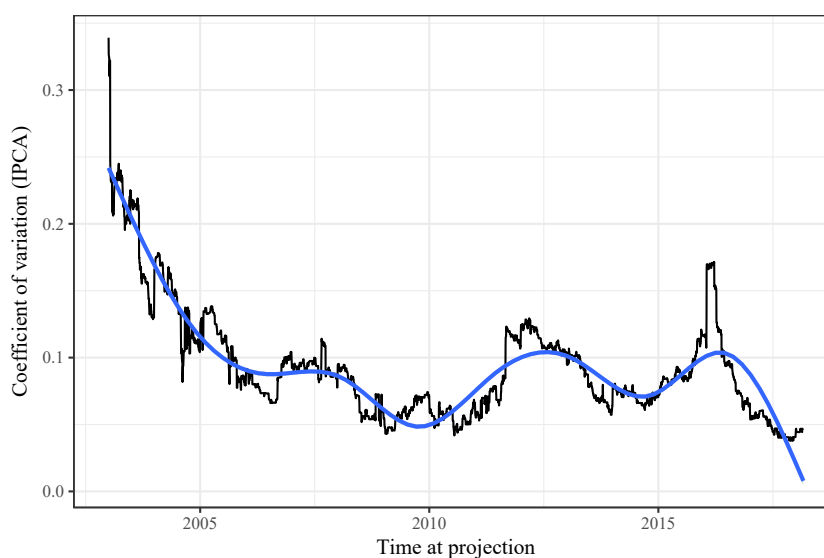
Figures 11, 12 and 13 present, respectively, the coefficient of variation in forecasts for accumulated inflation rate, gross domestic output growth and interest rates in the next two years. The coefficient of variation is the ratio between the standard deviation and the arithmetic mean of the series. The smooth line is a moving average of the series. Because forecasters aim for the rates at the end of the year, a fall in uncertainty is expected, as it is easier to predict the true rates, with exception of the inflation rate, which is a rolling forecast. Also, at each year change, the forecasts are updated one year in the future, hence the spikes in data. The horizontal axis represents the time where the projections were made (i.e; a data point where the time at projection lies within 2010 represent a projection for 2012, in the case of GDP and Selic rates, and so on)<sup>6</sup>. Could the information sets for economic indicators influence the behavior presented here? We know that information about IPCA is updated biweekly<sup>7</sup>. Information about GDP is updated in quarterly national accounts, and interest rates are updated daily.

<sup>5</sup> CDI stands for *Certificado de Depósito Interbancário*. The CDI rate is employed between banks in one-day interbank loans, often needed to balance the banks' checking accounts and fill reserves in the Brazilian central bank, actions that are mandatory for commercial financial institutions.

<sup>6</sup> As a simple statistical indicator, the coefficient of variation may exhibit inconsistencies, namely an upward bias when the mean approaches zero, or when data does not follow a Gaussian distribution. This is not the case here.

<sup>7</sup> IPCA-15, calculated by IBGE, presents inflation rates every 15 days.

Figure 11 – Variation of inflation rate forecasts, 2 years ahead.

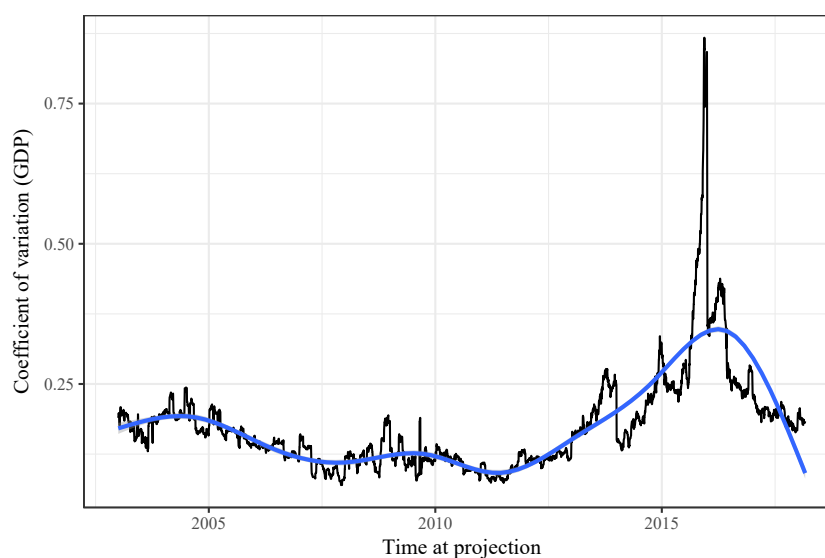


Source: BACEN (2018).

Uncertainty about inflation shown in Figure 11 exhibits well-defined cycles, and periods of higher uncertainty coincide with that of higher inflation rates. Historically, a downward trend can be seen from 2003 until 2008, reflecting a period of economic stabilization and growth. From 2010 on, uncertainty about inflation returned. Several factors can be credited to this fact, but the most widely accepted is the then-new economic policy developed at the time called *Nova Matriz Econômica* that focused on spending-driven economic development, deviating from one of the fundamentals of the Brazilian “macroeconomic tripod”<sup>8</sup> by promoting loose responses to inflation targeting. In fact, from 2010 to 2016 registered inflation rates (annualized IPCA) were higher than the target, revolving around the upper limit of tolerance bands (6.5%). At the time, the upper limit could be considered the new inflation target, as the central bank exerted no big effort to bring rates back to 4.5%. Another source of uncertainty was the administered prices, that is, prices subject to governmental control. Examples were oil, fuels and energy prices. The policy of price controls was a strategy to help control the momentum of inflation, at the expense of losses in energy companies and Petrobras, the state oil company.

<sup>8</sup> The macroeconomic tripod was a monetary policy directive adopted in Brazil shortly before the transition to free-floating exchange rates in 1999, and contained three elements: (i) an inflation targeting regime, (ii) free-floating exchange rates and (iii) national accounts surpluses.

Figure 12 – Variation of Brazilian GDP growth forecasts, 2 years ahead.

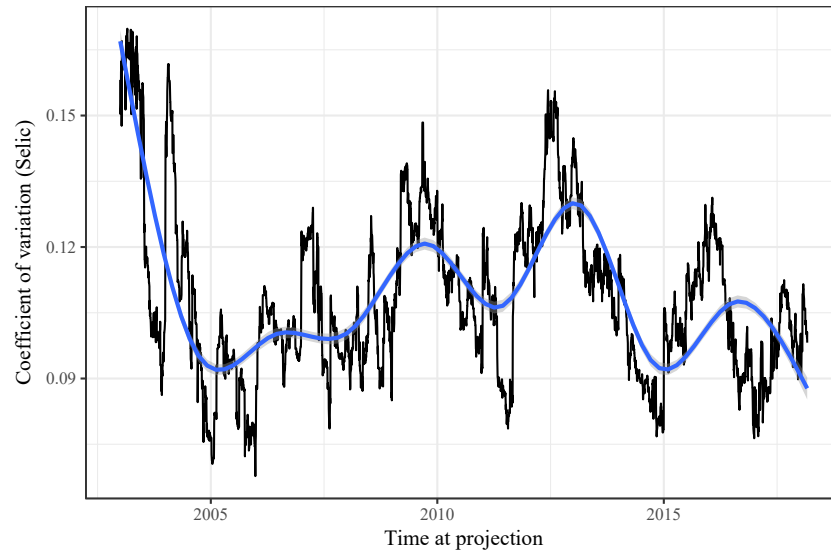


Source: BACEN (2018).

About GDP uncertainty, there are remarkable oscillations. First, there was uncertainty in the events of the 2008 financial crisis, and an upward trend starting from 2012 that peaked in the impeachment of then-President Dilma Rousseff. After, signs of economic recovery translated into a smaller disagreement among forecasters. Other economic policy measures helped diminish uncertainty about future growth. First, in fiscal policy, the establishment of a ceiling for government spending, and in monetary policy, the commitment to restore the inflation targeting system and the plan to pursue smaller interest rates in the long term. It seems that domestic factors were more important than foreign: Barboza and Zilberman (2018) studied a large number of proxies for economic uncertainty and estimated that the Brazilian economy could have performed better (with an improvement from 0.9% to 3.9% in 2015 industrial production) if not for the economic and political instabilities that led to the impeachment process.

Uncertainty about interest rates, despite looking more volatile in Figure 13 than in the earlier figures (due to scale), showed about the same volatility than the other indicators. Nonetheless, there were a lot more trend shifts in the period 2003-2018. A possible source of disagreement among forecasters comes from the fact that Selic rates are updated daily. Possibly, the constant changes in the information set available to forecasters can increase uncertainty itself. Similar to the information about inflation

Figure 13 – Variation of interest rate forecasts, 2 years ahead.



Source: BACEN (2018).

rates, visual inspection of the series suggests that uncertainty is often associated with increases in the interest rate targets.

## 2.5 Vector autoregression evidence

In an additional effort to characterize the business cycles in the Brazilian economy, for the period 2003-2018, we run a first-order, three-variable vector autoregression on GDP growth  $Y_t$ , inflation rate  $\pi_t$  and nominal interest rates  $i_t$ . The choice of lags followed the Schwarz information criterion (BIC), testing up to 12 lags. Upon inspection in Figures 2, 8 and 10, we find that these time series are non-stationary. Thus, first differences are taken. The equations that define the model follows:

$$\mathbf{y}_t = \mathbf{c} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{u}_t, \text{ with } \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}, \mathbf{y}_t = \begin{bmatrix} Y_t \\ \pi_t \\ i_t \end{bmatrix}, \mathbf{u}_t = \begin{bmatrix} e_t^Y \\ e_t^\pi \\ e_t^i \end{bmatrix} \text{ and } \mathbf{A} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} \\ \phi_{2,1} & \phi_{2,2} & \phi_{2,3} \\ \phi_{3,1} & \phi_{3,2} & \phi_{3,3} \end{bmatrix} \quad (2.1)$$

As common in vector autoregressions, there is a vector of endogenous variables  $\mathbf{y}_{3 \times 1}$ , constants  $\mathbf{c}_{3 \times 1}$ , a matrix of parameters to be estimated  $\mathbf{A}_{3 \times 3}$  and error vector  $\mathbf{u}_{3 \times 1}$ . Table 2 shows the results of the model, with parameters estimated by OLS.

Table 2 – Results of VAR(1) model

	Equation		
	$Y_t$	$\pi_t$	$i_t$
c	0.551 (0.584)	0.999*** (0.227)	-1.014*** (0.312)
$Y_{t-1}$	0.411*** (0.115)	-0.028 (0.045)	0.218*** (0.061)
$\pi_{t-1}$	-0.142 (0.350)	0.323** (0.136)	0.489** (0.187)
$i_{t-1}$	-0.488*** (0.181)	0.073 (0.070)	0.609*** (0.097)
Observations	58	58	58
R <sup>2</sup>	0.327	0.176	0.566
Adjusted R <sup>2</sup>	0.290	0.130	0.542
RSE (df = 54)	1.594	0.618	0.850
F Statistic (df = 3; 54)	8.758***	3.848**	23.444***

Note: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Source: The author.

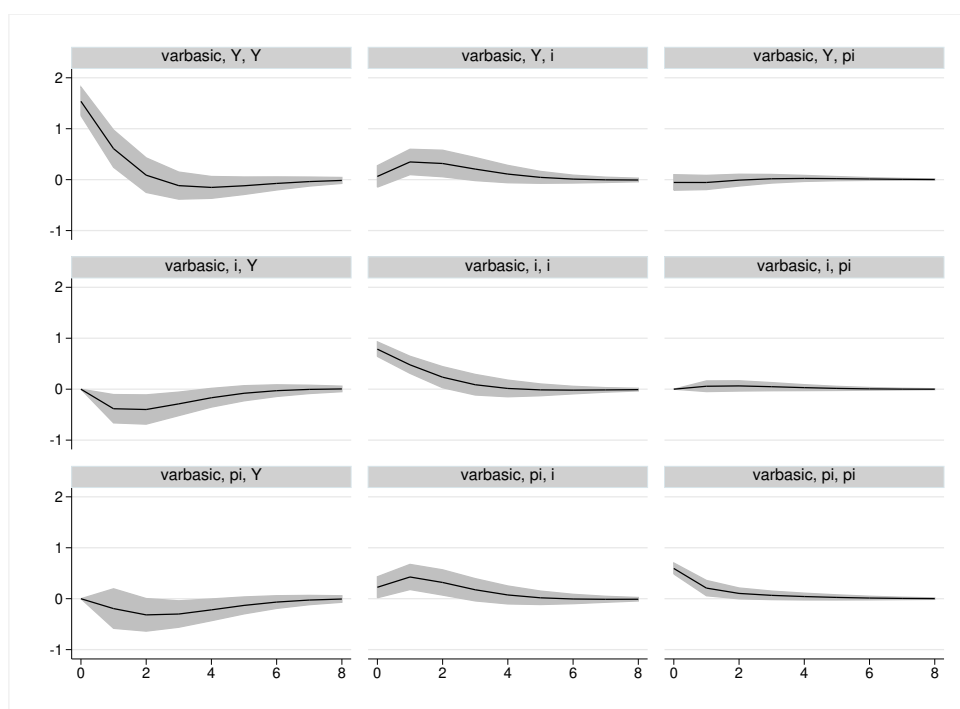
The results from Table 2 reveal the short-run dynamic behavior of the Brazilian economy. For the GDP equation, GDP growth reacts negatively to a shock in interest rates, as the cost of capital increases. On the other hand, the effect of inflation is not statistically significant. The interest rate equation is the most interesting as it shows the behavior of monetary policy. Interest rates are found to be moderately persistent (0.609).

Overall, the interpretation of VAR coefficients is difficult because of the endogenous nature of the system of equations. Therefore, we plot the orthogonalized impulse response functions of the model in Figure 14. Impulse response functions permits us to evaluate the response of the model as a whole to shock in a specific equation, through the analysis of the dynamic multipliers of the model. The notation in the plot legends means the origin and destination of the shock, respectively.

Figure 14 provides a clearer view of the recent business cycles. The vertical axis represents the magnitude of exogenous shocks in standard deviations, while the horizontal axis represents time, in quarters. Gray bands around the solid line represent the 95% confidence bands, calculated via bootstrap methods. The second row of plots correspond to the effect of monetary policy shocks in the system. The response of inflation to interest rate shocks is apparent in the last row: the peak of the response

happens right in the quarter following the shock. Monetary policy is found to actively respond to fluctuations in the output gap. In general, business cycles are absorbed by the system within a year (4 quarters).

Figure 14 – VAR(1) model impulse response functions



Source: The author.

Figure 15 shows the evolution of Brazilian net public debt, with monthly data spanning from January 2003 to December 2018. This data follows the definition of public sector net debt, containing non-financial public administrative units from municipalities, states and the federal government; pension funds; social security and non-financial state-owned enterprises (excluding the Eletrobras and Petrobras groups). The Brazilian central bank is also included as the former activities are directly financed by the national treasury. The net public debt also comprehends domestic and foreign debt.

The recent evolution of public debt shows two opposing trends. Prior to 2014, Brazil experienced long periods of economic growth, except for the 2008 financial crisis. In the meantime, continuous fiscal surpluses allowed the central government to maintain growth rates in spending compatible with GDP growth. According to Tinoco and Giambiagi (2018), in the period 2003-2008, the mean GDP growth was around 4%, while in the period 2009-2014 the rate was 2.8%. Following the 2008 financial crisis, Brazil employed a series of fiscal incentives to minimize the impacts of the crisis, such

Figure 15 – Net public debt share of Brazilian GDP, 2003-2018



Source: BACEN (2019).

as tax exemptions, more credit via development banks. Also, other measures were taken to control inflation rates, such as interventions in administered prices (fuels and energy). With the deterioration of the Brazil's economic fundamentals, fiscal surpluses became deficits from 2014 on, allowing for the debt-to-GDP rates to rapidly rise.

