



# Symanzik–Becchi–Rouet–Stora lessons on renormalizable models with broken symmetry: The case of Lorentz violation

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In memoriam of Raymond Stora — 1930–2015

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## Abstract

In this paper, we revisit the issue intensively studied in recent years on the generation of terms by radiative corrections in models with broken Lorentz symmetry. The algebraic perturbative method of handling the problem of renormalization of the theories with Lorentz symmetry breaking, is used. We hope to make clear the Symanzik's aphorism: "Whether you like it or not, you have to include in the lagrangian all counter terms consistent with locality and power-counting, unless otherwise constrained by Ward identities."<sup>1</sup>

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## 1. Introduction

Perturbative field models with symmetry breaking were investigated from the point of view of the theory of renormalization in the pioneering work of Symanzik [1,2] and treated in a way

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<sup>1</sup> The phrase was borrowed from text "Pedagogical Experiments in Renormalized Perturbation Theory," by Raymond Stora. Contribution to the Hesselberg Meeting on the Theory of Renormalization and Regularization, 24 February–1 March 2002.

that we can consider as definitive, by Becchi–Rouet–Stora [3–5]. However, several recent works, dealing in particular with field theories with Lorentz symmetry breaking, do not consider very carefully how the symmetry is broken, not taking into account the requirements that Symanzik–Becchi–Rouet–Stora have shown to be necessary. In this article we intend to write an updated review of the problem of renormalization of the theories with Lorentz symmetry breaking. All our analysis will be based on a general iterative scheme called “*Algebraic Renormalization*” [6–9].<sup>2</sup> In the algebraic approach, in order to study the renormalizability of models characterized by a system of Ward identities, without referring to any special regularization procedure, two steps must be followed. In the first step, for a power-counting renormalizable model, at the level of the radiative corrections, one investigates the preservation of the symmetries, or the determination of all possible anomalies. This amounts to find the solution of the cohomology of its symmetry group: trivial elements (co-boundaries) correspond to breakings which can be compensated by non-invariant counterterms, whereas the non-trivial elements are the possible anomalies. These cohomology conditions are a generalization of the Wess–Zumino consistency condition [10] used in order to compute the possible anomalies of the Ward identities in Yang–Mill theories. In a second step, we check the stability of the classical action – which ensures that the quantum corrections do not produce counterterms corresponding to the renormalization of parameters not already present in the classical theory.

Let us emphasize that the algebraic renormalization scheme is based on a set of theorems of renormalization theory, collected under the name of “*Quantum Action Principle*” (QAP) [11–13]. These theorems deal with the whole of Feynman graphs’ combinatorics and integrability, so that explicit graph considerations are unnecessary – unless one looks for explicit quantitative results for applications to physics, of course. Said Raymond: “*Use the theorems!*” [14].

This article is divided as follows: in Section 2, we give a short review on the renormalization of theories with explicit symmetry breaking. In Section 3, as a toy model, we analyze the Lorentz symmetry breaking in a scalar field model. Then, in Section 4, we revisit the quantum electrodynamics (QED) with violation of Lorentz and CPT symmetries. Among several issues, in particular using the BRST formalism, we reassess the possible generation of a Chern–Simons-like term induced by radiative corrections arising from a CPT and Lorentz violating term in the fermionic sector, a recurrent theme in the literature. It is important to emphasize that, concerning extended QED with a term which violates the Lorentz and CPT symmetries, most of the papers were devoted to discuss the gauge invariance of the model *only*, putting aside a more specific way how Lorentz invariance is broken. In Section 5, we finish with some reflections on the important works of Symanzik–Becchi–Rouet–Stora on renormalizable models with broken symmetry. This article is dedicated to Raymond Stora memory. The passion of Professor Stora for the fundamentals of Quantum Field Theory was what led him to become one of the leading researchers in the world of renormalization, culminating with the awards he has received: the Max Planck Medal (1998) and the Dannie Heineman Prize for Mathematical Physics (2009). The latter was a recognition of the important work he did with Carlo Becchi and Alain Rouet on a rigorous mathematical procedure for quantizing non-abelian gauge field theories, which is now known as BRST quantization.

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<sup>2</sup> It should be emphasized that, based on the method suggested by the Epstein–Glaser construction, the algebraic method of renormalization was “seeded” in the Lecture Notes by Professor Raymond Stora, “*Lagrangian Field Theory*,” in Particle Physics, Proceedings of the Les Houches Summer School, 1971, and edited by C. De Witt and C. Itzykson, Gordon & Breach, 1973.

## 2. Explicit symmetry breaking in a nutshell

In this section we present a sketch of the renormalization of models with broken symmetries, adapted from the works of Symanzik [1,2] and Stora and collaborators [3–5]. As in these works, we restrict ourselves to theories which are power-counting renormalizable and symmetries which are realized linearly.

First, suppose that a set of field transformation laws is given, infinitesimally, by

$$i[Q_\alpha, \Phi] = \delta_\alpha \Phi ,$$

where  $\Phi$  is a field or multiplet of fields transforming in a specific way under a symmetry group  $G$ . The charges  $Q_\alpha$  form a basis of the Lie algebra of  $G$ , satisfying commutation rules

$$[Q_\alpha, Q_\beta] = if_{\alpha\beta\gamma} Q_\gamma .$$

We may translate the transformations into the language of functional differential operators

$$W_\alpha := -i \int d^D x \delta_\alpha \Phi(x) \frac{\delta}{\delta \Phi(x)} ,$$

where  $D$  is the space–time dimension, fulfilling the same commutation rules as the charges:

$$[W_\alpha, W_\beta] = if_{\alpha\beta\gamma} W_\gamma . \tag{2.1}$$

These operators act on  $\Gamma[\Phi]$ , the vertex functional which generates the 1-particle irreducible and amputated Feynman graphs. In a perturbation expansion in powers of  $\hbar$  – equivalent to an expansion in the number of loops – the zeroth order or tree approximation functional  $\Gamma^{(0)}$  is just the classical action  $S[\Phi]$ , a local functional of the classical fields  $\Phi$ .

Now, suppose that, at the classical level, one adds to an action  $S_{\text{inv}}[\Phi]$ , invariant under the group transformations, *i.e.*, satisfying  $W_\alpha S_{\text{inv}} = 0$ , a *breaking term*

$$S_{\text{break}}[\Phi] = b^I \int d^D x B_I(x) ,$$

where the  $b^I$ 's play the role of “coupling constants” and the  $B_I$ 's are local functionals of  $\Phi$ , *i.e.*, local polynomials in  $\Phi$  and its derivatives. We assume that the  $B_I$ 's have power-counting dimension  $d \leq D$ , and transform under the symmetry transformations in a given representation  $R$  of the group:

$$\delta_\alpha B_I(x) = R_{\alpha I}^J B_J(x) ,$$

where the  $R_{\alpha I}^J$  are representation matrix elements of the generators  $Q_\alpha$ . Then the total action

$$S_{\text{tot}}[\Phi] = S_{\text{inv}}[\Phi] + S_{\text{break}}[\Phi] , \tag{2.2}$$

breaks the  $G$  symmetry:

$$W_\alpha S_{\text{tot}} = b^I R_{\alpha I}^J \int d^D x B_J \neq 0 . \tag{2.3}$$

In order to *control the breaking* and, in particular its power-counting and symmetry properties in all orders of perturbation theory, following Symanzik [1,2], we convert this action into one which is invariant under the original transformation, adding a term involving external fields  $\beta_I(x)$  of power-counting dimension  $D - d^3$ :

$$S[\Phi, \beta] = S_{\text{inv}}[\Phi] + \int d^D x (\beta^I(x) + b^I) B_I(x) , \tag{2.4}$$

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<sup>3</sup> This dimension will specify the renormalization procedure of the breaking operators  $B_I$  defined below.

where the  $\beta^I$ 's transform under  $G$  as

$$\delta_\alpha \beta^I(x) = -(\beta^J(x) + b^J) R_{\alpha I}{}^J B_J(x) .$$

The new action is invariant:

$$W_\alpha S[\Phi, \beta] = -i \int d^D x \left( \delta_\alpha \Phi(x) \frac{\delta}{\delta \Phi(x)} + \delta_\alpha \beta^I(x) \frac{\delta}{\delta \beta^I(x)} \right) S[\Phi, \beta] = 0 . \quad (2.5)$$

Obviously, at  $\beta(x) = 0$ ,  $S$  reduces to the action (2.2), and the identity (2.5) to the breaking identity (2.3).