Moraes and Divino (2016) examines the Brazilian public debt in detail. They show that there is a trend shift (for worse) in the conduction of the public debt. Also, they show statistics that illustrate this: from 2013 to 2015, the gross government debt jumped from 51% to 65% of GDP. In the meantime, there was widespread news on the loss of investment rating of Brazilian bonds, determined by rating agencies such as Moody's and Standard & Poors. In their empirical strategy, they conclude that fiscal shocks can induce inflation due to its effects on aggregate demand and gross fixed capital formation, perhaps the most interesting result: the composition of debt (inflation-indexed, interest-indexed or pre-fixed rate bonds) does not deeply affect real aggregates in response to a TFP shock. This happens as governments construct a portfolio heavily based on bonds indexed to interest rates (such as *Tesouro Selic* bonds). Finally, the optimal Taylor rule found by the authors is the one when the monetary authority does not respond to the output gap, but focusing on firm responses to inflation. While fiscal shocks contribute to greater government debt, monetary and TFP shocks can potentially reduce it, especially when debt is composed of inflation-indexed bonds,

as found in the simulations performed by Moraes and Divino (2016). Finally, we point out that the rise in public debt is a global trend: according to Chudik et al. (2018), there is an upward trend in the evolution of debt-to-GDP ratios worldwide, partly because of fiscal stimuli after the slow recovery from the 2008 financial crisis. Further, the authors argue that higher debt harms long-term economic growth.

Is the trajectory of public debt explosive? Table 3 shows linear, quadratic and exponential least-squares fits, on the timespan according to the latest upward trend, beginning in 2014. Analyzing the diagnostic statistics, especially the F statistic, the quadratic model  $\hat{d}_t = 62.08 - 0.5t + 0.002t^2$  seems to give a better fit to actual data. While the  $R^2$  statistic is also higher (0,806), this is not very informative because the model has an additional covariate in comparison to other models. For example, the quadratic model predicts that the debt-to-GDP ratio will reach around 80% by 2020 and surpassing the 100% mark soon, *coeteris paribus*, in an explosive trajectory.

Table 3 – Least squares fits on public debt

	Dependent variable:		
	$d_t$	$d_t$	$\log(d_t)$
	(1)	(2)	(3)
$t$	-0.057*** (0.009)	-0.500*** (0.018)	-0.001*** (0.0002)
$t^2$		0.002*** (0.0001)	
Constant	47.742*** (1.049)	62.081*** (0.764)	3.859*** (0.025)
Obs.	192	192	192
R <sup>2</sup>	0.161	0.806	0.164
Adjusted R <sup>2</sup>	0.156	0.804	0.160
Residual Std. Error	7.240	3.493	0.173
F Statistic	36.389***	391.710***	37.285***
Note:	*p<0.1; **p<0.05; ***p<0.01		

Source: The author.

Are debt and government consumption to GDP related in the long term? First, we will check for unit roots in both series. We run the Augmented Dickey Fuller unit root test with drift to both series. The null hypothesis is that the series has a unit root and the critical value is  $\tau_\alpha = -1.95$ . The statistics found for debt (2.32) and consumption (3.76) are unable to reject the null hypothesis at a 95% confidence interval. Now, we

can search for a co-integration relationship, using the Johansen trace co-integration test.

Table 4 – Johansen test results

Relationships	Test statistic	Critical value (95%)
r=1	5.71	9.24
r=0	27.3	19.96

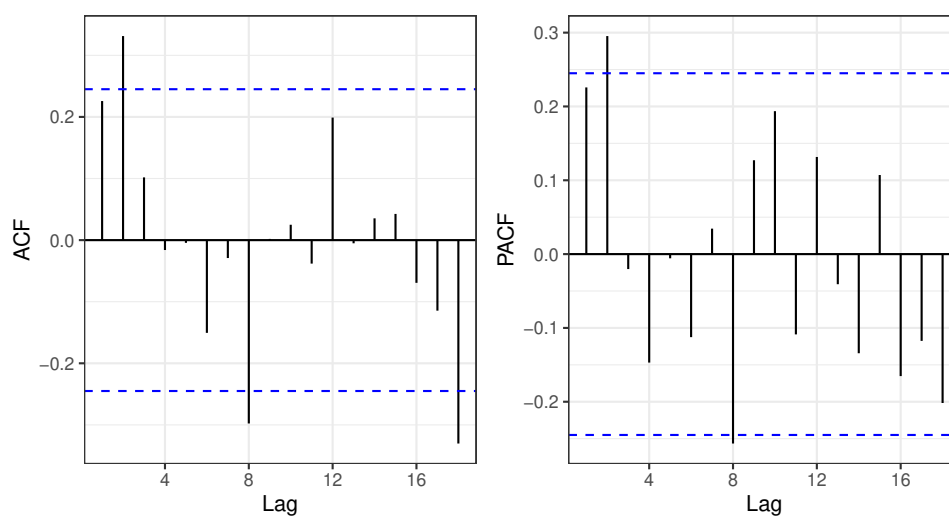
Source: The author.

Inspecting the results in Table 4, we were unable to find a long-term relationship between debt-to-GDP and government consumption to GDP ratios, providing us evidence that the evolution of debt should affect the economy through channels other than the fiscal policy shocks defined in our model. We believe this happens because of the trend shift in debt evolution. Having that said, we argue that the matter of public debt does not pose an immediate threat to our empirical model, because our control variable for the government sector presents more predictable trends, as shown by Figure 2. Nevertheless, we should follow the dynamics of debt closely, because of other transmission channels, such as the sovereign risk channel: the EMBI+ index – also known as *Risco Brasil* – registered an increase of 100 basis points from 2014 to 2017<sup>9</sup>.

To check if the AR(1) process is an accurate representation of the evolution of Brazilian government spending, we apply the Hodrick-Prescott filter with  $\lambda = 1600$  to  $G_t$ , extract the cycle series and examine the plot of its autocorrelation (ACF) and partial autocorrelation functions (PACF), presented in Figure 16.

Examination of the plots reveals interesting features, such as negative autocorrelation spikes every 8 quarters (2 years), that could be caused either by budget revisions or the electoral cycle. The ACF plot shows an oscillatory pattern, meaning that  $\rho_g$  could be negative. But the main information is that there are 2 statistically significant lags in the PACF plot (the first one at the margin), meaning that a second order autoregressive process would better fit empirical data. We should further evaluate the benefits of adding an AR(2) parameter to the fiscal policy rule.

<sup>9</sup> Available at: <<http://ipeadata.gov.br/ExibeSerie.aspx?serid=40940&module=M>>. The index is based on the spread from Brazilian short-term public bonds and U.S. Fed bonds.

Figure 16 – ACF and PACF plots of  $G_t$  cycles

Source: The author.

### 3 THEORETICAL AND EMPIRICAL FRAMEWORK

The real business cycles literature emphasizes the role of fluctuations in key macroeconomic variables for economic growth and development, such as government spending, taxation, interest and inflation rates. Also, the role of technological progress and human capital have been widely discussed. Since the seminal work of Kydland and Prescott (1982), the framework has been refined, with the addition of consumption habits, investment adjustment costs, different fiscal and monetary policy rules.

Recently, the role of uncertainty in economic activity has received greater attention (BLOOM, 2014; BAKER; BLOOM; DAVIS, 2013). With the development of more sophisticated estimation techniques (such as Bayesian methods), the study the dynamic behavior of economic variables not only in its first but also second moments became feasible. Then, a large number of research articles emerged (since the inception of this research, more than a thousand papers on economic uncertainty were indexed in Elsevier's *ScienceDirect* platform). Despite the mixed evidence on the issue of volatility-growth correlation, one of the goals of policymakers is to smooth out deviations from long-term, sustainable economic growth (i.e. monetary policy rules). This research agenda is especially relevant for emerging economies such as Brazil because the business cycles are more volatile.

Remarkably, uncertainty affects economic activity through investment decisions. The latest findings from the research on economic uncertainty delve into this topic: in example, Bianchi, Kung and Morales (2019) points out that uncertainty shocks greatly affect firms with a large number of intangible assets, due to the liquidation differences between physical and intellectual capital. With the notion that knowledge capital provides "a weak collateral", it is likely that firms in innovative sectors experience greater volatility. The other important feature of economic uncertainty is its complexity. In a study of European countries, Śmiech, Papież and Dąbrowski (2019) notes that metrics of consumer and stock market volatilities do not exhibit a strong correlation. As we will later show, one can build a set of economic fluctuations that is reasonably based on market fundamentals but, however, uncorrelated with each other. Real prices are

also affected by uncertainty: Watugala (2019) shows that global commodity futures contracts respond to swings in macroeconomic uncertainty.

In the baseline real business cycle models, financial markets are set aside, with the underlying assumption that financial fluctuations do not affect real economic aggregates. However, Bernanke, Gertler and Gilchrist (1999) point out that credit market shocks are not only a consequence but can also cause stress in the real economy. Evidence coming from the financial crises in the 1990s (Southeast Asian Tigers, Russia, Latin American countries) helped popularize this element in small and medium-sized models. Hence, our theoretical approach embodies the strand of the macroeconomic literature of dynamic stochastic general equilibrium models including the financial sector as a source (or amplifier) of business cycles.

Why financial frictions exist, in the first place? The central theoretical element of models with financial frictions is the existence of asymmetric information between lenders and borrowers, which poses agency problems. The problem of agency costs was first proposed by Townsend (1979). The author argues that the existence of asymmetric information between borrowers and lenders implies that financial institutions incur in monitoring costs for loans. This mechanism is at the heart of the argument in Christiano, Motto and Rostagno (2014): because there are such monitoring costs, banks charge greater interest rates on loans, increasing the threshold productivity shock required by entrepreneurs not to go bankrupt. Also, adverse selection is present, as loans are demanded, for the most part, by agents with inferior net worth.

While there is, to the best of our knowledge, no evidence regarding microeconomic uncertainty shocks, as we specified, for the Brazilian economy, a study that most closely matches our analytical framework is Melo and Silva (2019). In their paper, the authors model uncertainty as an autoregressive process that governs swings in total factor productivity. In their findings, volatility shocks caused a drop in output, consumption and investment, and a rise in labor supply and external debt. Other papers reveal the impact of financial frictions on Brazil's business cycles, which is the key transmission mechanism of entrepreneurial uncertainty. With a model based on Gerali et al. (2010), Aranha (2012) finds important evidence for Brazil: (i) a reduction in credit market frictions (measured by bank spreads and adjustment costs) results in an increase in

investment, consumption and GDP; and (ii) while those frictions help keeping banking spreads high, they have a negative impact on inflation rates. Kanczuk (2013) extends the Smets and Wouters (2007) model, incorporating credit markets for households and firms, to show how macroprudential policies can affect the Brazilian economy. Castro (2016) also focused on the topic, with a model that combines financial frictions and foreign capital flows, that is, direct investment as a dependent of sovereign risk, to establish if the fiscal policy should react to credit cycles. The authors suggest that this is only the case if macroprudential policies are independent and counter-cyclical.

Now, we discuss microeconomic shocks as a source of business cycles. This hypothesis states that shocks in productivity or sales, at firm or sector level, lead to aggregate output fluctuations, providing a microeconomic foundation to the real business cycles. However, this argument departs from the assumption that aggregate shocks are the result of an average of identical firms or sectors. In fact, empirical evidence from Gabaix (2011), studying the U.S. economy, shows that the distribution of firm sizes is skewed. Hence, exogenous shocks to large firms should account for a non-negligible amount of the business cycles. The author calls it the “granular hypothesis”. This is potential evidence for developing countries, as anecdotal evidence suggests that such economies have greater sectoral concentration, so one could expect this hypothesis to have greater explanatory power.

Acemoglu et al. (2012) refine the previous argument. The authors argue that microeconomic shocks lead to aggregate fluctuations through input-output relationships. This happens because productivity gains propagate along supply chains and generate spillovers to the whole economy. We call it the “network hypothesis”. The hypothesis, corroborated by the authors using U.S. input-output data, also states that microeconomic shocks translate into aggregate volatility “if there are significant asymmetries in the roles that sectors play as direct or indirect suppliers to others” (ACEMOGLU et al., 2012, p. 2004). Both the granular and the network hypotheses interact in the sense that large firms and markets have an important contribution to the business cycles. More papers emphasize the network characteristic of modern economies as a transmitter of economic fluctuations. Carvalho (2014) studied the production networks using U.S. input-output data and shows that indeed the productivity in central sectors of the economy and GDP growth are strongly correlated. These “central” sectors act

like hubs: they provide simple inputs (e.g.: raw materials) that firms from other sectors can resort to, in the event of a break in supply chains.

Acosta-Ormaechea and Morozumi (2017) further expands the knowledge between microeconomic uncertainty and long-run economic growth. Using cross-country evidence, they argue that while monitoring costs and credit frictions do affect capital accumulation, this effect is attenuated when bankruptcy costs are low. Hence, the theory predicts that if such costs are null, microeconomic uncertainty does not affect long-run economic growth at all.

### 3.1 The role of finance

Bernanke and Gertler (1995) summarize how credit markets can amplify the effects of monetary policy. The *external finance premium* is the difference between raising capital from internal (reinvesting profits) and external (i.e; equity markets) sources.

This premium, which is counter-cyclical, can be accessed via two channels. First, the *balance sheet channel*: if entrepreneurs have a greater net worth, they can self-finance investment projects partially or completely, or offer a higher amount of collateral as a guarantee, diminishing the external finance premium. Thus, the financial position of borrowers should affect the premium. Also, consider the position of borrowers after loan contracts are celebrated. If loans are negotiated at current, post-fixed interest rates, monetary policy shocks affect the firm's balance sheets in the short term. Now, firms reconsider taking new loans, expanding the external finance premium and depressing investment. Via this mechanism, policy shocks may have long-term effects.

Second, the *bank lending channel* predicts that banks will also experience a balance sheet effect in response to interest rate adjustments. According to Bernanke and Gertler (1995), while the plausibility of the balance sheet channel is well understood, the bank lending channel is controversial, because financial liberalization can diminish bank lending channel effects, as banks can offer new, less insured products.

Dorofeenko, Lee and Salyer (2008) employ a model with financial frictions, and while they found that microeconomic uncertainty shocks are small as compared to aggregate shocks, they affect the economy's steady state and the bankruptcy rates. In general, the theory predicts that the impact of microeconomic uncertainty shocks

on the real economy is largely affected by conditions in financial markets, i.e. the risk premium, hence the term “financial accelerator” coined by Bernanke, Gertler and Gilchrist (1999).

Gilchrist, Sim and Zakrajsek (2014) find that the impacts of microeconomic uncertainty shocks are highly sensitive to credit spreads (i.e., the risk premium). In a model that adds investment irreversibility to the financial accelerator model framework, fluctuations in the cost of capital can lead to liquidity problems for firms, because liquidation of capital in case of default lowers the corporate debt claims value. Chugh (2013) uses the financial accelerator framework to assess the impacts of uncertainty shocks, at the firm level, on the real economy. They find that these shocks account for approximately 5% of GDP volatility, which the authors interpret as a large value. Also, the microeconomic uncertainty shocks are about as large as aggregate productivity shocks.

Economic policy uncertainty can also influence business cycles. Chi and Li (2017) analyzes the impact of swings in economic policy uncertainty on bank loans in China. They regress loan amounts on an economic policy uncertainty index and banking sector controls, to conclude that greater policy uncertainty causes a reduction in quantity and size of loans, as they perceive increased risk, and this result is more pronounced in financially less-developed areas. Moreover, the effect size changes if banks are private, state-owned or have mixed ownership. As a policy recommendation, they advocate for better coordination between commercial banks and the monetary policy authority as a means of smoothing credit supply.

Valencia (2017) also shows that, in moments of greater uncertainty, banks adjust their capital-to-assets ratio, which varies according to the bank size, presenting a mechanism of self-insurance. This suggests that, in aggregate terms, the impact of this channel depends on the relevance of small and big banks. And how does the structure of the banking sector affect economic fluctuations? Larrain (2006) finds that greater access to banks contributes to a reduction in the volatility of industrial production: firms incur short-term loans to smooth output, in response to demand and inventory shocks. Thus, banks have a counter-cyclical effect on uncertainty shocks. Huang, Fang and Miller (2014) shows that a highly concentrated banking market cor-

relates with greater industrial growth volatility. However, a more concentrated banking sector can be beneficial to sectors highly dependent on external liquidity. Furthermore, Fendođlu (2017) finds that in emerging countries monetary policy tools based on domestic reserve requirements are helpful to counterbalance foreign capital inflows, and therefore preferred by the economic policy authorities.

A large number of papers investigate the relationship between financial markets and the mean and volatility of economic growth. The findings of Bekaert et al. (2005) and Bekaert, Harvey and Lundblad (2006) suggest that equity market liberalization is positively correlated with economic growth, and this effect is powered by better institutions. According to the authors, equity market liberalization, measured by national capital account openness, does not raise the volatility of gross domestic product. Coricelli and Masten (2004) analyzes the importance of credit markets in Eastern European countries, which experienced low economic growth and high volatility right after the transition from central planning to free-market economics. With respect to the banking sector, Pirozhkova (2017) shows that banks perform a portfolio reallocation in the presence of economic uncertainty: instead of offering risky loans, they resort to risk-free assets such as savings and public bonds.