The purpose of renormalization is to construct, perturbatively, a vertex functional  $\Gamma[\Phi, \beta]$  obeying the same functional identity (2.5), now expressed as the Ward identity

$$W_\alpha \Gamma[\Phi, \beta] = 0 . \quad (2.6)$$

This identity, taken at external field  $\beta = 0$ , yields the broken Ward identity

$$W_\alpha \Gamma[\Phi, 0] = b^J R_{\alpha, J}{}^I \int d^D x \left. \frac{\delta \Gamma[\Phi, \beta]}{\delta \beta^I(x)} \right|_{\beta^I(x)=0} , \quad (2.7)$$

where the r.h.s. represents the renormalization of the classical breaking on the r.h.s. of (2.3) [1–4].

At this point, two important remarks have to be done:

1. Since we have assigned to the external field  $\beta$  the power counting dimension  $D - d$ , we know from the QAP that the dimension of the renormalized breaking on the r.h.s. of (2.7) is an insertion of dimension  $d$ . In particular, if  $d < D$ , it is guaranteed to be a “soft” insertion, which means in particular that the asymptotic behaviour in momentum space of a Green function with this insertion is lower by a power  $D - d$  than the same green function without the insertion.
2. The broken Ward identity (2.7) explicitly shows that the renormalized breaking belongs to the same representation as its classical counterpart given in (2.3).

In conclusion, successfully fulfilling the renormalization program leading to the Ward identity (2.6) yields a perturbative quantization of the classical theory with full control of the dimension and covariance of the breaking. Of course, it remains the possibility of an anomaly, *i.e.*, the impossibility to fulfil the Ward identity.

The success of this program is guaranteed if the following two conditions are met [1–5]:

- 1) *Criterion of stability of the theory under small perturbations:* All possible counterterms  $\Delta_{\text{c.t.}}[\phi, \beta]$ , solutions of the invariance conditions  $W_\alpha \Delta_{\text{c.t.}} = 0$ , correspond to the renormalization of the parameters and fields of the classical theory defined by the action (2.4). If some solutions of the invariance condition do not meet this requirement, one has to suitably complete the classical action. As a consequence of the QAP, the  $\Delta_{\text{c.t.}}$ 's are integrated local functionals of the fields  $\Phi$  and  $\beta$ , of dimension limited by  $D$  due to power-counting renormalizability.
- 2) *Absence of anomaly:* Let  $\Delta_\alpha[\Phi, \beta]$  be integrated local functionals of dimension up to  $D$ . Then, the *consistency conditions*

$$W_\alpha \Delta_\beta - W_\beta \Delta_\alpha = i f_{\alpha\beta\gamma} \Delta_\gamma , \quad (2.8)$$

admit only “trivial” solutions of the form

$$\Delta_\alpha = W_\alpha \Delta , \quad (2.9)$$

for some integrated local functional  $\Delta[\phi, \beta]$  of dimension at most equal to  $D$ . If this condition is not met, we say we have an anomaly.

Condition 2) is based on the fact that, as a consequence of the QAP, the possible breakings of the Ward identity (2.6) are insertions of integrated local insertions whose lowest order are integrated local functionals  $\Delta_\alpha$  of dimension limited by  $D$ . The consistency condition (2.8) is then a consequence of the algebra (2.1). The fulfilment of (2.9) means that any possible breaking  $\Delta_\alpha$  can be reabsorbed in the action, at each order of perturbation theory, as non-invariant counterterm equal to  $-\Delta$ . As noticed first by Stora and his collaborators [3–5], solving this condition amounts to solving a problem of Lie algebra cohomology.

### 3. A toy model with hard Lorentz breaking

We study here a toy model of scalar fields with a *hard breaking* of Lorentz invariance, *i.e.*, a breaking of dimension 4, in a 4D space–time with Minkovsky metric  $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ . The scalar fields form an  $SO(N)$  multiplet  $\varphi^i(x)$ ,  $i = 1, \dots, N$ , transforming under the Lorentz transformations as

$$\delta\varphi^i(x) = -\epsilon^\mu{}_\nu x^\nu \partial_\mu \varphi^i(x) ,$$

where  $\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}$  are infinitesimal parameters. The invariant part of the action is the most general Lorentz invariant one, restricted by power-counting renormalizability to be

$$S_{\text{inv}}[\varphi] = \int d^4x \left( \frac{1}{2} \partial^\mu \varphi^i \partial_\mu \varphi^i - \frac{m^2}{2} \varphi^i \varphi^i - \frac{\lambda}{4!} (\varphi^i \varphi^i)^2 \right) , \tag{3.1}$$

with  $m^2$  and  $\lambda$  chosen as positive. The hard breaking term is given by

$$S_{\text{break}}[\varphi] = \frac{1}{2} c^{\mu\nu} \int d^4x \partial_\mu \varphi^i \partial_\nu \varphi^i ,$$

where the 9 arbitrary numbers  $c^{\mu\nu}$  are the elements of a symmetric traceless matrix (the trace part would correspond to a scalar term already present in  $S_{\text{inv}}$  as its kinetic term).

As explained in Section 2, we need to introduce an external field in order to control the group theory characteristics of the breaking. The characteristics here is that of a symmetric, traceless Lorentz tensor of rank two:

$$B_{\mu\nu} = \partial_\mu \varphi^i \partial_\nu \varphi^i - \frac{1}{4} \eta_{\mu\nu} \partial^\lambda \varphi^i \partial_\lambda \varphi^i .$$

The external field coupled to this breaking will thus be a symmetric traceless tensor field  $\gamma^{\mu\nu}(x)$ , transforming as

$$\delta\gamma^{\mu\nu}(x) = -\epsilon^\rho{}_\lambda x^\lambda \partial_\rho \gamma^{\mu\nu}(x) + \epsilon^\mu{}_\rho (\gamma^{\rho\nu}(x) + c^{\rho\nu}) + \epsilon^\nu{}_\rho (\gamma^{\rho\mu}(x) + c^{\rho\mu}) ,$$

under the Lorentz transformations. The functional identity (2.5) takes here the form

$$W_{(\epsilon)} S = -i \int d^4x \left( \delta\varphi^i(x) \frac{\delta S}{\delta\varphi^i} + \frac{1}{2} \delta\gamma^{\mu\nu}(x) \frac{\delta S}{\delta\gamma^{\mu\nu}(x)} \right) = 0 , \tag{3.2}$$

where the functional operators obey to the commutation rules

$$[W_{(\epsilon)}, W_{(\eta)}] = i W_{([\epsilon, \eta])} . \tag{3.3}$$

An action solution of (3.2) is

$$S = S_{\text{inv}} + \frac{1}{2} \int d^4x (\gamma^{\mu\nu} + c^{\mu\nu}) \partial_\mu \varphi^i \partial_\nu \varphi^i . \quad (3.4)$$

In order to see if the theory thus defined is renormalizable, let us look at the criteria enumerated at the end of Section 2.

*Condition of stability:*

The solutions of the condition  $W_{(\epsilon)} \Delta_{\text{c.t.}} = 0$  are, either any one of the three terms of the invariant action (3.1), or  $\gamma$ -dependent terms, given by:

$$\sum_{n=1}^{\infty} \frac{\alpha_n}{2n!} \int d^4x \hat{\gamma}^\mu{}_{\rho_1} \hat{\gamma}^{\rho_1}{}_{\rho_2} \cdots \hat{\gamma}^{\rho_n}{}_{\nu} \partial_\mu \varphi^i \partial_\nu \varphi^i , \quad (3.5)$$

$$\sum_{n=1}^{\infty} \frac{\beta_n}{2n!} \int d^4x \hat{\gamma}^\mu{}_{\rho_1} \hat{\gamma}^{\rho_1}{}_{\rho_2} \cdots \hat{\gamma}^{\rho_n}{}_{\nu} \partial_\mu (\varphi^i \partial_\nu \varphi^i) , \quad (3.6)$$

where  $\hat{\gamma}^{\mu\nu}(x) = \gamma^{\mu\nu}(x) + c^{\mu\nu}$  and  $\alpha_n, \beta_n$  are arbitrary parameters. This set of counterterms is infinite due to the zero dimensionality of the exterior field  $\gamma$ . But one observes that the sum of terms in (3.5) can be reduced to the form of the  $(\gamma + c)$  term of (3.4) by a non-linear redefinition of  $\gamma$ . However the counterterms in the sum (3.6) do not correspond to anything present in the action (3.4), and thus the latter should be completed with them – although their role is trivial: they turn out to be total derivatives when setting  $\gamma = 0$  at the end. In summary, all possible counterterms correspond to the renormalization of the given classical action: the parameters  $m$  and  $\lambda$ , the field  $\varphi^i$  and the external field  $\gamma^{\mu\nu}$ . Let us note that the renormalization of  $\gamma^{\mu\nu}$  amounts to a renormalization of the composite breaking operator  $B_{\mu\nu}$ .