### 3.2 Investment dynamics

Because most investment projects contain sunk costs, traditional calculations of net present value must take into account the alternative costs of other investment decisions (BERNANKE; ECONOMICS, 1983). In his theoretical model, agents can wait until new information about the market arrives. Otherwise, agents must calculate the expected value of committing to an investment project until its maturity. Irreversibility is largely related to the mobility of physical or human capital. In an example, a piece of highly specialized equipment that cannot be easily reallocated into another activity. This phenomenon also affects labor markets, as firms face sunk costs when dismissing workers, especially high-skilled ones. As an empirical example, Liu and Zhang (2019) shows that the impact of economic policy shocks to Chinese firms is heavier on industries with lower asset reversibility.

In this framework, Pindyck (1991, p. 1114) defines uncertainty as “fluctuations in prices that affect future cash flows, and consequently the net present value of investments”. Comparing risky and risk-free portfolios (assuming that risk is perfectly diversifiable, although it is not a necessary condition), the author then shows some interesting results that derive from the calculations of investment decisions under uncertainty. First, when uncertainty rises, the value of investment opportunities also goes up, meaning that firms can become more valuable even when markets are more volatile. Second, such results are independent of the risk preferences of managers. Third, higher real interest rates depress investment, which is a consistent result in neoclassical models of investment, but not because of a simple rise in the cost of capital: with greater interest rates, new investment projects are more expensive, but the value of finished projects stays the same.

In the context of DSGE models, examples of uncertainty/risk as discussed by Pindyck (1991) are explicit in time-to-build models (KYDLAND; PRESCOTT, 1982; JUNG, 2013). In this class of models, a representative agent commits to an investment project that matures many periods ahead. A result supported by the literature is that the longer the “time to build” it takes to finish an investment project, the more uncertain it is, and larger spreads of the investment rate of return with respect to the risk-free rates are required. This is aggravated if there are costs to stop and restart projects. Also, as pointed out by the author, a decrease in interest rates need not benefit long-term investments, because it also reduces capital costs for other projects.

In turn, Aiyagari (1994) focuses on a precautionary savings motive, combined with borrowing constraints. The author presents an example model where a continuum of agents is subject to random labor endowment shocks. In this case, uncertainty emerges from the productive process. Because markets are incomplete and agents try to smooth consumption, they accumulate capital. Thus, in the face of greater uncertainty agents increase aggregate savings (which we interpret as an investment, because of the macroeconomic identity). The author points out that earlier studies relied on the pure bequest motive as an explanation to inter-generational capital accumulation, but precautionary savings presents as a relevant explanation, with emphasis on lower-income households.

Studying the literature on U.S. income distributions (at the time the paper was published), Aiyagari (1994) shows that despite stock market returns were much higher than the risk-free rates, only 25% of the households owned stocks. Also, low-income households invested in low-risk assets, while the opposite happened in high-income households, probably because low-income households cannot absorb large negative shocks, despite individual risk preferences. According to the author, this presents evidence for incomplete markets and consequently uninsured idiosyncratic risk (uncertainty). Under a model with such characteristics, interest rates will be lower than the time preference rate (a function of the discount factor), increasing aggregate savings. As consumption is non-negative at each period  $t$ , a large negative shock in labor endowments results in larger loans. If agents expect a long sequence of low labor endowments, they accumulate capital to smooth consumption.

How the irreversible investments and the precautionary savings points on uncertainty contribute to our research hypothesis? We understand that our dynamic model incorporates elements present in both theoretical approaches. The mechanism of precautionary savings is present in household's behavior: when their income sequence becomes more uncertain (which depend on the returns of the firms they own), they accumulate savings. In this case, the theory predicts a positive correlation between aggregate savings and past microeconomic uncertainty shocks. On the other hand, entrepreneur's investment decisions are (i) partly irreversible and (ii) subject to frictions in financial markets, caused by asymmetric information between banks (lenders) and entrepreneurs (borrowers). Also, as predicted by Pindyck (1991), entrepreneurs can wait for subsequent periods to invest. In our model, this mechanism is described by the nominal income from holding finished capital. This nominal income is, in principle, a function of the price of unfinished capital  $q_t$  and the rental rate of finished capital  $(1 + r_{t+1}^k)$ .

From the perspective of the dynamics of investment, Bloom et al. (2007) shows that higher economic uncertainty diminishes the responsiveness of investment to demand shocks, because of the irreversibility of investment, as noted by Pindyck (1991). This means that economic agents are less responsive to policy shocks. The theoretical approach of Aiyagari (1994) predicts that in periods of higher uncertainty, agents engage in precautionary savings, resulting in higher long-term economic growth. This

is observed mostly in developed countries, where financial markets allow for a wider range of assets for agents to choose from. We should also note how credit cycles and the response of economic policy relates to economic uncertainty. Based on arguments and evidence reviewed so far, we can argue that agents take into account swings in credit supply when facing investment decisions, especially long-term ones. In addition, restrictions to equity markets raise the cost of external capital, as predicted by Bekaert et al. (2005). However, these phenomena enter in the demand curve of the credit markets.

In a survey of U.S. and German firms, Bachmann, Elstner and Sims (2012) find that uncertainty proxies based on survey data account for a greater share of output volatility than stock market-based proxies. Also, they present interesting evidence: both countries exhibit slightly different output dynamics in response to an uncertainty shock. While in the U.S. shocks have high persistence, in Germany shocks have a pattern of decline followed by quick recovery, called by the authors a “bust-boom cycle”. According to the Bachmann, Elstner and Sims (2012) and Bachmann and Bayer (2013), this provides evidence of the slightly higher cost of factor adjustments, that is, in moments of greater uncertainty firms prefer to postpone investments and observe the behavior of their competitors. Overall, the work of Bachmann suggests that in these countries the effects of “wait-and-see” dynamics on firm-level risk are small. This is not the only researcher that questions this dynamic: in influential research, Gilchrist, Sim and Zakrajsek (2014) showed that credit spreads are the real drivers of fluctuations in capital accumulation. However, the presence of labor regulations and capital constraints in Brazil result in greater factor adjustment costs (in comparison to developed economies). This suggests that investment dynamics and “wait-and-see” behavior could provide a sounder theoretical argument to explain some of the effects of microeconomic uncertainty shocks in the Brazilian economy.

Last, we consider the work of Aghion et al. (2010). The authors build a model containing short-term and long-term (more productivity-enhancing) investments, and the main result is that the long-term investment share is pro-cyclical when credit constraints are sufficiently tight, a common scenario in developing countries. However, this increases liquidity risks, therefore decreasing the share of long-term investments and economic growth.

The microeconomic uncertainty shocks we will discuss in this work are closely related to the ability of entrepreneurs to thrive. Therefore, we must discuss the ease of doing business in Brazil. In general, while Brazil supports a free enterprise economy, there is extensive government intervention and regulation, which manifests in the form of: (i) permits and obligations with federal, state and municipality entities; (ii) an extensive tax code that firms must comply with. According to the World Bank's 2018 Ease of Doing Business index, Brazil is ranked at 109th place, among 190 countries in the sample. Among the many aspects of the index, Brazil is poorly ranked when it comes to starting a business, paying taxes and dealing with international trade. We expect that these difficulties affect both the expected rate of survival of small business ventures and their variability, because entrepreneurs must evaluate both the short-term obligations and long-term macroeconomic expectations when starting a business.

### 3.3 Discussion of microeconomic uncertainty shocks

In this section, we will discuss the origins of microeconomic uncertainty shocks. First, we will discuss what are the conditions that favor the emergence of such shocks. microeconomic uncertainty shocks are commonly attributed to heterogeneities in firms' productive processes and outcomes, that are subject to fluctuations. In our work, this is represented by a continuum of entrepreneurs which shocks go in the same direction. Also, it is known that credit market imperfections exist, and they can amplify the effect of the uncertainty shocks in the aggregate economy (GERTLER; KIYOTAKI, 2010) due to an information problem because lenders do not observe the realized shock from borrowers. This phenomenon is called the "costly state verification problem" (TOWNSEND, 1979). This is an essential theoretical element, because, in the absence of credit constraints, there is no wedge in banks' zero profit condition, providing us evidence that any proxy variable for microeconomic uncertainty shocks must be correlated with financial markets (and not exogenously in production, in example).

In our analytical model, uncertainty arises from the dispersion of possible production plans of entrepreneurs. Earlier, we discussed how economic uncertainty affects investment, from several theoretical perspectives. In practice, there are many research initiatives on the topic. Morikawa (2019) studied the behavior of nearly 700

Japanese firms in the manufacturing sectors through the Survey of Production Forecast (SPF). A key finding from the author is that the dispersion of production forecasts was a predictor for macroeconomic indices such as employment and industrial production. Also, there was a co-movement between the firms' production fluctuations and measures of economic uncertainty such as the stock market volatility.

The proxy for microeconomic uncertainty shocks should contain a few important properties. First, it must be counter-cyclical, according to empirical evidence (BAKER; BLOOM; DAVIS, 2013). Second, according to Cesa-Bianchi and Fernandez-Corugedo (2014), the proxy should be able to indirectly generate fluctuations aggregate in output, consumption, investment and labor, based on evidence from Christiano, Motto and Rostagno (2014). Having all that said, it is admittedly difficult to obtain a definitive measure of economic uncertainty: firstly because it derives from a multitude of factors, such as economic conditions, agents' perception and behavior, and secondly because it is necessary to adopt some definition of uncertainty, and any choice may lead to different strands of literature and empirical results. Also, there are empirical limitations. According to Ludvigson, Ma and Ng (2015, p. 16), "common uncertainty proxies contain economically large components of their variability that do not appear to be generated by a movement in genuine uncertainty across the broader economy". We stick to the Knightian definition of uncertainty, namely the difficulty of agents to forecast the probability of occurrence of all possible outcomes from a set of events (and histories).

The works of Nicholas Bloom investigate the relationship between uncertainty and economic performance in greater detail. An important remark of the author is that "(...) uncertainty also appears to endogenously increase during recessions, as lower economic growth induces greater micro and macro uncertainty" (BLOOM, 2014, p. 153). Studying the U.S. economy, Bloom (2014) defines stylized facts for economic uncertainty: (i) macroeconomic uncertainty rises in recessions. That is, the volatility of almost all key economic indicators rises in recessions; (ii) microeconomic uncertainty also rises in recessions. The author states this fact based on data from firms and industrial production. Volatility rises in the sense that – in more uncertain times – some sectors perform better than others. A glance in Brazil reveals that the industry was the most affected sector in the last two recessions, as shown by Figure 4. It follows

that a proxy for microeconomic uncertainty shocks based on the Brazilian industrial sector can be a good candidate. Bloom (2014) also points out that these facts are not restricted to the U.S. economy, as the same economic rationale is also found in global economies, and impacts are even more intense in developing countries such as Brazil.

Now, we present each of the proposed measures of microeconomic uncertainty, drawing from the literature, to a posterior comparison. All time series are seasonally adjusted, and first differences are taken at any sign of unit roots.

Uncertainty evaluation based on the productivity of firms is the most commonly found in the literature (BACHMANN; ELSTNER; SIMS, 2012; CHUGH, 2013; CHRISTIANO; IKEDA, 2013; CHRISTIANO; MOTTO; ROSTAGNO, 2014). It is a known fact that the distribution of firm sales changes from times of economic growth to recessions, as the average goes down and the dispersion goes up (BLOOM, 2014). Publicly available firm-level data of private companies in Brazil are scarce. As a proxy variable, we use the moving standard deviation of monthly industry sales in the state of São Paulo, produced by FIESP (*Federação das Indústrias do Estado de São Paulo*).

Since economic uncertainty is understood as a result of multiple factors, some authors and entities proposed composite indices that can be used as a proxy for microeconomic uncertainty shocks. First, we consider the Economic Policy Uncertainty (EPU) index. The index has interesting properties: first, the selection of news coverage is statistically resistant to “media political bias”. This was tested by comparing two subsamples of newspapers segregated by political orientation. While the EPU index is primarily oriented to measure economic policy uncertainty (that is, shifts in fiscal and monetary policy) that refers to macroeconomic uncertainty, it contains useful elements for the study of microeconomic uncertainty shocks.

In Brazil, there is a similar effort, namely the IIE-Br (*Índice de Incerteza Econômica*) index, developed by IBRE/FGV. In a similar manner to Baker, Bloom and Davis (2013), the IIE-Br index is a weighted average of three components:

$$IIE_t = 0.7 * IIE_{media} + 0.2 * IIE_{expectations} + 0.1 * IIE_{market} \quad (3.1)$$

$IIE_{\text{media}}$  is based on the frequency of news mentioning keywords such as “uncertainty”, “crisis”, “risk” in Brazilian main newspapers<sup>1</sup>;  $IIE_{\text{expectations}}$  is built upon measures of disagreement among forecasters with respect to exchange and inflation rates (data available in Brazilian Central Bank FOCUS reports) and  $IIE_{\text{market}}$  is based on the volatility of Ibovespa, the Brazilian stock market index. The IIE-Br index is standardized such that it has mean 100 and standard deviation 10 in the last ten years.

There are a few metrics of economic uncertainty that derive exclusively from stock markets. The most popular is the VIX index, that measures the expected volatility of the S&P 500 index, calculated by the Chicago Board Options Exchange. For Brazil, we calculate the volatility of Ibovespa index by applying moving standard deviations to the monthly index, using backward-looking 3-month windows. The Ibovespa index is a weighted portfolio of the firms responsible for the largest part of daily negotiations. Stock market returns are very interesting because they reflect not only the performance of firms themselves but also agents expectations and perceptions about the market.

As our theoretical approach on entrepreneurs’ behavior is at the center of microeconomic uncertainty shocks, and directly involves financial frictions, we should also consider proxies that reflect credit risk. According to the model approach, microeconomic uncertainty shocks precedes fluctuations in credit spreads (that is, the difference between the interest rate offered by banks to lenders and the risk-free rate). Christiano, Motto and Rostagno (2014) also suggest that variables such as stock market value, credit to non-financial firms and the term structure of interest rates impact business cycles significantly.

We calculate the correlation of GDP growth the proposed uncertainty proxy variables. For comparison, we transform all variables to be distributed according to  $N \sim (0, 1)$ , the standardized normal distribution. We denote  $IBOV_t$  the moving dispersion of the Ibovespa index,  $EPU_t$  as the EPU index,  $INDS_t$  as the moving dispersion of industrial sales,  $SPR_t$  as the credit spread to non-financial firms and  $IIE_t$  the *Índice de Incerteza da Economia*. Table 5 presents the results:

<sup>1</sup> The full text in *Folha de São Paulo* and *Valor Econômico* is analyzed. In the other selected newspapers (*O Globo*, *Estado de São Paulo*, *Correio Braziliense* and *Zero Hora*), data is collected in the publications’ Twitter accounts.

Table 5 – Correlation between Brazilian GDP and uncertainty proxies

	$\Delta Y_t$	$\Delta Y_{t-1}$	$IBOV_t$	$EPU_t$	$INDS_t$	$SPR_t$	$IIE_t$
$IBOV_t$	<b>-0.178</b>	0.045	1				
$EPU_t$	-0.093	0.122	0.037	1			
$INDS_t$	-0.155	-0.047	0.041	0.190	1		
$SPR_t$	0.046	-0.069	0.341	-0.127	0.350	1	
$IIE_t$	0.060	-0.057	0.029	0.013	-0.008	0.323	1

Source: The author.

The reviewed economic theory tells us that uncertainty proxy should have a strong negative correlation with economic growth. Analyzing the results from Table 5, our proxy based on the dispersion of stock returns  $IBOV_t$  appears as having the largest negative correlation with GDP quarterly growth, followed by  $INDS_t$ , the uncertainty proxy based on industrial sales. This pattern appears both in contemporaneous and lagged correlations. This is corroborating evidence that proxies based on firm returns better approximate the business cycles, setting aside other factors that affect long-term economic growth. We find that stock market returns are easier to calculate, with richer data, and more robust in the sense that companies all around the country are included, while  $INDS_t$  is based solely on industries from the state of São Paulo. This finding is in agreement with studies such as Hillier and Loncan (2019), that found that political instabilities derived from *Operação Lava Jato* affected returns of Brazilian firms listed in U.S. markets. In general terms, the calculation above was impacted by the transformation of the series from monthly to quarterly data, due to the loss of information. Yet, redoing the calculations using monthly data for all variables and industrial production as a proxy for GDP growth yields similar results.

### 3.4 Endogenous business cycles

A few authors explored the matter of economic uncertainty drawing proposals to endogenize business cycles. But what is the economic thought behind it? The recent works of Rüdiger Bachmann investigate how endogenous mechanisms impact business fluctuations. Bachmann and Moscarini (2011) note that while the literature argues that aggregate economic activity is impacted by microeconomic uncertainty shocks, in the presence of credit market imperfections, the causality may run also in the opposite direction. According to the author, the literature focuses on the “real options”

theoretical approach, relying on the irreversibility that depresses investments because of the value of waiting, and that includes price adjustments. When price adjustment costs are low, firms may find advantageous to adopt larger price swings, attempting to estimate their demand curve in times of greater uncertainty. The authors used sales data from German firms to find that “(...) the dispersion in firm-level innovations in TFP, sales and real value-added is counter-cyclical, although the dispersion in investment rates is pro-cyclical” (BACHMANN; MOSCARINI, 2011, p. 2).

In another article, Bachmann and Bayer (2013) refines the previous argument. Calibrating a DSGE model and using survey data from U.S. and German firms, they summarize their work in three results: (i) unless firm productivity is really volatile, shocks at firm-level shocks lead to results similar as standard RBC models; (ii) uncertainty shocks alone are unable to generate co-movement between consumption and other macroeconomic aggregates, in contrast with precautionary savings hypothesis; (iii) a model with correlated risk (a measure analogous to our microeconomic uncertainty shocks) and TFP shocks better fit the empirical data than the specification with uncorrelated shocks.

To address this hypothesis in practice, Chugh (2013) proposes a “bundled shock” specification. For microeconomic uncertainty shocks to be certainly counter-cyclical, they can be linked to aggregate TFP shocks. This way, the standard deviation of entrepreneurial fluctuations in productivity declines when TFP raises. In the present framework, the time-varying dispersion of microeconomic uncertainty shocks is defined as  $S_t$  and a bundled shock could be defined as:

$$\log\left(\frac{S_t}{\bar{S}}\right) = \rho^S \log\left(\frac{S_{t-1}}{\bar{S}}\right) + \sigma^S \varepsilon_t^S + \underbrace{\sigma^B \varepsilon_t^A}_{\text{link to TFP shock}}$$

This equation assumes that microeconomic uncertainty follows a first-order autoregressive process, with the addition of a term, stating that microeconomic uncertainty shocks are a function of contemporaneous aggregate TFP shocks. For the link to be counter-cyclical we need  $\sigma^B < 0$ , that is, the sensitivity (or elasticity) of the entrepreneurial uncertainty to TFP shocks to be negative. The shocks itself  $\varepsilon_t^S$  and  $\varepsilon_t^A$  are uncorrelated. This requires an empirical link between both shocks, which is supported

by the works cited above (in the case of developed countries such as the United States and Germany).

## 4 ANALYTICAL MODEL

The research hypothesis directly addresses the matter of microeconomic uncertainty shock in the aggregate economy. The mechanisms that generate such shocks and their impact on the economy, while intuitively simple, are reasonably complex to be evaluated in the simple linear regression framework, due to the many linkages and transmission channels between firms, entrepreneurs, economic policy and output, whereas the dynamic stochastic general equilibrium framework provides an accurate depiction of agents, endogenous and exogenous imbalances in a simulated economy.

Dynamic stochastic models are proven to be a flexible approach, providing a bridge between well-known economic laws and recent discoveries. Agents' choices should not reject basic microeconomic laws, such as market dynamics and some kind of rational behavior. Conversely, it incorporates Keynesian elements such as price rigidities, investment dynamics other than the neoclassical cost-of-capital effects, active economic policy and intervention. In the realm of economic policy, dynamic models are a valuable and current tool for policymakers' decision-taking, not only for fiscal and monetary policy evaluation but also taxes, savings, social security (via overlapping generations models).

Some critiques of this class of models also apply. As Korinek (2015) warns, the DSGE framework, as in any other, has its benefits and limitations. The author remembers that DSGE models went under "heavy fire" after the 2008 financial crisis, and points out that a part of the Lucas critique (that motivated models that include expectations in the first place) could also be applied to DSGE models as of today:

"From a somewhat broader perspective, the Lucas critique is an application of the principle that if you leave something out of your model and that thing changes, you will get things wrong. DSGE models are neither necessary nor sufficient to deal with this broader problem – for example, the macroeconometric models at many central banks have explicitly incorporated inflation expectations in response to the Lucas critique without relying on full microfoundations" (KORINEK, 2015, p. 5).

To evaluate the role of microeconomic uncertainty shocks in business cycles in Brazil, we present a dynamic stochastic general equilibrium model based on Doro-

feenko, Lee and Salyer (2008). This is a model of a closed economy with a financial sector based on Carlstrom and Fuerst (1997). The key actors of the model are the entrepreneurs, which produce finished capital goods and are subject to production uncertainty and interact with banks to finance their activities. This interaction between entrepreneurs and banks will be the source of shocks that emerge from the financial sector and affect the real economy. Equity markets are absent from the modeled financial sector, hence debt is the only source of external finance. We enhance the model by adding fiscal and monetary policy, in order to evaluate the impacts of economic policy shocks in our model economy. The main variable of interest is  $S_t$ , the time-varying standard deviation of microeconomic uncertainty shocks. Innovations to  $S_t$  allow for the dispersion of the shocks to change over time.

The fundamentals of this model resemble a standard real business cycles (RBC) model, with the addition of financial market imperfections. The model incorporates the idea of the “financial accelerator”, which brings the external finance premium to investment dynamics. The relationship between entrepreneurs and banks are the key aspect behind microeconomic uncertainty shocks. Also, the financial accelerator described above depends on costly state verification, which introduces information asymmetries between financial parties.

We should also comment on why other elements found in other DSGE models are absent in our specification. The main reason is, obviously, simplicity. Even with recent advancements, numerical simulations present in DSGE models are computationally intensive and prone to inconsistencies in optimization algorithms<sup>1</sup>. In our view, the most important feature that is missing is international trade: in this work, we model a closed economy. Were it an open economy, we’d argue that entrepreneurs would be subject to the same constraints in credit markets, being foreign entrepreneurs exposed to an additional exchange rate risk (also absent from the model, consequently). Also, exports and imports can be faced as simple disaggregation of consumption and saving/investment, bringing no additional benefit in terms of interpretation of the model economy.

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<sup>1</sup> Also, we should wonder if a certain feature is key to explain any economic dynamics backed theoretically. Hence, we follow Occam’s razor as a good scientific principle.

An overview of the model follows. There is a continuum of agents distributed in the  $(0, 1)$  interval, divided between households and entrepreneurs. Entrepreneurs represent a fraction  $\eta$  of the economy, while households represent a fraction  $1 - \eta$ . To finance their activities, they interact with financial intermediaries called capital mutual funds (CMF) in Carlstrom and Fuerst (1997), which in here we simply refer as banks. In addition, some firms employ capital, household and entrepreneurial labor to produce a final, homogeneous good, and their productivity is subject to exogenous shocks.

#### 4.1 Financial contract

Here, we define the financial contract and its optimal conditions. We follow the specification of the contract in Dorofeenko, Lee and Salyer (2008). Financial contracts are one-period loans celebrated by two parties:

- *Entrepreneurs*: Risk-neutral agents that finance investment project combining internal (their own net worth) and external (loans from banks) resources.
- *Banks*: Risk-neutral financial intermediaries that operate in perfect competition. Banks collect resources from entrepreneurs when they: (i) go bankrupt; (ii) repay loans and (iii) buy capital from entrepreneurs willing to increase consumption.

In every period, entrepreneurial productivity is subject to an exogenous shock  $\omega_t$ . The realization of this shock at time  $t$  is known only by entrepreneurs – banks must pay a fraction  $\mu$  of the investment to observe the entrepreneurs' productivity. That is, they invest an amount  $i_t$  in capital goods with an expected return of  $\omega_t i_t$ . This productivity shock follows a log-normal distribution with mean one:  $\omega_t \sim \log \mathcal{N}(1, S_t)$ . Following the literature, we define the time-varying dispersion of entrepreneurial productivity  $S_t$  as a first-order autoregressive process:

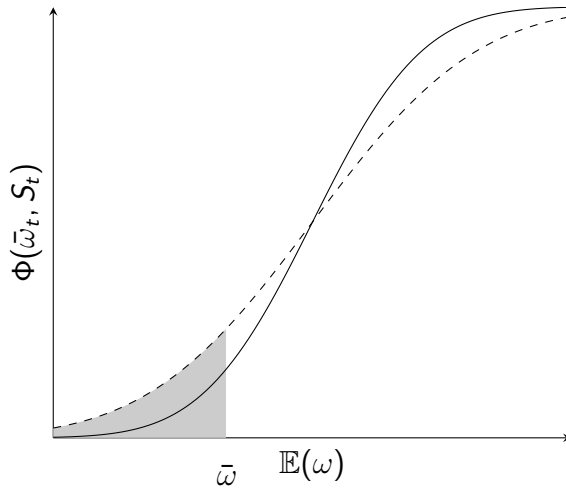
$$S_t = \bar{S}_t^{1-\zeta} S_{t-1}^\zeta \varepsilon_t^S \quad (4.1)$$

where  $\varepsilon_t^S \sim N(0, 1)$  is the *microeconomic uncertainty shock*.

Figure 17 provides intuition on the microeconomic uncertainty shock: it is a plot of the cumulative distribution function of  $\omega_t$ . The solid line is a hypothetical cumu-

lative distribution function (c.d.f.), while the dashed line is the c.d.f. after some positive shock in  $u_t$ . The vertical axis is the space of values for  $\Phi(\bar{\omega}_t, S_t)$ , leaving the entrepreneurial productivity  $S_t$  fixed, while the horizontal axis is the space of values for  $\omega_t \in \mathbb{R}$ . In the horizontal axis there are some example values Shocks to the dispersion of entrepreneurial productivity increase uncertainty about entrepreneurs' productivity (although they are mean-preserving), thus increasing the probability of bankruptcy  $\Phi(\omega, S_t)$  (which will be later defined) and changing the conditions of the financial contract:

Figure 17 – Effect of a microeconomic uncertainty shock



Source: The author.

An entrepreneur has one unit of labor that is always supplied and  $z_t$  units of capital at time  $t$ . Capital is rented to firms at a rate of return  $r_t$ . Hence, entrepreneurial income is  $w_t + r_t z_t$ . Also accounting for capital depreciation, net worth at time  $t$  is defined by the equation:

$$n_t = w_t + z_t(r_t + q_t(1 - \delta)) \quad (4.2)$$

where  $q_t$  is the price of finished capital. A solvent (i.e.,  $n_t > 0$ ) entrepreneur borrows  $i_t - n_t$  consumption goods from banks, with an obligation to pay back  $(1 + r_t^k)(i_t - n_t)$ ,

where  $r_t^k > r_t \forall t$  is the loan rate. According to their production, there will be a threshold shock  $\bar{\omega}_t$  that separates bankrupt from non-bankrupt entrepreneurs:

$$\omega_t < \frac{(1 + r^k)(i_t - n_t)}{i_t} \equiv \bar{\omega}_t \quad (4.3)$$

This leads us to two possible scenarios:

1.  $\omega_t \geq \bar{\omega}_t$ : The entrepreneur produces  $\omega_t i_t$  units of capital and pays back  $(1 + r^k)(i_t - n_t)$  to banks.
2.  $\omega_t < \bar{\omega}_t$ : The entrepreneur goes bankrupt and the bank will take all assets, upon facing monitoring costs  $\mu$ .

We should now define the shared of capital production to be distributed to entrepreneurs and banks:

$$f(\bar{\omega}_t, S_t) = \int_{\bar{\omega}_t}^{\infty} \omega \phi(\bar{\omega}_t, S_t) d\omega - [1 - \Phi(\bar{\omega}_t, S_t)] \bar{\omega}_t \quad (4.4)$$

$$m(\bar{\omega}_t, S_t) = \int_{-\infty}^{\bar{\omega}_t} \omega \phi(\bar{\omega}_t, S_t) d\omega + [1 - \Phi(\bar{\omega}_t, S_t)] \bar{\omega}_t - \mu \Phi(\bar{\omega}_t, S_t) \quad (4.5)$$

where  $f(\bar{\omega}_t, S_t)$  is the share of capital output obtained by the entrepreneur and  $m(\bar{\omega}_t, S_t)$  is the share received by banks. Note the integrands: while  $f(\bar{\omega}_t, S_t)$  is calculated above the threshold shock,  $m(\bar{\omega}_t, S_t)$  is calculated below the shock. We also have the property that  $f(\bar{\omega}_t, S_t) + m(\bar{\omega}_t, S_t) = 1 - \mu \Phi(\bar{\omega}_t, S_t)$ .

The optimal financial contract is a choice of investment and a threshold productivity shock that gives the entrepreneur maximum return, once banks are willing to offer such resources (an incentive compatibility constraint). Defining  $\Phi(\bar{\omega}_t, S_t)$  as the cumulative distribution function of  $\omega_t$  and  $\phi(\bar{\omega}_t, S_t)$  as its probability distribution function, the optimal contract is the solution to the optimization problem:

$$\max_{\{i_t, \bar{\omega}_t\}} q_t i_t f(\bar{\omega}_t, S_t) \quad (4.6)$$

$$\text{s.t. } q_t i_t m(\bar{\omega}_t, S_t) \geq i_t - n_t \quad \forall t \quad (4.7)$$

Solution of the problem above leads to the following first order conditions:

$$\frac{\partial L}{\partial \bar{\omega}} : q_t i_t \frac{\partial f(\bar{\omega}_t, S_t)}{\partial \bar{\omega}} = -\lambda_t q_t i_t \frac{\partial m(\bar{\omega}_t, S_t)}{\partial \bar{\omega}} \quad (4.8)$$

$$\frac{\partial L}{\partial i_t} : q_t f(\bar{\omega}_t, S_t) = -\lambda_t [1 - q_t m(\bar{\omega}_t, S_t)] \quad (4.9)$$

The first condition can be rewritten as:

$$1 - \frac{1}{\lambda_t} = \frac{\phi(\bar{\omega}_t, S_t)}{1 - \Phi(\bar{\omega}_t, S_t)} \quad (4.10)$$

Solving for  $\lambda_t$  and plugging into the second condition, we can find an expression for  $q_t$ :

$$\begin{aligned} \frac{1}{q_t} &= \left[ (f(\bar{\omega}_t, S_t) + m(\bar{\omega}_t, S_t)) + \frac{\phi(\bar{\omega}_t, S_t) \mu f(\bar{\omega}_t, S_t)}{\frac{\partial f(\bar{\omega}_t, S_t)}{\partial \bar{\omega}}} \right] \\ &= \left[ 1 - \mu \Phi(\bar{\omega}_t, S_t) + \frac{\phi(\bar{\omega}_t, S_t) \mu f(\bar{\omega}_t, S_t)}{\frac{\partial f(\bar{\omega}_t, S_t)}{\partial \bar{\omega}}} \right] \end{aligned} \quad (4.11)$$

The second equilibrium condition can also be rewritten as:

$$i_t = \frac{1}{1 - q_t m(\bar{\omega}_t, S_t)} n_t \quad (4.12)$$

Let's examine the properties of this investment model. We can interpret Equation 4.12 as the aggregate investment supply function, decreasing in the price of capital  $q_t$  and increasing in net worth  $n_t$ , while  $\bar{\omega}_t$  is uniquely defined by Equation 4.11, once we set  $\bar{\omega}_t$  and  $S_t$  as fixed in our comparative statics exercise. Since Equation 4.12 is linear in its arguments, simple aggregation is possible. To answer how microeconomic uncertainty shocks affect investment demand, note that in Equation 4.12 a rise in  $\bar{\omega}_t$  implies a rise in  $m(\bar{\omega}_t, S_t)$ , leading to a fall in  $i_t$ , *ceteris paribus*.

Now, we move on to a detailed description of the agents of the model.