*Absence of anomalies:*

The form of the consistency condition follows from the commutation rules (3.3) for (rigid) Lorentz symmetry:

$$W_{(\epsilon)} \Delta_{(\eta)} - W_{(\eta)} \Delta_{(\epsilon)} = \Delta_{([\epsilon, \eta])} .$$

Its general solution has been proved [15] to be of the form  $\Delta_\epsilon = W_\epsilon \Delta$  with  $\Delta$  an integrated local functional of dimension  $\leq 4$ . There is thus no anomaly.

*Counting the number of parameters and of renormalizations:*

As we have seen in the discussion of stability under small perturbations, the theory depends on the following physical parameters: the mass  $m$ , the coupling constant  $\lambda$  and the 9 breaking parameters  $c^{\mu\nu}$ . Beyond these, the theory also depends on the parameter corresponding to the renormalization of the field  $\varphi$  and on the infinite number of parameters  $\alpha_n$  corresponding to the non-linear renormalization of the external field  $\gamma^{\mu\nu}$ , see (3.5). On the other hand, since the  $c^{\mu\nu}$  are given in the definition of the Ward identity operator  $W_\epsilon$ , they are not renormalized. Thus we have only 2 renormalizations, those of  $m$  and  $\lambda$ , beyond the field amplitude renormalizations  $\varphi^i$  and  $\gamma^{\mu\nu}$ .

This situation has to be contrasted with that of the theory with Lorentz invariance hardly broken without control through external fields, as we are going to show later on.

Before going on, a remark concerning the infinity of parameters  $\alpha_n$  is due. This seems to imply a non-renormalizability of the theory, since an infinite number of normalization conditions

is indeed needed in order to fix them. The normalization conditions, beyond the usual ones which fix  $m$ ,  $\lambda$  and the field amplitude of  $\varphi$ , may be chosen, *e.g.*, as

$$\Gamma_{2,n} = 1, \quad n = 1, \dots, \infty,$$

where  $\Gamma_{2,n}$  is the vertex function with  $n$  insertion of the breaking operator  $\int d^4x c^{\mu\nu} B_{\mu\nu}$  and 2 (amputated) external lines  $\varphi^i$  at some conveniently chosen momentum. Each of the counterterms in the sum (3.5) will contribute to the breaking, at zero external field, with a coefficient of order  $n$  in  $c^{\mu\nu}$ . Therefore, in a realistic situation where the breaking is expected to be very small, only a few terms of low order will effectively contribute!

*Hard breaking without controlled covariance:*

Let us now turn to the case of a breaking of Lorentz symmetry where we do not introduce the controlling external field  $\gamma^{\mu\nu}$ , as, *e.g.*, in [38]. The reason for introducing external fields is two-fold, as we explain in Section 2: controlling the power-counting dimension of the breaking and controlling its covariance. In the case considered here, no control of dimension is needed since the dimension of a hard breaking is by definition already the maximum one allowed by power-counting renormalizability, namely 4. In order to clarify some point which may still appear unclear, it is interesting to have a look at what would happen if no controlling external field is introduced. We would thus start with the action (3.4) with  $\gamma^{\mu\nu} = 0$ :

$$S = S_{\text{inv}} + c^{\mu\nu} \int d^4x \partial_\mu \varphi^i \partial_\nu \varphi^i. \tag{3.7}$$

There is now no Ward identity like (3.2), so that the independent counterterms which will be generated are all possible field polynomial of power-counting dimension up to 4, restricted only by  $SO(N)$  invariance. Beyond the 12 terms of the action (3.7), there is the possibility of the dimension 3 terms  $d^\mu \int d^4x \varphi^i \partial_\mu \varphi^i$  depending on 4 parameters  $d^\mu$ . This total of 16 counterterms do correspond to an equal number of 16 renormalizations.

How would one explain this difference in a context of Feynman graphs calculations? On the one hand, calculating Green functions with the Feynman rules defined by the action (3.7), one will sooner or later encounter ultraviolet singularities whose renormalization will generate 16 arbitrary finite parameters corresponding to the 16 ambiguities of the subtraction [16] or regularization [17] procedure.<sup>4</sup> On the other hand, if one introduces the external fields and takes into account, at each order of perturbation theory, of the conditions imposed by the fulfilment of the Ward identity, the dimension 3 terms  $d^\mu \int d^4x \varphi^i \partial_\mu \varphi^i$  will not appear and the independent parameters will be reduced to those we had above – without counting the parameters  $\alpha_n$  corresponding to the renormalization of the breaking operator.

**4. The QED with soft Lorentz breaking**

The quantum electrodynamics (QED) with violation of Lorentz and CPT have been studied intensively in recent years. Among several issues, the possible generation of a Chern–Simons-like term induced by radiative corrections arising from a CPT and Lorentz violating term in the fermionic sector has been a recurrent theme in the literature. We particularly mention the following works [18–39] (and references cited therein), where many controversies have emerged

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<sup>4</sup> For simplification, this procedure is assumed to preserve the explicit  $SO(N)$  symmetry.

from the discussion whether this Chern–Simons-like term could be generated by means of radiative corrections arising from the axial coupling of charged fermions to a constant vector  $b_\mu$  responsible for the breakdown of Lorentz Symmetry.

In this section, we reassess the discussion on the radiative generation of a Chern–Simons-like term induced from quantum corrections in the extended QED. We show, to all orders in perturbation theory, that a CPT-odd and Lorentz violating Chern–Simons-like term, *definitively*, is not radiatively induced by the axial coupling of the fermions with the constant vector  $b_\mu$ . The proof of this fact is based on general theorems of perturbative quantum field theory (see [6–8] and references therein), where the Lowenstein–Zimmermann subtraction scheme in the framework of Bogoliubov–Parasiuk–Hepp–Zimmermann–Lowenstein (BPHZL) renormalization method [40] is adopted. The former has to be introduced, owing to the presence of massless gauge field, so as to subtract infrared divergences that should arise from the ultraviolet subtractions.

#### 4.1. The model at the classical level

We start by considering an action for extended QED with a term which violates the Lorentz and CPT symmetries in the matter sector only. In the tree approximation, the classical action of extended QED with one Dirac spinor that we are considering here is given by:

$$\Sigma^{(s-1)} = \Sigma_S + \Sigma_{SB} + \Sigma_{IR} + \Sigma_{gf} + \Sigma_{ext} , \quad (4.1)$$

where

$$\Sigma_S = \int d^4x \left\{ i\bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \right\} ,$$

is the symmetric part of  $\Sigma$  under gauge and Lorentz transformations. The term

$$\Sigma_{SB} = - \int d^4x b_\mu \bar{\psi}\gamma_5\gamma^\mu\psi , \quad (4.2)$$

is the symmetry-breaking part of  $\Sigma$  that breaks the manifest Lorentz covariance by the presence of a constant vector  $b_\mu$  which selects a preferential direction in Minkowski space–time, breaking its isotropy, as well as it breaks CPT. In turn,

$$\Sigma_{IR} = \int d^4x \frac{1}{2}M^2(s-1)A_\mu A^\mu ,$$

is the Lowenstein–Zimmermann mass term for the photon field. A Lowenstein–Zimmermann mass term of the  $M^2(s-1)$  type is there in order to enable a momentum space subtraction scheme without introducing spurious infrared singularities. The Lowenstein–Zimmermann parameter  $s$  lies in the interval  $0 \leq s \leq 1$  and plays the role of an additional subtraction variable (as the external momentum) in the BPHZL renormalization program, such that the theory describing a really massless particle is recovered for  $s = 1$ . At this point, a comment about the Lowenstein–Zimmermann mass term for the photon field is now in order: the gauge invariance properties are not spoiled by the photon mass; this is a peculiarity of the *abelian case* [6]. This was studied in details for the QED in Ref. [41] using the BPHZ scheme.