## 4.2 Households

Households are risk-neutral and have infinite life. They interact with firms by selling labor (in exchange of wages  $w_t$ ) and renting capital (at a rate  $r_t$ ). Following

Dorofeenko, Lee and Salyer (2008), we choose a simple quasi-linear functional form for household utility. Households maximize the discounted sum of lifetime expected utility, by choosing the sequence of allocations of the final consumption good and leisure  $\{c_t, l_t\}_{t=0}^{\infty}$  starting from the information set at time  $t = 0$ , according to the problem:

$$\max_{\{c_t, l_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \nu(1 - l_t)] \quad (4.13)$$

$$\text{s.t. } w_t l_t + r_t k_t \geq c_t + q_t i_t + T_t \quad \forall t \quad (4.14)$$

$$k_{t+1} = (1 - \delta)k_t + i_t \quad \forall t \quad (4.15)$$

where  $\beta \in (0, 1)$  is the discount factor,  $\nu$  is the elasticity of labor,  $w_t$  is the real wage,  $r_t$  is the capital rental rate and  $k_t$  is the capital stock,  $q_t$  is the price of capital goods,  $i_t$  is the flow of new capital goods and  $T_t$  are lump-sum transfers to the government. The price of the final consumption good is normalized to unity. The second constraint is a standard law of motion for the households' capital stock, with a depreciation rate  $\delta \in (0, 1)$ . To solve the constrained optimization problem above, we plug the second constraint into the first and form a Lagrangian. The first order conditions are (solutions are available in Section A):

$$\nu c_t = w_t \quad (4.16)$$

$$\frac{q_t}{c_t} = \beta \mathbb{E}_t \left[ \frac{q_{t+1}(1 - \delta) + r_{t+1}}{c_{t+1}} \right] \quad (4.17)$$

The resulting equilibrium gives relationships that are standard in the literature of RBC models: the first condition states that the marginal rate of substitution between leisure and consumption equals real wages, while the second is the Euler equation, that describes household consumption dynamics.

### 4.3 Firms

Firms combine capital, household and entrepreneurial labor to produce the final consumption good, using a constant returns-to-scale Cobb-Douglas technology according to the equation:

$$Y_t = A_t F(K_t, H_t, H_t^e) = A_t K_t^{\alpha_K} H_t^{\alpha_H} (H_t^e)^{1-\alpha_K-\alpha_H} \quad (4.18)$$

Total factor productivity (TFP) evolves according to a first-order autoregressive process in logarithms:

$$A_{t+1} = A_t^{\rho^A} \varepsilon_{t+1}^A \quad (4.19)$$

where  $\rho_t$  is the persistence of TFP shocks, and  $\varepsilon_{t+1}^A$  is an i.i.d shock that follows a standard normal distribution. Equilibrium conditions of firms are found by maximizing production subject to households' input supply:

$$w_t = \alpha_H \frac{Y_t}{H_t} \quad (4.20)$$

$$r_t = \alpha_K \frac{Y_t}{K_t} \quad (4.21)$$

$$w_t^e = (1 - \alpha_K - \alpha_H) \frac{Y_t}{H_t^e} \quad (4.22)$$

where  $A_t$  is total factor productivity,  $K_t$  is the capital stock,  $H_t^2$  is aggregate household labor supply and  $H_t^e$  is entrepreneurial labor supply at time  $t$ . According to equations above, wages and the capital rental rate should equal their marginal rates of transformation in equilibrium.

### 4.4 Entrepreneurs

Entrepreneurs maximize the discounted sum of their lifetime utility, according to the problem below:

<sup>2</sup> Household labor  $H_t$  is linked to the definition of labor  $l_t$  by equation  $H_t = (1 - \eta)l_t$ .

$$\max_{\{c_t^e\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e \quad (4.23)$$

$$\text{s.t. } z_{t+1} = n_t \left[ \frac{f(\bar{\omega}_t, S_t)}{1 - q_t m(\bar{\omega}_t, S_t)} \right] - \frac{c_t^e}{q_t} \quad \forall t \quad (4.24)$$

$$n_t = w_t + z_t(r_t + q_t(1 - \delta)) \quad \forall t \quad (4.2)$$

where  $\gamma \in (0, 1)$  is an additional discount factor, implying that entrepreneurs are more impatient than households. Entrepreneurs' choice is constrained by the law of motion of capital and definition of net worth, and their consumption is defined after the returns from investment are realized. This is necessary in order to prevent a scenario where entrepreneurs accumulate enough capital to finance their projects entirely with internal resources<sup>3</sup>.

Solution of the problem above leads to the following equilibrium condition:

$$q_t = \beta\gamma \mathbb{E}_t \left[ (q_{t+1}(1 - \delta) + r_{t+1}) \left( \frac{q_{t+1} f(\bar{\omega}_t, S_t)}{1 - q_{t+1} m(\bar{\omega}_t, S_t)} \right) \right] \quad (4.25)$$

## 4.5 Economic policy

We introduce economic policy in the model, starting with fiscal policy. There is a government that consumes final goods and finance expenditures via lump-sum transfers from households. Hence, ricardian equivalence holds every period. Government consumption follows a first-order autoregressive process:

$$G_t = \bar{G}^{1-\rho^g} G_{t-1}^{\rho^g} + \varepsilon_t^G \quad (4.26)$$

## 4.6 Equilibrium

We introduce the definition of equilibrium with the market clearing conditions. First, the clearing conditions in labor markets, recalling that  $\eta$  is the share of entrepreneurs in the economy, and labor supply  $(1 - l_t)$  is normalized to unity:

<sup>3</sup> The other way to turn this issue tractable is to assume that a small fraction of entrepreneurs consume all their assets and leave the market in every period. This is observed in Bernanke, Gertler and Gilchrist (1999)

$$H_t = (1 - \eta)l_t \quad (4.27)$$

$$H^e = \eta \quad (4.28)$$

Next, the market clearing conditions in the final goods market. The first condition simply states that output equals the sum of aggregate household consumption, invest and government consumption.

$$C_t + I_t + G_t = Y_t \quad (4.29)$$

$$C_t = (1 - \eta)c_t + \eta c_t^e \quad (4.30)$$

$$I_t = \eta i_t \quad (4.31)$$

Now, the equilibrium law of motion of capital stock:

$$K_{t+1} = (1 - \delta)K_t + I_t[1 - \mu\Phi(\bar{\omega}_t, S_t)] \quad (4.32)$$

A competitive equilibrium is defined by the optimal policy rules from the sequence of variables  $\{K_{t+1}, Z_{t+1}, H_t, H_t^e, q_t, n_t, i_t, \bar{\omega}_t, c_t, c_t^e\}$  and states  $\{K_t, Z_t, A_t, S_t, g_t\}$  that satisfy the equations discussed earlier. Substitutions that derived from the firms' equilibrium conditions are made when possible. First, the equilibrium conditions that derive from the households' optimization problem:

$$\nu c_t = \alpha_H \frac{Y_t}{H_t} \quad (4.16)$$

$$\frac{q_t}{c_t} = \beta \mathbb{E}_t \left[ \frac{1}{c_{t+1}} \left( q_{t+1}(1 - \delta) + \alpha_K \frac{Y_{t+1}}{K_{t+1}} \right) \right] \quad (4.17)$$

The conditions at the optimal financial contract:

$$q_t = \left[ 1 - \mu\Phi(\bar{\omega}_t, S_t) + \frac{\phi(\bar{\omega}_t, S_t)\mu f(\bar{\omega}_t, S_t)}{f'(\bar{\omega}_t)} \right]^{-1} \quad (4.11)$$

$$i_t = \frac{1}{1 - q_t m(\bar{\omega}_t, S_t)} n_t \quad (4.12)$$

The conditions associated with the entrepreneurs' optimization problem:

$$q_t = \beta\gamma\mathbb{E}_t \left[ \left( q_{t+1}(1-\delta) + \alpha_K \frac{Y_{t+1}}{K_{t+1}} \right) \left( \frac{q_{t+1}f(\bar{\omega}_t, S_t)}{1 - q_t m(\bar{\omega}_t, S_t)} \right) \right] \quad (4.25)$$

$$n_t = \alpha_{H^e} \frac{Y_t}{H_t^e} + Z_t \left( q_t(1-\delta) + \alpha_K \frac{Y_t}{K_t} \right) \quad (4.2)$$

$$Z_{t+1} = \eta n_t \left[ \frac{f(\bar{\omega}_t, S_t)}{1 - q_t m(\bar{\omega}_t, S_t)} \right] - \eta \frac{c_t^e}{q_t} \quad (4.24)$$

The equilibrium defined by the processes that govern TFP, microeconomic uncertainty and government spending:

$$A_{t+1} = A_t^{\rho^A} \varepsilon_{t+1}^A \quad (4.19)$$

$$S_t = \bar{S}^{1-\zeta} S_{t-1}^\zeta + \varepsilon_t^S \quad (4.1)$$

$$G_t = \bar{G}^{1-\rho^G} G_{t-1}^{\rho^G} + \varepsilon_t^G \quad (4.26)$$

Finally, a definition of household welfare that will be used in further analyses. Here, welfare is the discounted infinite sum of consumption decisions:

$$\mathcal{W}_t = U(c_t, l_t) + \beta\mathbb{E}_t[U(c_{t+1}, l_{t+1})], \quad (4.33)$$

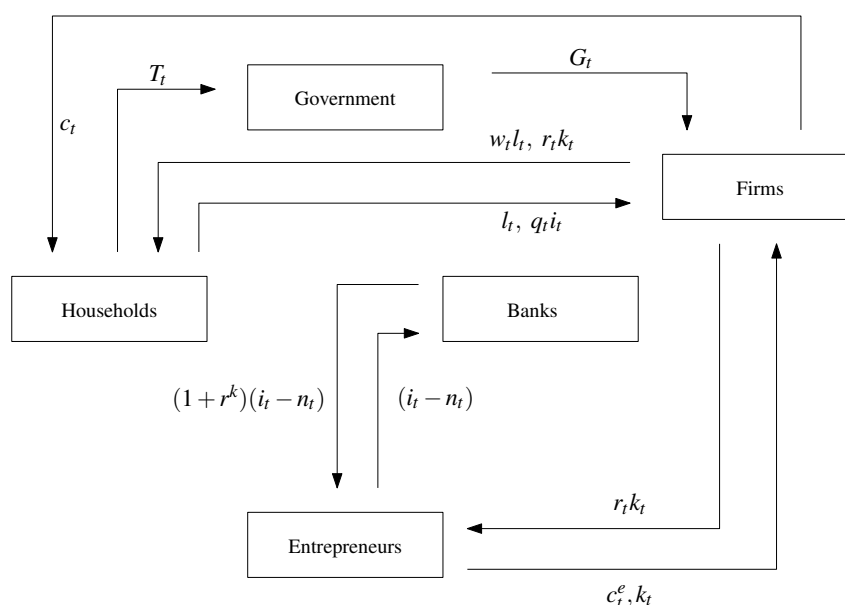
$$\text{with } U(c_t, l_t) = \ln(c_t) + \nu(1 - l_t)$$

Figure 18 shows all the interactions between agents, in terms of the flows of inputs, outputs and goods. In the center of the diagram are the agents directly involved in the financial frictions, that according to the theoretical framework amplify the microeconomic uncertainty shocks throughout the economy.

## 4.7 Data and procedures

In summary, the model has 10 real variables, 16 parameters, 10 equilibrium equations and 4 exogenous shocks. The time unit of the model is a quarter. In this work, we perform simulations with the aim to approximate actual data and ensure the robustness of our findings. We used quarterly data from the Brazilian central bank Time Series Management System (SGS) for macroeconomic aggregates,

Figure 18 – Diagram of flows of the model



Source: The author.

namely  $C_t, C_t^e, l_t, G_t, K_t, H_t$ . Consistent, seasonally adjusted quarterly data for the aggregates span from the third quarter of 1993 to present dates. Data on credit market variables are available at the *Relatório de Estabilidade Financeira*<sup>4</sup> reports from the Brazilian central bank, available every semester since 2002. In conformity with other data sources, our period of analysis spans from the first quarter of 2003 to the last quarter of 2018.

To perform the simulations, we input the steady state equilibrium equations into Dynare, a toolbox for DSGE models available for MATLAB and Octave<sup>5</sup>. Table 6 provides the list of parameters of the model to be specified and data sources for calibration. To calibrate the model, we used empirical data for our period of interest using the data sources noted above, whenever feasible. Otherwise, we resorted to our main references of analytical models with financial frictions (CARLSTROM; FUERST, 1997; DOROFEENKO; LEE; SALYER, 2008; CHRISTIANO; MOTTO; ROSTAGNO, 2014; CESA-BIANCHI; FERNANDEZ-CORUGEDO, 2014) and the existing literature for DSGE models with financial frictions that study the Brazilian economy (CASTRO et al., 2011; CAVALCANTI; VEREDA, 2011; ARANHA, 2012; KANCZUK, 2013; AREOSA; COELHO, 2015; DIVINO; KORNELIUS, 2015).

<sup>4</sup> Available at: <<http://www.bcb.gov.br/publicacoes/ref>>. Accessed in Aug. 14, 2019.

<sup>5</sup> The full computer code is available at Section A in the Appendix.

We should comment in detail about how parameters were retrieved. The discount factor  $\beta$  was calculated as follows: first, we calculated average inflation  $\pi$  and interest rates  $Rn$  (annualized Selic) using data from BACEN, then we applied a relationship found in the canonical real business cycle model:

$$\beta = \frac{\bar{\pi}}{(1 + \bar{R}n)} \quad (4.34)$$

For the capital share of production  $\alpha_K$ , Cavalcanti and Vereda (2011) point out that a great deal of research about the Brazilian economy attributes levels around 0.4, and we choose to employ this value. For the household labor share of production  $\alpha_H$ , we set a value of 0.599 to leave a share of 0.001 for the entrepreneurial labor production (defined in the model as  $1 - \alpha_K - \alpha_H$ ).

Table 6 – Calibration of analytical model parameters

Param.	Value	Description	Reference
$\beta$	0.890	Discount factor	Relative to real interest rates (BACEN)
$\alpha^K$	0.400	Capital share	According to literature
$\alpha^H$	0.599	Household labor share	Relative to $\alpha^K$
$\gamma$	0.870	Entr. discount factor	Carlstrom and Fuerst (1997)
$\delta$	0.005	Depreciation rate	Annual rate of 2%
$\eta$	0.100	Share of entrepreneurs	Carlstrom and Fuerst (1997)
$\nu$	2.520	Elast. of household labor	Dorofeenko, Lee and Salyer (2008)
$\mu$	0.300	Monitoring costs	Cesa-Bianchi and Fernandez-Corugedo (2014)
$\rho^A$	0.996	Persistence of TFP	AR(1) on output gap (BACEN)
$\sigma^A$	0.072	Std. dev. of TFP shock	AR(1) residuals on output gap (BACEN)
$\sigma^N$	0.100	Std. dev. of net worth shock	Dorofeenko, Lee and Salyer (2008)
$\rho^G$	0.995	Persistence of govt. spending	AR(1) on govt. consumption (BACEN)
$\zeta$	0.950	Persistence of unc. shock	AR(1) on scaled uncertainty index
$\bar{S}$	0.210	S.s. micro uncertainty	Oliveira (2012)
$\bar{G}$	0.250	S.s. govt. spending	Average $\frac{G_t}{Y_t}$ ratio (BACEN)

Source: The author.

To set the entrepreneurial discount factor  $\gamma$ , we follow Carlstrom and Fuerst (1997). They set this parameter to ensure a rate of return to capital such that entrepreneurs are willing to accumulate capital – considering that they are more impatient than households. The capital depreciation rate  $\delta$  follows the literature for Brazil (an annual rate of 2%). Next, we draw some parameters that are specific from this class of models from the literature  $(\eta, \nu, \mu)$ . Here, we draw special attention to the monitoring costs parameter  $\mu$ : this controls the weight of costly state verification in the optimal contract setting. The values chosen in the literature span from 0.15 to 0.30,

when studying developed economies. We opt for higher values to reflect imperfections in developing credit markets.

To calibrate the productivity shocks  $\rho_A$  and  $\sigma_A$ , we fit a first-order autoregressive process on GDP deviations from its long-term trend, a common measure of total factor productivity<sup>6</sup>. We also employ this strategy to determine the persistences of fiscal policy and uncertainty shocks. For the steady state microeconomic uncertainty  $\bar{S}$ , we obtain the value 0.21 from Oliveira (2012) which is the estimate of the external finance premium for a large sample of Brazilian firms (Panel A) from 1994 to 2010, excluding monetary policy shocks. Finally, the steady state government spending  $\bar{G}$  is obtained by calculating the average government consumption to GDP ratio, using BACEN data.

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<sup>6</sup> An alternative measurement for TFP can be found by extracting the residuals from a least-squares fit of GDP on gross fixed capital formation and labor force.

## 5 RESULTS AND DISCUSSION

In this Section we will present the outputs from our analytical model numeric simulations. First, we introduce plots of impulse response functions (IRF) of the model. The impulse response analysis describe how our model variables behave upon exogenous shocks (microeconomic uncertainty shocks, fiscal and monetary policies) in terms of deviations from their steady-state values, up to 40 quarters (10 years) after the shock<sup>1</sup>. Next, we further diagnose the correlation between aggregates to find if they conform to existing literature and empirical data. Finally, we perform a welfare analysis of the household in response to the exogenous shocks to determine possible welfare losses due to microeconomic uncertainty shock.

### 5.1 Impulse response analysis

For the impulse response analysis, we employ a second-order approximation of policy functions around the non-stochastic steady state, as in Schmitt-Grohé and Uribe (2004). This is necessary as we expect that the effects of microeconomic uncertainty shocks are limited in a first-order approximation as (i) uncertainty shock are mean-preserving; (ii) only one of the possible states of the model economy is considered and (iii) evaluation of welfare functions under first-order approximations are imprecise. This also implies that we require a different treatment of impulse response functions, because in a higher-order approximation we have a multitude of optimal policy functions, according to each of the possible states generated by microeconomic uncertainty shocks at any given time (histories). We also apply the pruning procedure implemented by Andreasen, Fernández-Villaverde and Rubio-Ramírez (2018) to treat impulse response functions that exhibit explosive behavior, a problem commonly found in higher-order approximations. The pruning procedure consists of discarding terms with order greater than the required when computing the approximated solution of the system.

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<sup>1</sup> Plots for government consumption appear only in the impulse responses to fiscal policy shocks, because fiscal expenditures are exogenously determined (according to Equation 4.26).

The second-order approximation works as follows. Consider a state-space representation of a general DSGE model:

$$\mathbf{y}_t = \mathbf{y}^s + \mathbf{A}\mathbf{y}_{t-1}^h + \mathbf{B}\mathbf{u}_t \quad (5.1)$$

where  $\mathbf{y}^t$  is the vector of endogenous variables,  $\mathbf{y}^s$  are their steady-state values and  $\mathbf{y}_t^h = \mathbf{y}_t - \mathbf{y}^s$ . Matrices  $\mathbf{A}$  and  $\mathbf{B}$  store, respectively, the coefficients for the decision rules (that is, the optimal policy function parameters) with respect to state variables and exogenous shocks. To find the second-order approximation, a Taylor expansion around the steady state is performed:

$$\mathbf{y}_t = \mathbf{y}^s + \frac{1}{2}\Delta^2 + \mathbf{A}\mathbf{y}_{t-1}^h + \mathbf{B}\mathbf{u}_t + \frac{1}{2}\mathbf{C}(\mathbf{y}_{t-1}^h \otimes \mathbf{y}_{t-1}^h) + \frac{1}{2}\mathbf{D}(\mathbf{u}_t \otimes \mathbf{u}_t) + \mathbf{E}(\mathbf{y}_{t-1}^h \otimes \mathbf{u}_t) \quad (5.2)$$

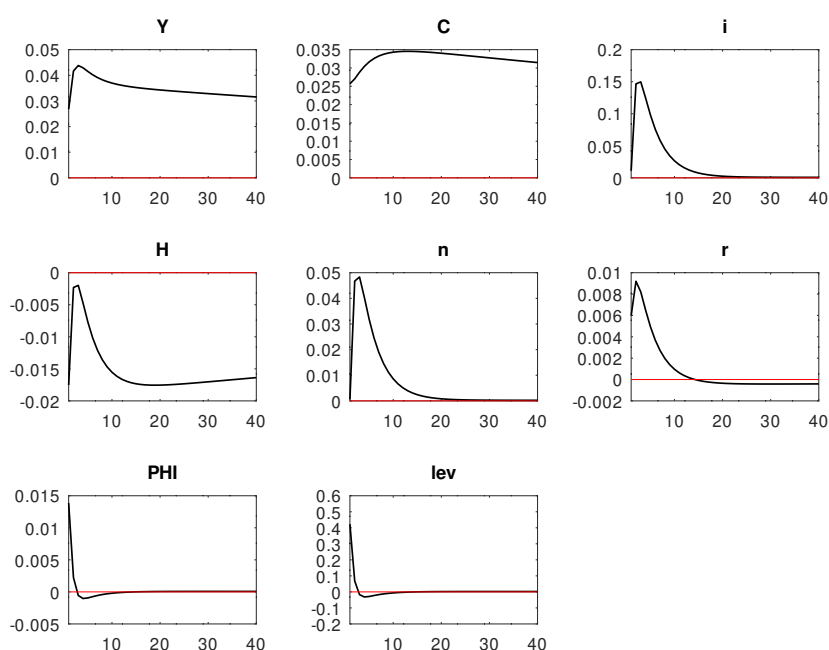
The equation above introduces new terms:  $\Delta$  is the shift effect of the variance of expected shocks,  $\mathbf{C}$  is a parameter matrix associated with the Kronecker product of the vector of state variables,  $\mathbf{D}$  is a parameter matrix associated with the Kronecker product of exogenous variables and  $\mathbf{E}$  is associated with the Kronecker product of state and exogenous variables.

The variables depicted in the impulse response analyses are, respectively, aggregate output  $Y_t$ , household consumption  $C_t$ , investment  $I_t$ , aggregate labor supply  $H_t$ , aggregate entrepreneurial net worth  $n_t$ , the risk-free interest rate  $r_{t+1}$ , the probability of entrepreneurial default  $\Phi(\bar{\omega}_t, S_t)$ , and the leverage of entrepreneurs  $l_t$ , defined by the debt-to-assets ratio  $\frac{i_t}{n_t}$ . In the impulse response analysis, all shocks have unit variance, and the vertical axis represents percent deviations from steady state values, while the horizontal plot represents time (in quarters) after the shock.

Figure 19 shows the response of the main model variables to a total factor productivity (TFP) shock. First, we find that impacts to output and household consumption are long-lasting, due to the persistence of technology shocks found in empirical data for Brazil. A unit standard deviation, positive TFP shock causes a 4% increase in output, and the effect persists many quarters after the perturbation. Shocks to investment, labor inputs, entrepreneurs' net worth and price of capital exhibit a quicker speed of correction to the steady state. Note the rise in both investment and the price of capi-

tal: in the partial equilibrium of capital markets, this indicates a rise in the demand for capital. The impacts of TFP shocks on entrepreneurial activity are interesting, as the bankruptcy rates exhibit an upward spike by an order of 1.5%. We understand that the rise in the cost of capital diminishes net worth accumulation, “raising the bar” for the threshold value between solvent and insolvent entrepreneurs. In this case, the optimal solution from the point of view of entrepreneurs is to increase leverage.

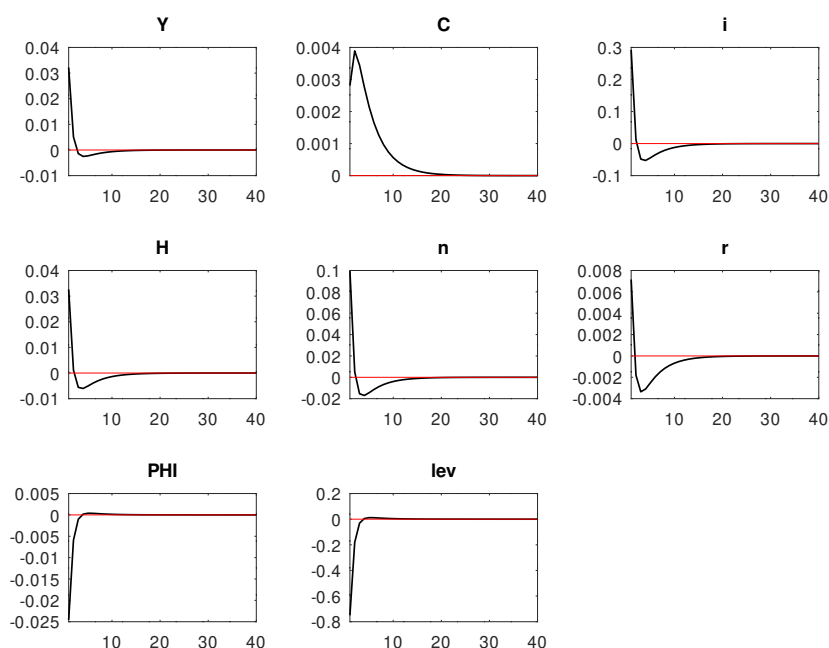
Figure 19 – Responses to a TFP shock



Source: The author.

In Figure 20 we plot responses to a positive shock to entrepreneurial net worth shock. In comparison to the other model perturbations, we observe that the rate of adjustment to the steady-state values is higher. A unit net worth shock causes a positive increase in output of 3.2% in the first period. The output increase is not consumption-driven, as the impact on aggregate consumption is of 0.3%. In fact, the stimulus is investment-driven: the impact on investment is of 3% immediately after the shock. Interest rates experience only a modest increase, because with additional net worth entrepreneurs can finance a greater deal of their projects internally. The demand for external finance falls, causing a fall in the price of capital. Also, the increased assets allow for the entrepreneurs to pay the rented capital, thus reducing the probability of bankruptcy.

Figure 20 – Responses to a net worth shock

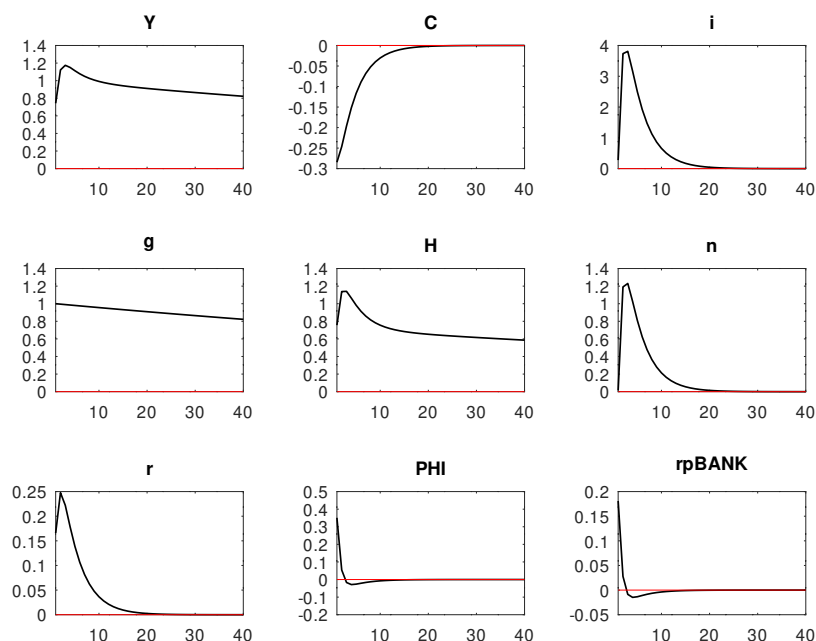


Source: The author.

Figure 21 presents the responses to a positive fiscal policy shock. We observe that a fiscal policy shock induces a rise in output, investment and labor aggregates. This is straightforward as the government is increasing the demand for final goods, hence firms must increase production to clear markets. This increases the demand for capital, explaining the increase in interest rates. Consequently, household consumption decreases following a fiscal policy shock because a larger fraction of final goods is being directed to the government. An interesting result emerges: we have found that fiscal stimulus increases the bankruptcy hazard rate. Our numerical analysis in the case of fiscal policy shocks is limited as the volatility of economic aggregates due to fiscal shocks does not quite match other results found in the Brazilian business cycle literature. For instance, Vereda and Cavalcanti (2010) found an impact close to 0,9% in output from a unit standard deviation fiscal policy shock, while the impact in Cavalcanti and Vereda (2011) is of 0,3%. Using vector autoregressions, Prince, Marral and Holland (2017) found an impact of approximately 40%.

The following techniques benefit from the estimation of the impulse response functions from our analytical model. In Table 7 we compute the model autocorrelation

Figure 21 – Responses to a fiscal policy shock



Source: The author.

function and display the simulated coefficients from  $t - 1$  to  $t - 4$ , representing four quarters in the simulation. From the results, we find that the simulated output series is very persistent, diverging from empirical data. Upon closer examination of economic aggregates, the process that governs fiscal policy seems to generate the most persistence. This corresponds to the Brazilian government consumption time series, which shows a strong serial correlation.

Next, in Table 8 we present the model variance decomposition of all exogenous shocks previously defined. Using this technique, we can find the contribution of each shock to any endogenous variable in the model. According to the results, our fiscal policy shock is the greatest source of changes in real variables, including output. Then again, this could be related to the scaling of the steady state government spending parameter. Two notable exceptions are the aggregate household consumption and real wages, which are somewhat affected by technology shocks. Meanwhile, the impact of shocks to entrepreneurial net worth on the endogenous variables is negligible. Moving on to the role of uncertainty shocks in the model variance results from the Table shows that while risk shocks have a sound impact in real aggregates in the impulse response

Table 7 – Model autocorrelation functions of selected variables

Variable	Order: 1	2	3	4
$K_t$	0.999	0.997	0.995	0.992
$H_t$	0.852	0.676	0.526	0.406
$q_t$	0.661	0.563	0.523	0.496
$n_t$	0.853	0.677	0.527	0.408
$G_t$	0.995	0.990	0.985	0.980
$\bar{w}$	0.925	0.874	0.830	0.789
$w_t$	0.941	0.834	0.736	0.657
$Y_t$	0.996	0.989	0.983	0.976
$r_t$	0.907	0.738	0.584	0.460
$l_t$	0.866	0.687	0.531	0.408
$C_t$	0.897	0.794	0.709	0.641
$r_t^k$	0.713	0.624	0.582	0.553
$f(\bar{w}_t, S_t)$	0.925	0.875	0.831	0.790
$m(\bar{w}_t, S_t)$	0.931	0.881	0.837	0.795
$A_t$	0.996	0.992	0.988	0.984
$\bar{S}_t$	0.950	0.902	0.857	0.814

Source: The author.

functions, it induces variability directly on the financial sector of the model. Specifically, it is interesting how microeconomic uncertainty shocks affect capital formation via the price of capital  $q_t$  and the loan rate  $r_t^k$ .

Table 8 – Model variance decomposition of selected variables

Variable	$\varepsilon^A$	$\varepsilon^N$	$\varepsilon^G$	$\varepsilon^S$
$K_t$	0.19	0	99.41	0.40
$H_t$	0.08	0	99.88	0.04
$q_t$	0.06	0.25	35.66	64.02
$n_t$	0.15	0.27	96.42	3.15
$G_t$	0	0	100.00	0
$\bar{w}$	0	0.01	2.06	97.92
$w_t$	37.16	0.03	59.26	3.54
$Y_t$	0.17	0	99.80	0.03
$r_t$	0.13	0.05	94.18	5.63
$l_t$	0.15	0.25	96.08	3.53
$C_t$	39.83	0.02	56.73	3.42
$r_t^k$	0.05	0.21	29.62	70.12
$f(\bar{w}_t, S_t)$	0	0.01	2.00	97.99
$m(\bar{w}_t, S_t)$	0	0.01	1.47	98.52
$A_t$	100.00	0	0	0
$\bar{S}_t$	0	0	0	100.00

Source: The author.