Finally, in order to quantize the system a gauge-fixing is added

$$\Sigma_{gf} = \int d^4x \left( \Pi\partial_\mu A^\mu + \frac{\xi}{2}\Pi^2 + \bar{c}\square c \right) , \quad (4.3)$$

together with the term,  $\Sigma_{\text{ext}}$ , coupling the non-linear Becchi–Rouet–Stora–Tyutin (BRST) transformations to external sources

$$\Sigma_{\text{ext}} = \int d^4x \left( \bar{\Omega} \mathfrak{s}\psi - \mathfrak{s}\bar{\psi} \Omega \right). \quad (4.4)$$

*Continuous symmetries:*

The infinitesimal BRST transformations are given by:

$$\begin{aligned} \mathfrak{s}\psi &= ic\psi, \quad \mathfrak{s}\bar{\psi} = -ic\bar{\psi}, \\ \mathfrak{s}A_\mu &= -\frac{1}{e}\partial_\mu c, \quad \mathfrak{s}c = 0, \\ \mathfrak{s}\bar{c} &= \frac{1}{e}\Pi, \quad \mathfrak{s}\Pi = 0, \end{aligned} \quad (4.5)$$

where  $c$  is the ghost field,  $\bar{c}$  is the antighost field and  $\Pi$  is the Lautrup–Nakanishi field [42], respectively. The latter plays the role of the Lagrange multiplier field. Although not massive, the Faddeev–Popov ghosts are free fields, they decouple, therefore, no Lowenstein–Zimmermann mass term has to be introduced for them.

The BRST invariance of the action is expressed in a functional way by the Slavnov–Taylor identity

$$\mathcal{S}(\Sigma^{(s-1)}) = 0, \quad (4.6)$$

where the Slavnov–Taylor operator  $\mathcal{S}$  is defined, acting on an arbitrary functional  $\mathcal{F}$ , by

$$\mathcal{S}(\mathcal{F}) = \int d^4x \left\{ -\frac{1}{e}\partial^\mu c \frac{\delta\mathcal{F}}{\delta A^\mu} + \frac{1}{e}\Pi \frac{\delta\mathcal{F}}{\delta \bar{c}} + \frac{\delta\mathcal{F}}{\delta \bar{\Omega}} \frac{\delta\mathcal{F}}{\delta \Psi} - \frac{\delta\mathcal{F}}{\delta \Omega} \frac{\delta\mathcal{F}}{\delta \bar{\Psi}} \right\}. \quad (4.7)$$

The corresponding linearized Slavnov–Taylor operator reads

$$\mathcal{S}_{\mathcal{F}} = \int d^4x \left\{ -\frac{1}{e}\partial^\mu c \frac{\delta}{\delta A^\mu} + \frac{1}{e}\Pi \frac{\delta}{\delta \bar{c}} + \frac{\delta\mathcal{F}}{\delta \bar{\Omega}} \frac{\delta}{\delta \Psi} + \frac{\delta\mathcal{F}}{\delta \Psi} \frac{\delta}{\delta \bar{\Omega}} - \frac{\delta\mathcal{F}}{\delta \Omega} \frac{\delta}{\delta \bar{\Psi}} - \frac{\delta\mathcal{F}}{\delta \bar{\Psi}} \frac{\delta}{\delta \Omega} \right\}. \quad (4.8)$$

The following nilpotency identities hold:

$$\mathcal{S}_{\mathcal{F}}\mathcal{S}(\mathcal{F}) = 0, \quad \forall \mathcal{F}, \quad (4.9)$$

$$\mathcal{S}_{\mathcal{F}}\mathcal{S}_{\mathcal{F}} = 0 \text{ if } \mathcal{S}(\mathcal{F}) = 0. \quad (4.10)$$

In particular,  $(\mathcal{S}_{\Sigma}^{(s-1)})^2 = 0$ , since the action  $\Sigma^{(s-1)}$  obeys the Slavnov–Taylor identity (4.6). The operation of  $\mathcal{S}_{\Sigma^{(s-1)}}$  upon the fields and the external sources is given by

$$\begin{aligned} \mathcal{S}_{\Sigma^{(s-1)}}\phi &= s\phi, \quad \phi = \{\Psi, \bar{\Psi}, A_\mu, c, \bar{c}, \Pi\}, \\ \mathcal{S}_{\Sigma^{(s-1)}}\Omega &= -\frac{\delta\Sigma^{(s-1)}}{\delta\bar{\Psi}}, \quad \mathcal{S}_{\Sigma^{(s-1)}}\bar{\Omega}_+ = \frac{\delta\Sigma^{(s-1)}}{\delta\Psi}. \end{aligned}$$

In addition to the Slavnov–Taylor identity (4.6), the classical action  $\Sigma^{(s-1)}$  (4.1) is characterized by the gauge condition, the ghost equation and the antighost equation:

$$\frac{\delta\Sigma^{(s-1)}}{\delta\Pi} = \partial^\mu A_\mu + \xi\Pi, \quad (4.11)$$

$$\frac{\delta\Sigma^{(s-1)}}{\delta\bar{c}} = \square c, \quad (4.12)$$

$$-i\frac{\delta\Sigma^{(s-1)}}{\delta c} = i\square\bar{c} + \bar{\Omega}\Psi - \bar{\Psi}\Omega. \quad (4.13)$$

The action  $\Sigma^{(s-1)}$  (4.1) is invariant also with respect to the rigid symmetry

$$W_{\text{rigid}} \Sigma^{(s-1)} = 0, \quad (4.14)$$

where the Ward operator,  $W_{\text{rigid}}$ , is defined by

$$W_{\text{rigid}} = \int d^4x \left\{ \Psi \frac{\delta}{\delta \Psi} - \bar{\Psi} \frac{\delta}{\delta \bar{\Psi}} + \Omega \frac{\delta}{\delta \Omega} - \bar{\Omega} \frac{\delta}{\delta \bar{\Omega}} \right\}.$$

On the other hand, the Lorentz symmetry is broken by the presence of the constant vector  $b_\mu$ . The fields  $A_\mu$  and  $\Psi$  transform under infinitesimal Lorentz transformations  $\delta x^\mu = \epsilon^{\mu\nu} x^\nu$ , with  $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$ , as

$$\begin{aligned} \delta_L A_\mu &= -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda A_\mu + \epsilon_\mu{}^\nu A_\nu \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} A_\mu, \\ \delta_L \Psi &= -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \Psi - \frac{i}{4} \epsilon^{\mu\nu} \sigma_{\mu\nu} \Psi \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} \Psi, \end{aligned} \quad (4.15)$$

where  $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ .

It should be noticed that the Lorentz breaking (4.2) is not linear in the dynamical fields, therefore will be renormalized. It is however a “soft breaking,” since its UV power-counting dimension is less than 4, namely 3. According to Symanzik [1,2], a theory with soft symmetry breaking is renormalizable if the radiative corrections do not induce a breakdown of the symmetry by terms of UV power-counting dimension equal to 4 – called hard breaking terms. Concretely, according to the Weinberg’s Theorem [43], this means that the symmetry of the theory in the asymptotic deep euclidean region of momentum space is preserved by the radiative corrections. In order to control the Lorentz breaking and, in particular, its power-counting properties, following Symanzik [1,2], and [15] for the specific case of Lorentz breaking, we introduce an external field  $\beta_\mu(x)$ , of dimension 1 and transforming under Lorentz transformations according to

$$\delta_L \beta_\mu(x) = -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \beta_\mu(x) + \epsilon_\mu{}^\nu (\beta_\nu(x) + b_\nu) \equiv \frac{1}{2} \epsilon^{\alpha\beta} \delta_{L\alpha\beta} \beta_\mu(x). \quad (4.16)$$

The functional operator which generates these transformations reads

$$W_{L\alpha\beta} = \int d^4x W_{L\alpha\beta}(x) = \int d^4x \sum_{\varphi=A_\mu, \Psi, \bar{\Psi}, \beta} \delta_{L\alpha\beta} \varphi(x) \frac{\delta}{\delta \varphi(x)}. \quad (4.17)$$

Redefining the action by adding a term in  $\beta_\mu$ :

$$\tilde{\Sigma}^{(s-1)} = \Sigma^{(s-1)} - \int d^4x \beta_\mu \bar{\Psi} \gamma_5 \gamma^\mu \Psi, \quad (4.18)$$

one easily checks the classical Ward identity

$$W_{L\alpha\beta} \tilde{\Sigma}^{(s-1)} = 0, \quad (4.19)$$

which, at  $\beta_\mu = 0$ , reduces to the broken Lorentz Ward identity

$$W_{L\alpha\beta} \Sigma^{(s-1)} = \epsilon_\mu{}^\nu b_\nu \int d^4x \bar{\Psi} \gamma_5 \gamma^\mu \Psi. \quad (4.20)$$