## 5.2 The role of microeconomic uncertainty shocks

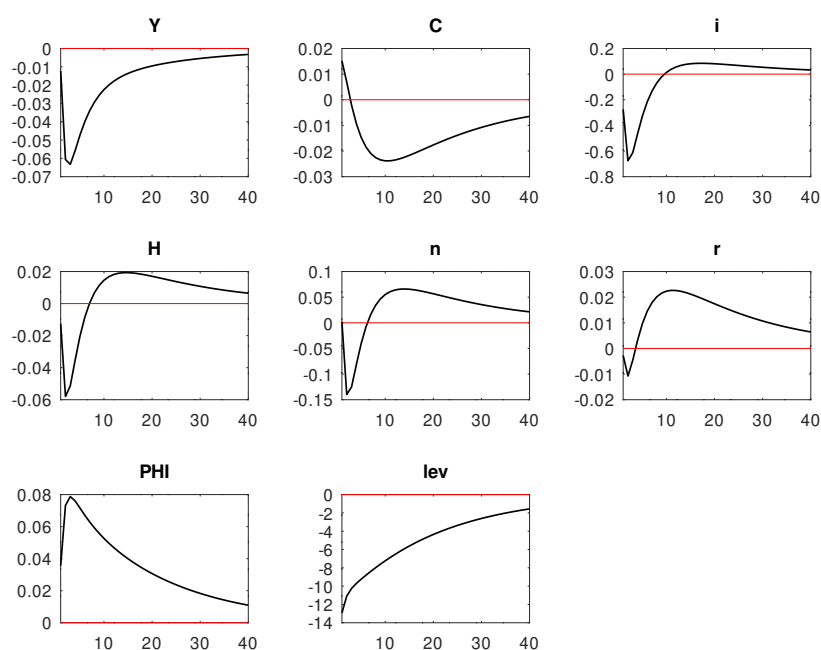
Figure 22 shows the responses of the model variables to a positive (increasing) shock in microeconomic uncertainty, our main exogenous shock of interest. The plots reveal that greater uncertainty diminishes output and especially investment. A unit standard deviation shock in uncertainty depresses growth by 6%, a semester after the shock. This is a number smaller than those found in large-scale models such as Christiano, Motto and Rostagno (2014), that attribute around 20% of variations in business cycles to uncertainty shocks similar to the ones defined here, emerging from the microeconomic scale, in fact, they are closer to the 5% mark found in Chugh (2013). Also, we learn that final goods consumption increases: entrepreneurs are more impatient than households, so as investment decreases their optimal decision is to prefer present consumption. The effect in solvency is also expressive: a unit standard deviation shock to micro uncertainty cause a 8% rise in the probability of default of entrepreneurs.

In addition, note that leverage sharply declines following a microeconomic uncertainty shock, revealing that in times of greater uncertainty entrepreneurs prefer to finance more of their projects internally. This happens because greater microeconomic uncertainty raises the external finance premium, as predicted by the literature of financial accelerator models. We also find that aggregate investment decreases, and the price of capital initially responds to uncertainty shocks with a fall, followed by a persistence rise. In the partial equilibrium of capital markets, this is similar to what happens after a TFP shock, but here there is a decline in capital supply. This leads to another observation: financial variables responses to the uncertainty shock all exhibit a sine-wave pattern. A possible explanation to the fact is that entrepreneurs aggressively readjust their production plans to avoid bankruptcy and exit from the market, overshooting the requirements to return to the equilibrium condition. Similar to labor markets, we expect that the speed of adjustment is lower, the higher the parameters associated with nominal rigidities are.

As recalled by Christiano, Motto and Rostagno (2014), Baker, Bloom and Davis (2013) pointed out that cyclical changes in the dispersion of firm returns were a powerful proxy for economic uncertainty. Our simulations shows how micro uncertainty

spreads to short-term economic growth via intertemporal resource allocations, which is in line with this branch of the literature. Analysis of the impulse response functions showed that uncertainty shocks caused a net fall in both consumption and investment, a finding that conforms with similar research on dynamic models for Brazil such as Lopes (2014) and Melo and Silva (2019).

Figure 22 – Responses to a microeconomic uncertainty shock

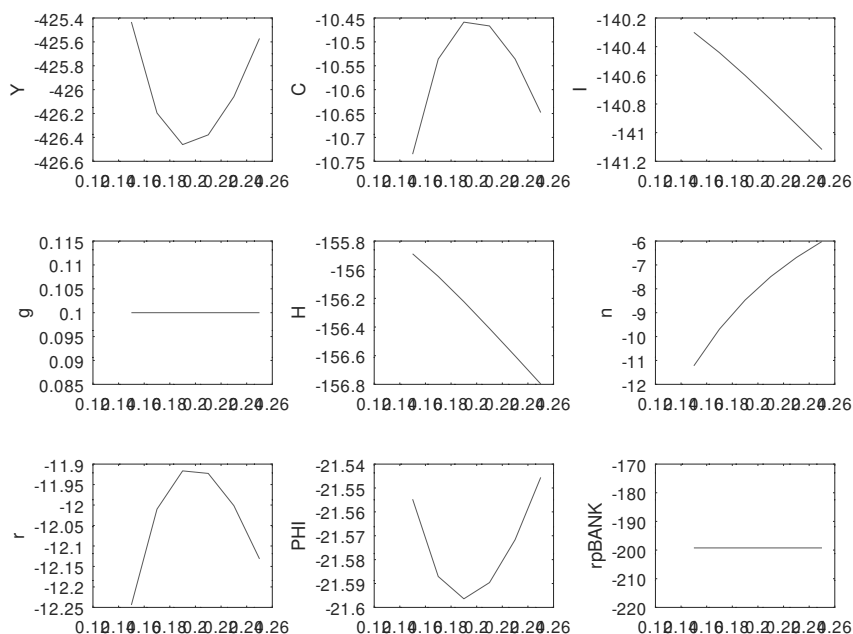


Source: The author.

Now, we follow the insight from Cesa-Bianchi and Fernandez-Corugedo (2014) and analyze how steady state values of macroeconomic aggregates change according to the steady state levels of microeconomic uncertainty. Results are plotted in Figure 23, where the steady state levels of aggregates are in the vertical axes, and steady state levels of microeconomic uncertainty in the horizontal axes, ranging from 0.15 to 0.25 (a reasonable set of values around the steady-state calibrated value  $\bar{S}$ ).

We have found that steady state levels are decreasing in uncertainty, in most economic aggregates. Consumption, investment and labor supply are decreasing, a fact that is predicted by the literature as a precautionary savings mechanism, although the fall in consumption happens after an “optimal” point. Entrepreneurial net worth is increasing in uncertainty because of the increasing cost of external finance. Because

Figure 23 – Steady states and microeconomic uncertainty shocks



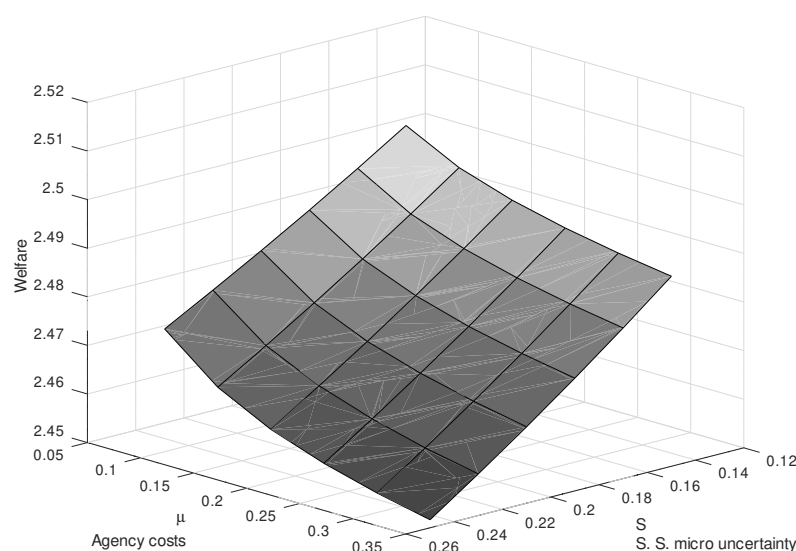
Source: The author.

investment is decreasing, we conclude that entrepreneurs are less leveraged when economic uncertainty is higher. The model also predicts that steady state interest rates are lower when steady state levels of microeconomic uncertainty are higher. This result should not be confused with the impacts of microeconomic uncertainty shocks on interest rates, where in fact interest rates go up and then oscillate back to the steady state. On the contrary, we combine this with the evidence that investment levels are decreasing to conclude that in greater steady state levels of uncertainty, the supply of capital decreases. Last, but not less important, the Figure shows that steady state values of economic output are actually higher when uncertainty is higher. Because consumption is decreasing, the effect is not demand-driven, but possibly due to a precautionary savings effect.

### 5.3 Welfare and sensitivity analysis

In this section, we underscore how household welfare is impacted by different scenarios of financial monitoring costs and steady state microeconomic uncertainty. To achieve this goal, we evaluate Equation 4.33 in the steady states of our simulated

Figure 24 – Welfare, agency costs and microeconomic uncertainty shocks

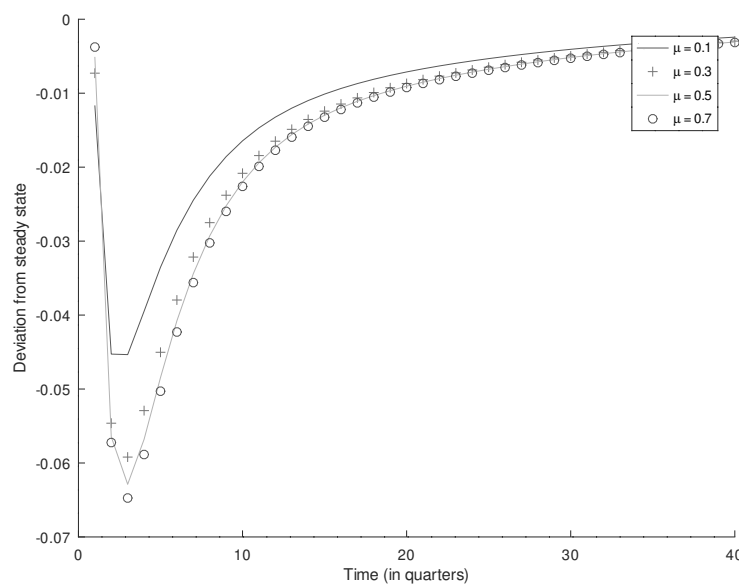


Source: The author.

economy in a range of scenarios. In Figure 24, the surface represents household welfare steady-state values as a function of monitoring costs and uncertainty settings in a space of calibrations for the monitoring cost parameter  $\mu$ , ranging from 0.1 to 0.35 in steps of 0.05, and for the steady state microeconomic uncertainty  $\bar{S}$ , ranging from 0.15 to 0.25 in steps of 0.02, *ceteris paribus*. Simulations show that, as predicted by the literature, our model economy face welfare losses emerging from our costly state verification problem. The fall in welfare due to monitoring costs is exponentially smoothed, while the fall due to average uncertainty is seemingly linear. This is a novel finding for the Brazilian economy: the marginal social benefit of reducing credit market rigidities is greater than the benefit of reducing the overall entrepreneurial output uncertainty. We understand that the latter is a greater effort because entrepreneurs rely on the macroeconomic and political overview – they also need to invest in demand forecasting and other technologies to augment production certainty.

Finally, we study if (and how) monitoring costs amplify the impact of microeconomic uncertainty shocks in our simulated Brazilian economy. This is a common product of general equilibrium models based on financial frictions. Here, we compare the impulse response functions of a microeconomic uncertainty shock to economic output using various calibrations for our agency cost parameter  $\mu$ . Figure 25 shows the

Figure 25 – IRF of output to uncertainty shock given steady-state monitoring costs



Source: The author.

results of the simulation. The values for  $\mu$  are depicted in the legend. Overall, the Figure indicates that information asymmetries between parties in the financial markets induce greater economic growth volatility. The economic intuition is that, in a context of limited information, firms have a harder time finding a production plan that smoothes out their discounted future cash flows. On the supply of credit side, we can interpret monitoring costs become an implicit risk-aversion measure for banks.

## 6 CONCLUSION

The aim of this work was to analyze the impact of microeconomic uncertainty shocks in the Brazilian economy, with the use of a real business cycle model with financial frictions, calibrated for the 2003-2018 period. Our work explored the underlying investment model in our general equilibrium framework to investigate the effects of economic uncertainty – emerging from a large number of competitive agents – in capital accumulation.

We draw a number of conclusions from our research: first, agency costs strongly impact our model economy. As highlighted by Figure 25, higher monitoring costs produces steady states where banks set higher interest rates, affecting long-term investment projects and economic growth. Also, the average life of entrepreneurs is impacted by higher aggregate bankruptcy rates. As in other general equilibrium models, they work like capital adjustment costs: asymmetric information between lenders and borrowers increases the cost of external finance, with uncertainty shocks affecting bankruptcy rates and the supply of capital.

Second, microeconomic uncertainty shocks impacts intertemporal choice and capital accumulation in the modeled Brazilian economy. Analyzing the responses from impulses to uncertainty, we found that uncertainty negatively impacts investment, as expected from the literature discussion. A broad literature points out that this effect would be amplified in economies with greater credit constraints. Also, in more uncertain times agents cease to invest and engage in present consumption, although in our model a short-term reduction in consumption is also observed.

Third, microeconomic uncertainty shocks are welfare reducing, as the steady state levels of household consumption of final goods are decreasing in uncertainty. Model simulations showed that reducing information problems in credit markets provide a greater marginal benefit to household welfare than trying to shift the mean or variance distribution of entrepreneurs time-varying productivity. The welfare problem, in a sense, derives from the capital accumulation problem: in steady states of higher uncertainty, the infinite-horizon consumption plan of final goods by families is affected. This leaves

a broad avenue of research, as household welfare can be input in the decision rule of an policymaker that wants to address the issue of economic uncertainty.

Surely, these conclusions are bound by the limitations of the empirical strategy: there isn't a comparison of simulated and observed economic time series: that would require the estimation of a DSGE-VAR via Bayesian or maximum likelihood methods. Such methods, however, lie outside the scope of this research. Nonetheless, the key findings of this work suggest that authorities should pursue policies that minimize agency costs in the Brazilian capital market. An example of initiative in this sense is the *BC+ Agenda*<sup>1</sup> that is currently being implemented by the Brazilian monetary authority, in a package that includes initiatives such as a "positive score" for agents that pay their obligations within due dates, a secondary market for public bonds and ideas for an international bank settlement system.

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<sup>1</sup> Available at: <[https://www.bcb.gov.br/acessoinformacao/bcmais\\_competitividade](https://www.bcb.gov.br/acessoinformacao/bcmais_competitividade)>. Accessed in Sep. 15, 2019.

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## APPENDIX A – Appendix

### Solutions of the optimization problems

The household maximization problem is:

$$\max_{\{c_t, l_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \nu(1 - l_t)] \quad (\text{A.1})$$

$$\text{s.t. } w_t l_t + r_t k_t \geq c_t + q_t i_t + T_t \quad \forall t \quad (\text{A.2})$$

$$k_{t+1} = (1 - \delta)k_t + i_t \quad \forall t \quad (\text{A.3})$$

We solve the second constraint for  $i_t$ , input the solution into the first constraint and form the Lagrangian:

$$\mathcal{L}(c_t, l_t, k_{t+1}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \nu(1 - l_t)] - \lambda_t [c_t + q_t [k_{t+1} - (1 - \delta)k_t] + T_t]$$

Solution of the optimization problem requires that the first derivatives of the Lagrangian with respect to its arguments are set to zero (the first order conditions):

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} = 0 &: \frac{\beta^t}{c_t} - \lambda_t = 0 \rightarrow \lambda_t = \frac{\beta^t}{c_t} \\ \frac{\partial \mathcal{L}}{\partial l_t} = 0 &: -\beta^t \nu + \lambda_t w_t = 0 \end{aligned}$$

We input the definition of  $\lambda_t$  found in FOC (1) into FOC (2) to find:

$$\begin{aligned} -\beta^t \nu + \frac{\beta^t}{c_t} w_t &= 0 \\ \nu &= \frac{w_t}{c_t} \\ \nu c_t &= w_t \end{aligned}$$

This is the first equilibrium condition of households. To find the second, we evaluate FOC (3):

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 : \lambda_t q_t + \lambda_{t+1} [q_{t+1}(1 - \delta) + r_{t+1}] = 0$$

We recall that  $\lambda_t = \frac{\beta^t}{c_t}$  and plug it in the condition above:

$$\begin{aligned} \frac{\beta^t}{c_t} q_t &= \beta^{t+1} \mathbb{E}_t \left[ \frac{q_{t+1}(1 - \delta) + r_{t+1}}{c_{t+1}} \right] \\ \frac{q_t}{c_t} &= \beta \mathbb{E}_t \left[ \frac{q_{t+1}(1 - \delta) + r_{t+1}}{c_{t+1}} \right] \end{aligned}$$