The external field  $\beta_\mu(x)$  being coupled to a gauge invariant expression (the axial current:  $j_5^\mu = \bar{\psi} \gamma_5 \gamma^\mu \psi$ ), we take it to be BRST invariant in order to preserve gauge invariance,

$$s \int d^4x \beta_\mu \bar{\Psi} \gamma_5 \gamma^\mu \Psi = 0 \implies s \beta_\mu(x) = 0. \quad (4.21)$$

Therefore, it follows that the action  $\widetilde{\Sigma}^{(s-1)}$  (4.18) satisfies the same Slavnov identity (4.6) as the action  $\Sigma^{(s-1)}$  (4.1), namely:

$$\mathcal{S}(\widetilde{\Sigma}^{(s-1)}) = 0, \quad (4.22)$$

together with the conditions:

$$\frac{\delta \widetilde{\Sigma}^{(s-1)}}{\delta \Pi} = \partial^\mu A_\mu + \xi \Pi, \quad (4.23)$$

$$\frac{\delta \widetilde{\Sigma}^{(s-1)}}{\delta \bar{c}} = \square c, \quad (4.24)$$

$$-i \frac{\delta \widetilde{\Sigma}^{(s-1)}}{\delta c} = i \square \bar{c} + \bar{\Omega} \Psi - \bar{\Psi} \Omega, \quad (4.25)$$

$$W_{\text{rigid}} \widetilde{\Sigma}^{(s-1)} = 0. \quad (4.26)$$

*Discrete symmetries:*

The discrete symmetries of the theory are the following ones:

*Charge conjugation C:* Assuming the Dirac representation of the  $\gamma$ -matrices [44], the charge conjugation transformations read:

$$\begin{aligned} \psi &\xrightarrow{C} \psi^c = C \bar{\psi}^T, \\ \bar{\psi} &\xrightarrow{C} \bar{\psi}^c = -\psi^T C^{-1}, \\ A_\mu &\xrightarrow{C} A_\mu^c = -A_\mu, \\ C \gamma_\mu C &= \gamma_\mu^T, \\ C \gamma_5 C &= -\gamma_5^T = -\gamma_5, \end{aligned} \quad (4.27)$$

where  $C$  is the charge conjugation matrix, with  $C^2 = -1$ . All terms of the action  $\widetilde{\Sigma}^{(s-1)}$  (4.18) are invariant under charge conjugation.

*Parity P:*

$$\begin{aligned} x &\xrightarrow{P} (x^0, -\vec{x}), \\ \psi &\xrightarrow{P} \gamma^0 \psi, \\ \bar{\psi} &\xrightarrow{P} \bar{\psi} \gamma^0, \\ A_\mu &\xrightarrow{P} A^\mu. \end{aligned} \quad (4.28)$$

All terms of the action  $\widetilde{\Sigma}^{(s-1)}$  (4.18) are invariant under parity, unless the Lorentz breaking term  $\Sigma_{\text{SB}}$  (4.2).

*Time reversal T:*

$$\begin{aligned} \psi &\xrightarrow{T} T \psi, \\ \bar{\psi} &\xrightarrow{T} \bar{\psi} T, \\ A_\mu &\xrightarrow{T} A_\mu, \\ T \gamma^\mu T &= \gamma_\mu^T = \gamma^{\mu*}, \\ T \gamma_5 T &= \gamma_5. \end{aligned} \quad (4.29)$$

Table 1

UV ( $d$ ) and IR ( $r$ ) dimensions, ghost number ( $FP$ ) and Grassmann parity ( $GP$ ).

	$A_\mu$	$\psi$	$c$	$\bar{c}$	$\Pi$	$\Omega$	$\beta_\mu$	$s-1$	$s$
$d$	1	3/2	0	2	2	5/2	1	1	1
$r$	1	2	0	2	2	2	1	1	0
$FP$	0	0	1	-1	0	-1	0	0	0
$GP$	0	1	1	1	0	0	0	0	0

The broken Lorentz term  $\Sigma_{SB}$  (4.2) is non-invariant under time reversal, whereas the other terms in the action  $\tilde{\Sigma}^{(s-1)}$  (4.18) remain invariant. As a consequence, the action  $\tilde{\Sigma}^{(s-1)}$  (4.18), has CPT symmetry broken by the Lorentz breaking term,  $\Sigma_{SB}$  (4.2):

$$\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi \xrightarrow{CPT} -\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi . \quad (4.30)$$

*UV and IR dimensions:*

Switching off the coupling constant  $e$  and the constant vector  $b^\mu$  defines the free part of the action (4.1) and the tree-level propagators for all the fields. In momentum space, they read:

$$\Delta_{\psi\psi}(k) = i \frac{\not{k} + m}{k^2 - m^2} , \quad (4.31)$$

$$\Delta_{AA}^{\mu\nu}(k, s) = -i \left\{ \frac{1}{k^2 - M^2(s-1)^2} \left( \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) + \frac{\xi}{k^2 - \xi M^2(s-1)^2} \frac{k^\mu k^\nu}{k^2} \right\} , \quad (4.32)$$

$$\Delta_{A\Pi}^\mu(k) = \frac{k^\mu}{k^2} , \quad \Delta_{\Pi\Pi}(k) = 0 , \quad (4.33)$$

$$\Delta_{\bar{c}c}(k) = -i \frac{1}{k^2} . \quad (4.34)$$

In order to establish the ultraviolet (UV) and infrared (IR) dimensions of any fields,  $X$  and  $Y$ , we make use of the UV and IR asymptotical behaviour of their propagator,  $\Delta_{XY}(k, s)$ ,  $d_{XY}$  and  $r_{XY}$ , respectively:

$$d_{XY} = \overline{\text{deg}}_{(k,s)} \Delta_{XY}(k, s) ,$$

$$r_{XY} = \underline{\text{deg}}_{(k,s-1)} \Delta_{XY}(k, s) ,$$

where the upper degree  $\overline{\text{deg}}_{(k,s)}$  gives the asymptotic power for  $(k, s) \rightarrow \infty$  whereas the lower degree  $\underline{\text{deg}}_{(k,s-1)}$  gives the asymptotic power for  $(k, s-1) \rightarrow 0$ . The UV ( $d$ ) and IR ( $r$ ) dimensions of the fields,  $X$  and  $Y$ , are chosen to fulfil the following inequalities:

$$d_X + d_Y \geq 4 + d_{XY} \quad \text{and} \quad r_X + r_Y \leq 4 + r_{XY} . \quad (4.35)$$

In summary, the UV ( $d$ ) and IR ( $r$ ) dimensions – which are those involved in the Lowenstein–Zimmermann subtraction scheme [40] – as well as the ghost numbers ( $FP$ ) and the Grassmann parity ( $GP$ ) of all fields are collected in Table 1. Notice that the statistics is defined as follows: the integer spin fields with odd ghost number, as well as, the half integer spin fields with even ghost number anticommute among themselves. However, the other fields commute with the formers and also among themselves.

#### 4.2. The model at the quantum level

In this section, we present the perturbative quantization of the extended QED theory, using the algebraic renormalization procedure (see [7–9] for a review of the method and references to the original literature). Our aim is to prove that the full quantum theory has the same properties as the classical theory, *i.e.*, demonstrate that, at the quantum level, the Slavnov–Taylor identity, related to the gauge symmetry (4.22), and the Ward identity associated to the Lorentz symmetry (4.19), are satisfied to all orders of perturbation theory:

$$\mathcal{S}(\Gamma^{(s-1)})|_{s=1} = 0, \tag{4.36}$$

$$W_{L\alpha\beta}\Gamma^{(s-1)}|_{s=1} = 0. \tag{4.37}$$

In order to study the renormalizability of models characterized by a system of Ward identities, without referring to any special regularization procedure, two steps must be followed [7–9]: In the first step, we compute the possible anomalies of the Ward identities through an analysis of the Wess–Zumino consistency condition. Next, we check the stability of the classical action – which ensures that the quantum corrections do not produce counterterms corresponding to the renormalization of parameters not already present in the classical theory.

#### 4.3. The Wess–Zumino consistency condition: in search for anomalies

At the quantum level the vertex functional,  $\Gamma^{(s-1)}$ , which coincides with the classical action,  $\tilde{\Sigma}^{(s-1)}$  (4.18), at 0th order in  $\hbar$ ,

$$\Gamma^{(s-1)} = \tilde{\Sigma}^{(s-1)} + \mathcal{O}(\hbar), \tag{4.38}$$

has to satisfy the same constraints as the classical action does, namely Eq. (4.19) and Eqs. (4.22)–(4.26).