■

## Steady state

As usual in steady-state analysis, we drop time subscripts. As in Dorofeenko, Lee and Salyer (2008), we further simplify the solutions by scaling the expressions by the share of entrepreneurs of the economy. This further prove that this share does not alter the steady-state relationships. We should find the values  $\{c, c^e, k, \bar{\omega}, h, q, z, n, i, g\}$  that satisfy the equations below. First, as in Carlstrom and Fuerst (1997), the contribution of entrepreneurial labor is considered very small, and the aggregate production function simplifies to:

$$Y = AK^\alpha [(1 - \eta)l]^{1-\alpha} \quad (\text{A.4})$$

In addition, we consider  $y = \frac{Y}{\eta}$ ,  $h = \frac{(1-\eta)l}{\eta}$ ,  $k = \frac{K}{\eta}$ , and set the steady state productivity shifter  $A$  to unity. From the market clearing condition, it follows that:

$$c + c^e + i + g = k^\alpha h^{1-\alpha} \quad (\text{A.5})$$

From Equation 4.17, we can find that:

$$\begin{aligned} q &= \frac{\alpha\beta}{1 - \beta(1 - \delta)} k^{\alpha-1} h^{1-\alpha} \\ &= \frac{\alpha\beta}{1 - \beta(1 - \delta)} \frac{y}{k} \end{aligned} \quad (\text{A.6})$$

$$h = \frac{1 - \alpha y}{\nu c} \quad (\text{A.7})$$

Taking the law of motion of capital stock  $k_{t+1} = (1 - \delta)k_t + i_t[1 - \mu\Phi(\bar{\omega}_t, S_t)]$  and solving for  $k$  at the steady-state, we find:

$$k = \frac{1 - \mu\Phi(\bar{\omega}_t, S_t)}{\delta} i \quad (\text{A.8})$$

The following results derive from the entrepreneurs' maximization problem. First, we plug Equation 4.17 into the definition of net worth to find:

$$n = z \left( q(1 - \delta) + \alpha \frac{y}{k} \right) = z \frac{q}{\beta} \quad (\text{A.9})$$

Following Carlstrom and Fuerst (1997), we take the entrepreneurial law of motion and further impose that  $\gamma \left( \frac{qf(\bar{\omega}_t, S_t)}{1 - m(\bar{\omega}_t, S_t)} \right)$ , that is, the internal rate of return of entrepreneurs is offset by their discount factor, leading to the following relationship:

$$z = n \frac{1}{q\gamma} - \frac{c^e}{q} \quad (\text{A.10})$$

The optimal conditions of the financial contract, evaluated at the steady state:

$$q = \frac{1}{1 - \mu\Phi(\bar{\omega}_t, S_t) + \phi(\bar{\omega}_t, S_t) \frac{f(\bar{\omega}_t, S_t)}{f(\bar{\omega}_t, S_t)'}} \quad (\text{A.11})$$

$$i = \frac{1}{1 - q(1 - \mu\Phi(\bar{\omega}_t, S_t) - f(\bar{\omega}_t, S_t))} n \quad (\text{A.12})$$

In order to find all the unknowns, we follow Dorofeenko, Lee and Salyer (2008), which introduce a definition of risk premium  $\xi$  for entrepreneurs:

$$q\omega \frac{i}{i - n} = \xi \quad (\text{A.13})$$

Combining the risk premium and the relationship found in Equation A.12:

$$\frac{n}{i} = 1 - qm(\bar{\omega}_t, S_t) \quad (\text{A.14})$$

Rearranging and substituting from the previous expression leads to:

$$\omega = \xi m(\bar{\omega}_t, S_t) \quad (\text{A.15})$$

We take advantage of the fact that the probability of an entrepreneur going bankrupt can be calibrated from data:

$$\Phi(\bar{\omega}_t, S_t) = \text{empirical bankruptcy rate} \quad (\text{A.16})$$

Now, steady state values for  $\bar{\omega}_t$  and  $S_t$  can be found by solving the two equations above.

## Analytical model statistics

Table 9 – Moments of the distribution of selected variables

Variable	Mean	Std. Dev.	Variance
$Y_t$	0.608	10.108	102.166
$C_t$	0.352	0.642	0.412
$I_t$	0.068	7.437	55.301
$g_t$	0.250	10.013	100.251
$H_t$	0.433	7.421	55.074
$n_t$	0.022	2.391	5.717
$r_t$	0.136	0.476	0.227
$\Phi(\bar{\omega}_t, S_t)$	0.024	0.443	0.196

Source: The author.

Table 10 – Variance-covariance matrix of selected variables

Variables	$Y_t$	$C_t$	$I_t$	$G_t$	$H_t$	$n_t$	$r_t$	$\Phi(\bar{\omega}_t, S_t)$
$Y_t$	1	-0.189	0.316	0.998	0.994	0.314	0.307	0.025
$C_t$	-0.189	1	-0.611	-0.210	-0.292	-0.611	-0.747	-0.415
$I_t$	0.316	-0.611	1	0.284	0.387	0.994	0.921	-0.051
$G_t$	0.998	-0.210	0.284	1	0.994	0.283	0.289	0.056
$H_t$	0.994	-0.292	0.387	0.994	1	0.385	0.386	0.054
$n_t$	0.314	-0.611	0.994	0.283	0.385	1	0.928	-0.016
$r_t$	0.307	-0.747	0.921	0.289	0.386	0.928	1	0.342
$\Phi(\bar{\omega}_t, S_t)$	0.025	-0.415	-0.051	0.056	0.054	-0.016	0.342	1

Source: The author.

## Dynare code

```

% Angelo Salton
% Dynare 4.5.6, Octave 4.4.1 (linux_amd64)
% encoding: UTF-8 CRLF

var
  K      ${K}$ (long_name='capital stock')
  ke     ${K^e}$ (long_name='ent. capital stock')
  H      ${H}$ (long_name='labor')
  He     ${H^e}$ (long_name='ent. labor')
  h      ${h}$ (long_name='scaled labor')
  q      ${q}$ (long_name='price of capital')
  n      ${n}$ (long_name='ent. net worth')
  i      ${i}$ (long_name='investment')
  g      ${g}$ (long_name='gov. spending')
  omegab ${\bar{\omega}}$ (long_name='threshold unc. shock')
  ce     ${c^e}$
  cc     ${c^e}$
  w      ${w}$ (long_name='real wage')
  we     ${w^e}$ (long_name='real ent. wage')
  Y      ${Y}$ (long_name='agg. output')
  r      ${r}$ (long_name='capital rental rate')
  I      ${I}$ (long_name='agg. investment')
  Cc     ${C^c}$ (long_name='agg. h.h. consumption')
  Ce     ${C^e}$ (long_name='agg. ent. consumption')
  C      ${C}$ (long_name='agg. consumption')
  Rb     ${R^b}$ (long_name='Rb')
  rpBANK ${r^{\textit{BANK}}}$ (long_name='bank risk prem.')
  rpENT  ${r^{\textit{ENT}}}$ (long_name='ent. risk prem.')
  lev    ${l}$ (long_name='ent. leverage')
  rif    ${r^{\textit{if}}}$
  PHI    ${\Phi}$
  phi    ${\phi}$
  f      ${f}$ (long_name='ent. share of profits')
  m      ${m}$ (long_name='bank share of profits')
  A      ${A}$ (long_name='TFP')
  St     ${S}$ (long_name='time-varying unc.')
  M      ${M}$
  welf

;

varexo
  eA ${\varepsilon^A}$ (long_name='tfp shock')
  eN ${\varepsilon^N}$ (long_name='net worth shock')
  eG ${\varepsilon^G}$ (long_name='gov. spending shock')
  eS ${\varepsilon^S}$ (long_name='unc. shock')
;

parameters
  alphaK ${\alpha^K}$ (long_name='capital share')
  alphaH ${\alpha^H}$ (long_name='h.h. labor share')
  beta   ${\beta}$ (long_name='discount rate')
  gamma  ${\gamma}$ (long_name='add. ent. discount rate')
  delta  ${\delta}$ (long_name='depreciation rate')
  eta    ${\eta}$ (long_name='ent. share')
  S      ${\bar{S}}$ (long_name='s.s. uncertainty')
  nu     ${\nu}$ (long_name='elast. of h.h. labor')

```

```

mu      ${\mu}$ (long_name='monitoring cost')
p       ${p}$ (long_name='price level')
rhoA    ${\rho^A}$ (long_name='tfp persistence')
rhoG    ${\rho^G}$ (long_name='gov. spend. persistence')
StdeA   ${\sigma^A}$ (long_name='std. dev. tfp shock')
StdeN   ${\sigma^N}$ (long_name='std. dev. net worth shock')
zeta    ${\varepsilon}$ (long_name='unc. shock persistence')
G       ${\bar{G}}$ (long_name='s.s. gov. spending')
;

% calibration
alphaK  = 0.4;
alphaH  = 1-alphaK-0.0001;
beta    = 0.89;
delta   = 0.02/4;
gamma   = beta-0.02;
eta     = 0.1;
rhoA    = 0.996;
StdeA   = 0.0723;
StdeN   = 0.1;
S       = 0.21;
nu      = 2.52;
mu      = 0.3;
p       = pi;
rhoStdA = 0.83;
StdeStdA = 0.19;
zeta    = 0.95;
G       = 0.25;
rhoG    = 0.995;

model;
H = (1-eta)*h;
He = eta;
Cc = (1-eta)*cc;
Ce = eta*ce;
C = (1-eta)*cc + eta*ce;
I = eta*i;
Y = (1-eta)*cc + eta*ce + eta*i + g;
nu*cc = w;
w = alphaH*A*(K(-1)^alphaK)*(H^(alphaH-1))*(He^(1-alphaK-alphaH));
q/cc = beta*(1/cc(+1))*(q(+1)*(1-delta)+r(+1));
ke = i*f - ce/q;
K = (1-delta)*K(-1) + I*(1-mu*PHI);
i = (1/(1-q*m)) * n;
q = (beta*gamma) * (r(+1) + q(+1)*(1-delta)) * ((q(+1)*f(+1))/(1-q(+1)*m(+1))
↪ );
q = 1/(1 - mu*PHI - (mu*phi*f)/(1-PHI));
n = we + (ke(-1))*(q*(1-delta)+r) + StdeN*eN;
Y = A*(K(-1)^alphaK)*(H^alphaH)*(He^(1-alphaK-alphaH));
r = alphaK*A*(K(-1)^(alphaK-1))*(H^alphaH)*(He^(1-alphaK-alphaH));
we = (1-alphaK-alphaH)*A*(K(-1)^alphaK)*(H^alphaH)*He^(-(alphaK+alphaH));
Rb = (q*i*omegab)/(i-n);
rpBANK = Rb-1;
lev = i/n;
rif = q*f*i/n;
rpENT = q*(1+r)-Rb;
PHI = normcdf((log(omegab)-M)/St);
phi = normpdf((log(omegab)-M)/St) / (omegab*St);

```

```

m = normcdf((log(omegab)-M)/St - St) - PHI*mu + (1-PHI)*omegab;
f = 1-mu*PHI-m;
A = (1-rhoA) + rhoA*A(-1) + StdeA*eA;
St = S^(1-zeta)*St(-1)^zeta+eS;
g = G^(1-rhoG)*g(-1)^rhoG+eG;
M = -.5*St^2;
welf = log(cc) + nu*h + beta*welf(+1);
end;

initval;
omegab = 0.603892;
St = S;
M = -.5*St^2;
PHI = normcdf((log(omegab)-M)/St);
phi = normpdf((log(omegab)-M)/St) / (omegab*St);
m = normcdf((log(omegab)-M)/St - St) - PHI*mu + (1-PHI)*omegab;
f = 1-mu*PHI- m;
q = 1/(1-mu*0.01+(gamma-1)*f);
r = q*((1-beta*(1-delta))/beta);
H = .3;
He = eta;
h = .3/(1-eta);
K =
↪ (alphaK/r)^(1/(1-alphaK))*(He^(1-alphaK-alphaH))*(H^(alphaH/(1-alphaK)));
Y = A*(K^alphaK)*(H^alphaH)*(He^(1-alphaK-alphaH));
i = (delta/(eta*(1-mu*0.01)))*K;
n = (1-m*q)*i;
ke = (beta/q)*(eta*n-(1-alphaK-alphaH)*Y);
ce = q*(f*i-(ke/eta));
cc = (Y - eta*ce - eta*i)/(1-eta);
A = 1;
g = G;
welf = 100;
end;

%write_latex_parameter_table;

resid;
steady;
check;

shocks;
var eA = 1;
var eN = 1;
var eG = 1;
var eS = 1;
end;

stoch_simul(order=2,pruning,irf=40,replic=200) Y C i g H n r PHI lev;

% welfare w.r.t. agency costs and uncertainty shocks -----

% ranges of values for parameters
age = 0.1:0.05:0.35;
unc = 0.15:0.02:0.25;

wel = [];

```

```

for i = 1:length(age);
    for j = 1:length(unc);
        set_param_value('mu',age(i));
        set_param_value('S',unc(j));
        stoch_simul(order=2,pruning,nograph,noprint);
        wel(i,j) = oo_.mean(33);
    end
end

surf(unc,age,wel);
xlabel({'S','S. S. micro uncertainty'});
ylabel({'\mu','Agency costs'});
zlabel('Welfare');
%saveas(gcf, 'welf1', 'eps');
close all;

% steady state values w.r.t. unc. shocks (as in CF2017) -----
ssv = zeros(length(unc),length(oo_.mean));

for i = 1:length(unc);
    set_param_value('S',unc(i));
    stoch_simul(order=2,pruning,nograph);
    ssv(i,:) = oo_.mean;
end

% optional: represent in percent changes
%pctc = diff(ssv)./ssv(1:end-1,:);
%ssv2 = [ones(length(oo_.mean)); pctc]

% in this array you can select variables to be plotted
varss = {'Y','C','I','g','H','n','r','PHI','lev'}

inds = find(ismember(M_.endo_names,varss))

% check dimensions of 'varss'
for i = 1:9;
    subplot(3,3,i);
    plot(unc,ssv(:,i));
    %xlabel('S');
    ylabel(varss{i});
end
saveas(gcf, 'unc1', 'eps');

% agency costs sensitivity analysis -----
mrks = ['-+', '-o', '-*', '-.']

mus = [0.1, 0.3, 0.5, 0.7];
x = struct();

for i = 1:length(mus);
    set_param_value('mu',mus(i));
    stoch_simul(order=2,pruning,irf=40,nograph,noprint);
    x(i) = oo_.irfs;
end

% output -----
figure();

```

```

hold all;

for i = 1:length(mus);
    plot(x(i).Y_eS, mrks(i));
end;

hold off;
legend('\mu = 0.1', '\mu = 0.3', '\mu = 0.5', '\mu = 0.7');
ylabel('Deviation from steady state');
xlabel('Time (in quarters)');
saveas(gcf, 'agc_y', 'eps');
close all;

% consumption -----
figure();
hold all;

for i = 1:length(mus);
    plot(x(i).C_eS, mrks(i));
end;

hold off;
legend('\mu = 0.1', '\mu = 0.3', '\mu = 0.5', '\mu = 0.7');
ylabel('Deviation from steady state');
xlabel('Time (in quarters)');
saveas(gcf, 'agc_c', 'eps');
close all;

% net worth -----
figure();
hold all;

for i = 1:length(mus);
    plot(x(i).n_eS, mrks(i));
end;

hold off;
legend('\mu = 0.1', '\mu = 0.3', '\mu = 0.5', '\mu = 0.7');
ylabel('Deviation from steady state');
xlabel('Time (in quarters)');
saveas(gcf, 'agc_n', 'eps');
close all;

clear x;

% capital share sensitivity analysis -----
csh = [0.3, 0.35, 0.4, 0.45];
x = struct();

for i = 1:length(csh);
    set_param_value('alphaK', csh(i));
    stoch_simul(order=2, pruning, irf=40, nograph, noprint);
    x(i) = oo_.irfs;
end

% output -----
figure();
hold all;

```

```

for i = 1:length(csh);
    plot(x(i).Y_eS, mrks(i));
end;

hold off;
legend('\alpha = 0.3', '\alpha = 0.35', '\alpha = 0.4', '\alpha = 0.45');
ylabel('Deviation from steady state');
xlabel('Time (in quarters)');
saveas(gcf, 'csh_y', 'eps');
close all;

% consumption -----
figure();
hold all;

for i = 1:length(csh);
    plot(x(i).C_eS, mrks(i));
end;

hold off;
legend('\alpha = 0.3', '\alpha = 0.35', '\alpha = 0.4', '\alpha = 0.45');
ylabel('Deviation from steady state');
xlabel('Time (in quarters)');
saveas(gcf, 'csh_c', 'eps');
close all;

% net worth -----
figure();
hold all;

for i = 1:length(csh);
    plot(x(i).n_eS, mrks(i));
end;

hold off;
legend('\alpha = 0.3', '\alpha = 0.35', '\alpha = 0.4', '\alpha = 0.45');
ylabel('Deviation from steady state');
xlabel('Time (in quarters)');
saveas(gcf, 'csh_n', 'eps');
close all;

```