In accordance with the Quantum Action Principle [11–13], the Slavnov–Taylor identity (4.6) and the Lorentz symmetry Ward identity get quantum breakings:

$$\mathcal{S}(\Gamma^{(s-1)})|_{s=1} = \Delta \cdot \Gamma^{(s-1)}|_{s=1} = \Delta_g + \mathcal{O}(\hbar \Delta_g), \tag{4.39}$$

$$W_{L\alpha\beta}\Gamma^{(s-1)}|_{s=1} = \Delta_{L\alpha\beta} \cdot \Gamma^{(s-1)}|_{s=1} = \Delta_{L\alpha\beta} + \mathcal{O}(\hbar \Delta_{L\alpha\beta}), \tag{4.40}$$

where  $\Delta_g \equiv \Delta_g|_{s=1}$  and  $\Delta_{L\alpha\beta} \equiv \Delta_{L\alpha\beta}|_{s=1}$  are integrated local functionals, taken at  $s = 1$ , with ghost number one and, UV and IR dimensions bounded by  $d \leq 4$  and  $r \geq 4$ , respectively.

The validity of the Lorentz Ward identity has been proved in [15] by using the Whitehead’s Lemma for semi-simple Lie groups, which states the vanishing of the first cohomology of such kind of group [3,5]. Here, see details in [37], this means that  $\Delta_{L\alpha\beta}$  in (4.40) can be written as

$$\Delta_{L\alpha\beta} = W_{L\alpha\beta} \widehat{\Delta}_L, \tag{4.41}$$

where  $\widehat{\Delta}_L$  is an integrated local insertion of UV and IR dimensions bounded by  $d \leq 4$  and  $r \geq 4$ , respectively. Therefore,  $\widehat{\Delta}_L$  can be reabsorbed in the action as a noninvariant counterterm, order by order, establishing the Lorentz Ward identity (4.37) at the quantum level.

The nilpotency identity (4.9) together with

$$\mathcal{S}_{\Gamma^{(s-1)}} = \mathcal{S}_{\tilde{\Sigma}^{(s-1)}} + \mathcal{O}(\hbar), \tag{4.42}$$

implies the following consistency conditions for the breaking  $\Delta_g$ :

$$\mathcal{S}_{\tilde{\Sigma}^{(s-1)}} \Delta_g = 0, \tag{4.43}$$

and beyond that, the breaking  $\Delta_g$  also satisfies the constraints:

$$\frac{\delta \Delta_g}{\delta \Pi} = \frac{\delta \Delta_g}{\delta \bar{c}} = \int d^4x \frac{\delta \Delta_g}{\delta c} = W_{\text{rigid}} \Delta_g = W_{L\alpha\beta} \Delta_g = 0. \tag{4.44}$$

The Wess–Zumino consistency condition (4.43) constitutes a cohomology problem in the sector of ghost number one. Its solution can always be written as a sum of a trivial cocycle  $\mathcal{S}_{\Sigma} \widehat{\Delta}_g^{(0)}$ , where  $\widehat{\Delta}_g^{(0)}$  has ghost number zero, and of nontrivial elements belonging to the cohomology of  $\mathcal{S}_{\Sigma^{(s-1)}}$  (4.8) in the sector of ghost number one:

$$\Delta_g^{(1)} = \widehat{\Delta}_g^{(1)} + \mathcal{S}_{\Sigma^{(s-1)}} \widehat{\Delta}_g^{(0)}. \tag{4.45}$$

However, considering the Slavnov–Taylor operator  $\mathcal{S}_{\Sigma^{(s-1)}}$  (4.8) and the quantum breaking (4.39), it results that  $\Delta_g^{(1)}$  exhibits UV and IR dimensions bounded by  $d \leq 4$  and  $r \geq 4$ .

From the antighost equation in (4.44):

$$\int d^4x \frac{\delta \widehat{\Delta}_g^{(1)}}{\delta c} = 0, \tag{4.46}$$

it follows that  $\widehat{\Delta}_g^{(1)}$  can be written as

$$\widehat{\Delta}_g^{(1)} = \int d^4x \mathcal{T}_\mu \partial^\mu c, \tag{4.47}$$

where  $\mathcal{T}_\mu$  is a rank-1 tensor with ghost number zero, with UV and IR dimensions bounded by  $d \leq 3$  and  $r \geq 3$ , respectively. The tensor  $\mathcal{T}_\mu$  can be split into two pieces:

$$\mathcal{T}_\mu = r_v \mathcal{V}_\mu + r_p \mathcal{P}_\mu, \tag{4.48}$$

where  $\mathcal{V}_\mu$  is a vector and  $\mathcal{P}_\mu$  is a pseudo-vector, with  $r_v$  and  $r_p$  being coefficients to be determined. By considering the UV and IR dimensional constraints to be satisfied by  $\mathcal{T}_\mu$  (4.48) together with the conditions upon the Slavnov–Taylor breaking  $\widehat{\Delta}_g^{(1)}$  (4.45), given by (4.43) and (4.44), it follows that:

$$\mathcal{T}_\mu = r_v \partial^\rho F_{\rho\mu} + r_p \epsilon_{\mu\nu\rho\sigma} A^\nu F^{\rho\sigma}. \tag{4.49}$$

Consequently, substituting (4.49) into (4.47), the breaking  $\widehat{\Delta}_g^{(1)}$  reads:

$$\widehat{\Delta}_g^{(1)} = -\frac{r_p}{2} \int d^4x c \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}, \tag{4.50}$$

which is the (Abelian) Adler–Bardeen–Bell–Jackiw anomaly [45]. Therefore, up to noninvariant counterterms, which are  $\mathcal{S}_{\Sigma^{(s-1)}}$ -variations of the integrated local insertions  $\widehat{\Delta}_g^{(0)}$ :

$$\Delta_g^{(1)} = \mathcal{S}_{\Sigma^{(s-1)}} \widehat{\Delta}_g^{(0)} - \frac{r_p}{2} \int d^4x c \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}. \tag{4.51}$$

The anomaly coefficient  $r_p$  does not get renormalizations [46,7], it is identically zero if it vanishes at the one loop order, so it is sufficient to verify its vanishing at this order. However, that is the case, due to the fact that the potentially dangerous axial current  $j_5^\mu = \bar{\psi} \gamma_5 \gamma^\mu \psi$  is only coupled to the external field  $\beta_\mu$  – and not to any quantum field of the theory – which means that no gauge anomaly can be produced [6,27,33]. Hence it follows that the Slavnov–Taylor identity (4.36) is established at the quantum level.

Finally, in which concerns anomalies, the presence of a CPT violating interaction term by coupling an axial fermion current ( $j_5^\mu = \bar{\psi} \gamma_5 \gamma^\mu \psi$ ) with a constant vector field  $b_\mu$ , does not

induce neither a Lorentz anomaly nor a gauge anomaly – independent of any regularization scheme.

#### 4.4. The stability condition: in search for counterterms

In order to verify if the action in the tree-approximation ( $\tilde{\Sigma}^{(s-1)}$ ) is stable under radiative corrections, we perturb it by an arbitrary integrated local functional (counterterm)  $\tilde{\Sigma}^{c(s-1)}$ , such that

$$\widehat{\Sigma}^{(s-1)} = \tilde{\Sigma}^{(s-1)} + \varepsilon \tilde{\Sigma}^{c(s-1)}, \quad (4.52)$$

where  $\varepsilon$  is an infinitesimal parameter. The functional  $\tilde{\Sigma}^c \equiv \tilde{\Sigma}^{c(s-1)}|_{s=1}$  has the same quantum numbers as the action in the tree-approximation at  $s = 1$ .

The deformed action  $\widehat{\Sigma}^{(s-1)}$  must still obey all the conditions presented above, henceforth,  $\tilde{\Sigma}^{c(s-1)}$  is subjected to the following set of constraints:

$$\mathcal{S}_{\Sigma^{(s-1)}} \tilde{\Sigma}^{c(s-1)} = 0, \quad (4.53)$$

$$\frac{\delta \tilde{\Sigma}^{c(s-1)}}{\delta \Pi} = \frac{\delta \tilde{\Sigma}^{c(s-1)}}{\delta \bar{c}} = \frac{\delta \tilde{\Sigma}^{c(s-1)}}{\delta c} = 0, \quad (4.54)$$

$$W_{\text{rigid}} \tilde{\Sigma}^{c(s-1)} = 0, \quad (4.55)$$

$$W_{L\alpha\beta} \tilde{\Sigma}^{c(s-1)} = 0. \quad (4.56)$$

The most general invariant counterterm  $\tilde{\Sigma}^{c(s-1)}$  – the most general field polynomial – with UV and IR dimensions bounded by  $d \leq 4$  and  $r \geq 4$ , with ghost number zero and fulfilling the conditions displayed in Eqs. (4.53)–(4.56), reads:

$$\begin{aligned} \tilde{\Sigma}^{c(s-1)} \Big|_{r \geq 4}^{d \leq 4} = & \int d^4x \left\{ \alpha_1 i \bar{\psi} \gamma^\mu (\partial_\mu + ie A_\mu) \psi + \alpha_2 \bar{\psi} \psi + \alpha_3 F^{\mu\nu} F_{\mu\nu} + \right. \\ & \left. + \alpha_4 (\beta_\mu(x) + b_\mu) \bar{\psi} \gamma_5 \gamma^\mu \psi \right\}. \end{aligned} \quad (4.57)$$

The coefficients  $\alpha_1, \dots, \alpha_4$  are arbitrary, and they are fixed, order by order in perturbation theory, by the four normalization conditions:

$$\begin{aligned} \Gamma_{\bar{\psi}\psi}(\not{p}) \Big|_{\not{p}=m} &= 0, \quad \frac{\partial}{\partial \not{p}} \Gamma_{\bar{\psi}\psi}(\not{p}) \Big|_{\not{p}=m} = 1, \\ \frac{\partial}{\partial p^2} \Gamma_{A^T A^T}(p^2) \Big|_{p^2=\kappa^2} &= 1, \quad -\frac{1}{4} \text{Tr}[\gamma^\mu \gamma^5 \Gamma_{\beta_\mu \bar{\psi}\psi}(0, \not{p})] \Big|_{\not{p}=m} = 1. \end{aligned} \quad (4.58)$$

It shall be stressed here that, a Chern–Simons-like term of the type

$$\Sigma_{\text{CS}} = \int d^4x \alpha_5 \left\{ \epsilon_{\mu\nu\alpha\beta} \beta^\mu(x) A^\nu \partial^\alpha A^\beta \Big|_4^4 + \epsilon_{\mu\nu\alpha\beta} b^\mu A^\nu \partial^\alpha A^\beta \Big|_3^3 \right\}, \quad (4.59)$$

in spite of fulfilling the conditions (4.54)–(4.56), its first term breaks gauge invariance by violating the Slavnov–Taylor identity (4.53), whereas its second term violates IR dimension constraint ( $r_{\Sigma_{\text{CS}}} \geq 4$ ), it has IR dimension equal to three. Therefore, the Chern–Simons-like term  $\Sigma_{\text{CS}}$  (4.59) can never be generated by radiative corrections if the renormalization procedure is performed correctly: First, by taking care of the IR divergences – for instance, through the

Lowenstein–Zimmermann method [40] – that show up, thanks to the presence of a photon, which is massless; Second, by properly treating and controlling the Lorentz symmetry breaking through the Symanzik method [1,2]. Anyway, even though the external field  $\beta_\mu(x)$  was not introduced in order to control the Lorentz breaking, the Chern–Simons-like term – which is a soft Lorentz breaking (UV dimension less than four) – would not be radiatively generated as explained above, nevertheless, any gauge invariant hard Lorentz breaking (UV dimension equal to four) could be induced by radiative corrections. In short, a CPT-odd and Lorentz-violating Chern–Simons-like term, *independent of any regularization scheme*, is definitely *not* radiatively induced by coupling an axial fermion current ( $j_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi$ ) with a constant vector field  $b_\mu$ .

## 5. Conclusions

We revisit the issue intensively studied in recent years on the generation of terms by radiative corrections in models with broken Lorentz symmetry. We have exemplified all of this in two examples. In the first example, we have discussed the case of the hard breaking for a very simple model involving scalar fields and tried to give some insight on the way Symanzik’s method of introducing controlling exterior fields is working. We have explicitly shown the difference of theory’s behaviour, the external field being present or being not present. An interesting collateral result is the presence in this model of an infinite number of counterterms – all compatible with power-counting renormalizability – which could jeopardize renormalizability in the sense of needing an infinite number of normalization conditions to fix them, which physically corresponds to an infinite set of measurements. But we have argued that the Lorentz violation, if physically present, must be very small, and thus only a few of these counterterms are practically relevant. In the second example, we reassess the discussion on the radiative generation of a Chern–Simons-like term induced from quantum corrections in the extended QED. We show, to all orders in perturbation theory, that a CPT-odd and Lorentz violating Chern–Simons-like term, *definitively*, is not radiatively induced by the axial coupling of the fermions with the constant vector  $b_\mu$ . The proof of this fact is based on general theorems of perturbative quantum field theory, where the Lowenstein–Zimmermann subtraction scheme in the framework of Bogoliubov–Parasiuk–Hepp–Zimmermann–Lowenstein (BPHZL) renormalization method is adopted.

It is true that we need new ideas to go beyond the Standard Model. Such an idea is the Lorentz symmetry breaking. If it is present in our Universe has been the subject of much discussion. So far, no trace was found. Experience is the final judgement of a theory; therefore to be checked experimentally, the Lorentz symmetry breaking remains a theoretical construction, regardless of how seductive the idea can be. However, even as a theoretical construction, the idea of the Lorentz symmetry breaking should be well grounded. But it seems that this has not happened in the recent literature on the subject. In particular, in this article we analyze the issue intensively studied in recent years on the generation of terms by radiative corrections in models with broken Lorentz symmetry. Unfortunately, several recent works, dealing the subject, do not consider very carefully how the Lorentz symmetry is broken, not taking into account the requirements that Symanzik–Becchi–Rouet–Stora have shown to be necessary. The young researchers who study a QFT with broken Lorentz symmetry should read the pioneer articles of Symanzik–Becchi–Rouet–Stora and “devour them.” Exactly what we do over the years! And that is why we hope to have conveyed the impression that the reconsideration of the fundamental works on renormalization of quantum field models developed mainly in the 1970’s, especially the papers of Symanzik–Becchi–Rouet–Stora on renormalizable models with broken symmetry, provide us with a theoretical tool susceptible to avoid some “bad” conclusions associated with

models with broken Lorentz symmetry. It is important to emphasize that, the main characteristics of this method is the *control* of the breaking and, in particular, its power-counting properties, converting the initial action containing terms that violate the Lorentz symmetry into one which is invariant under the original transformation adding external fields (the Symanzik sources). Without this control, the study of the stability (here meant additive renormalization) tells us that *any* term that breaks the Lorentz symmetry, compatible with the power-counting, must *necessarily* be present in the starting lagrangian. On the other hand, if we include in the initial lagrangian all terms that break the symmetry Lorentz, compatible with the locality and power-counting, no breaking control is required (see Ref. [38]). Therefore, paraphrasing Symanzik, *whether you like it or not, you have to include in the initial lagrangian all terms that violate the Lorentz symmetry consistent with locality and power-counting, unless otherwise constrained by a break control!*

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## References

- [1] K. Symanzik, Renormalizable models with simple symmetry breaking: 1. Symmetry breaking by a source term, *Commun. Math. Phys.* 16 (1970) 48.
- [2] K. Symanzik, Renormalization of theories with broken symmetry, in: D. Bessis (Ed.), *Cargèse Lectures in Physics 1970*, vol. 5, Gordon & Breach, 1972, p. 179.
- [3] R. Stora, Renormalizable models with broken symmetries, in: G. Velo, A.S. Wightman (Eds.), *Renormalization Theory*, D. Reidel Publ. Co., Dordrecht Holland, 1976, p. 299.
- [4] J.H. Lowenstein, A. Rouet, R. Stora, W. Zimmermann, Renormalizable models with broken symmetries, in: E.R. Caianiello (Ed.), *Renormalization and Invariance in Quantum Field Theory*, Plenum Press, 1981, p. 75.
- [5] C. Becchi, A. Rouet, R. Stora, Renormalizable theories with symmetry breaking, in: E. Tirapegui (Ed.), *Field Theory, Quantization and Statistical Physics*, D. Reidel Publ. Co., Dordrecht Holland, 1981, p. 3.
- [6] O. Piguet, A. Rouet, Symmetries in perturbative quantum field theory, *Phys. Rep.* 76 (1981) 1.
- [7] O. Piguet, S.P. Sorella, *Algebraic Renormalization: Perturbative Renormalization, Symmetries and Anomalies*, *Lect. Notes Phys.*, vol. M28, 1995.
- [8] A. Borech, S. Emery, O. Moritsch, M. Schweda, T. Sommer, H. Zerrouki, *Applications of Noncovariant Gauges in the Algebraic Renormalization Procedure*, World Scientific, 1998.
- [9] D.H.T. Franco, O. Piguet, Algebraic renormalization, *Scholarpedia* 8 (11) (2013) 8336.
- [10] J. Wess, B. Zumino, Consequences of anomalous Ward identities, *Phys. Lett. B* 37 (1971) 95.
- [11] Y.-M.P. Lam, Perturbation lagrangian theory for scalar fields: Ward–Takahasi identities and current algebra, *Phys. Rev. D* 6 (1972) 2145.
- [12] J.H. Lowenstein, Differential vertex operators in lagrangian field theory, *Commun. Math. Phys.* 24 (1971) 1.
- [13] F. Brenneke, M. Dütsch, The quantum action principle in the framework of causal perturbation theory, in: B. Fauser, J. Tolksdorf, E. Zeidler (Eds.), *Quantum Field Theory: Competitive Models*, 2009, p. 177.
- [14] Raymond Stora, *Private communication*, 1973.
- [15] H. Balasin, M. Schweda, M. Stierle, O. Piguet, The cohomology problems of rigid Lorentz transformations in axial gauge theories, *Phys. Lett. B* 215 (1988) 328.
- [16] W. Zimmermann, Composite operators in the perturbation theory of renormalizable interactions, in: *Lect. Notes Phys.*, vol. 558, 2000, p. 244, 278;  
W. Zimmermann, Normal products and the short distance expansion in the perturbation theory of renormalizable interactions, *Ann. Phys.* 77 (1973) 536, 570.
- [17] H. Epstein, V. Glaser, The role of locality in perturbation theory, *Ann. Henri Poincaré* A19 (1973) 211.
- [18] S. Coleman, S.L. Glashow, High-energy tests of Lorentz invariance, *Phys. Rev. D* 59 (1999) 11608.

- [19] R. Jackiw, V.A. Kostelecký, Radiatively induced Lorentz and CPT violation in electrodynamics, *Phys. Rev. Lett.* 82 (1999) 3572.
- [20] J.-M. Chung, Lorentz and CPT-violating Chern–Simons term in the functional integral formalism, *Phys. Rev. D* 60 (1999) 127901.
- [21] J.-M. Chung, Radiatively-induced Lorentz and CPT violating Chern–Simons term in QED, *Phys. Lett. B* 461 (1999) 138.
- [22] M. Pérez-Victoria, Exact calculation of the radiatively induced Lorentz and CPT violation in QED, *Phys. Rev. Lett.* 83 (1999) 2518.
- [23] W.F. Chen, Understanding radiatively induced Lorentz-CPT violation in differential regularization, *Phys. Rev. D* 60 (1999) 085007.
- [24] J.-M. Chung, Phyllis Oh, Lorentz and CPT-violating Chern–Simons term in the derivative expansion of QED, *Phys. Rev. D* 60 (1999) 067702.
- [25] J.-M. Chung, B.K. Chung, Induced Lorentz- and CPT-violating Chern–Simons term in QED: Fock–Schwinger proper time method, *Phys. Rev. D* 63 (2001) 105015.
- [26] C. Adam, F.R. Klinkhamer, Causality and radiatively induced CPT violation, *Nucl. Phys. B* 513 (2001) 245.
- [27] G. Bonneau, Regularisation: many recipes, but a unique principle: Ward identities and normalisation conditions. The case of CPT violation in QED, *Nucl. Phys. B* 593 (2001) 398.
- [28] M. Pérez-Victoria, Physical (ir)relevance of ambiguities to Lorentz and CPT violation in QED, *J. High Energy Phys.* 04 (2001) 032.
- [29] W.F. Chen, Issues on radiatively induced Lorentz and CPT violation in quantum electrodynamics, arXiv:hep-th/0106035.
- [30] O.W. Greenberg, CPT violation implies violation of Lorentz invariance, *Phys. Rev. Lett.* 89 (2002) 231602.
- [31] Y.A. Sitenko, K.Yu. Rulik, On the effective Lagrangian in spinor electrodynamics with added violation of Lorentz and CPT symmetries, *Eur. Phys. J. C* 28 (2003) 405.
- [32] B. Altschul, Gauge invariance and the Pauli–Villars regulator in Lorentz- and CPT-violating electrodynamics, *Phys. Rev. D* 70 (2004) 101701.
- [33] G. Bonneau, Extended QED with CPT violation: clarifying some controversies, *Nucl. Phys. B* 764 (2007) 83.
- [34] A.P. Baêta Scarpelli, Marcos Sampaio, M.C. Nemes, B. Hiller, Gauge invariance and the CPT and Lorentz induced Chern–Simons-like term in extended QED, *Eur. Phys. J. C* 56 (2008) 571.
- [35] F.A. Brito, J.R. Nascimento, E. Passos, A.Yu. Petrov, The ambiguity-free four-dimensional Lorentz-breaking Chern–Simons action, *Phys. Lett. B* 664 (2008) 112.
- [36] J. Alfaro, A.A. Andrianov, M. Cambiaso, P. Giacconi, R. Soldati, Bare and induced Lorentz and CPT invariance violations in QED, *Int. J. Mod. Phys. A* 25 (2010) 3271, arXiv:0904.3557 [hep-th].
- [37] O.M. Del Cima, J.M. Fonseca, D.H.T. Franco, O. Piguet, Lorentz and CPT violation in QED revisited: a missing analysis, *Phys. Lett. B* 688 (2010) 258.
- [38] O.M. Del Cima, J.M. Fonseca, D.H.T. Franco, A.H. Gomes, O. Piguet, All orders renormalizability of a Lorentz and CPT violating quantum electrodynamics, *Phys. Rev. D* 85 (2012) 065023.
- [39] J.F. Assunção, T. Mariz, Radiatively induced CPT-odd Chern–Simons term in massless QED, *Europhys. Lett.* 110 (2015) 41002.
- [40] J.H. Lowenstein, BPHZ renormalization, in: G. Velo, A.S. Wightman (Eds.), *Renormalization Theory*, D. Reidel Publ. Co., Dordrecht Holland, 1976, p. 95.
- [41] J.H. Lowenstein, B. Schroer, Gauge invariance and Ward identities in a massive vector meson model, *Phys. Rev. D* 6 (1972) 1553;  
J.H. Lowenstein, B. Schroer, Comment on the absence of radiative corrections to the anomaly of the axial-vector current, *Phys. Rev. D* 7 (1973) 1929.
- [42] N. Nakanishi, Covariant quantization of the electromagnetic field in the Landau gauge, *Prog. Theor. Phys.* 35 (1966) 1111;  
B. Lautrup, *Mat. Fys. Medd. Dan. Vid. Selsk* 35 (11) (1967).
- [43] S. Weinberg, High-energy behavior in quantum field theory, *Phys. Rev.* 118 (1960) 838.
- [44] C. Itzykson, J.-B. Zuber, *Quantum Field Theory*, Phys. Ser., McGraw–Hill, New York, 1980.
- [45] S.L. Adler, Axial vector vertex in spinor electrodynamics, *Phys. Rev.* 117 (1969) 2426;  
J.S. Bell, R. Jackiw, A PCAC puzzle:  $\pi_0 \rightarrow \gamma\gamma$  in the sigma model, *Nuovo Cimento* 60 (1969) 47;  
J. Schwinger, On gauge invariance and vacuum polarization, *Phys. Rev.* 82 (1951) 664.
- [46] S.L. Adler, W.A. Bardeen, Absence of higher order corrections in the anomalous axial vector divergence equation, *Phys. Rev.* 182 (1969) 1517;  
W.A. Bardeen, in: *Renormalization of Yang–Mills Fields and Application to Particle Physics*, C.N.R.S., Marseille, 1972, p. 29, report 72/p470;

- W.A. Bardeen, in: Proceedings of the 16th International Conference on High Energy Physics, in: Fermilab, vol. II, 1972, p. 295;
- A. Zee, Axial vector anomalies and the scaling property of field theory, *Phys. Rev. Lett.* 29 (1972) 1198;
- A. Blasi, O. Piguet, S.P. Sorella, Landau gauge and finiteness, *Nucl. Phys. B* 356 (1991) 154.