FABRÍCIO SEPÚLVEDA GOMES

THE IMPACT OF CLIMATE CHANGE ON THE ECONOMY OF THE PARNAÍBA RIVER BASIN

Dissertation submitted to the Applied Economics Graduate Program of the Universidade Federal de Viçosa in partial fulfillment of the requirements for the degree of of *Magister Scientiae*.

Adviser: Ian Michael Trotter

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This work is dedicated to all my friends and people who supported me during this journey of learning. Throughout the master's degree I was able to make eternal friendships. I'm only here because of the support of my parents, friends and family. This achievement is dedicated to everyone who believed in me. My mother, Ivanilda Sepúlveda Gomes, has always helped me in what she can. She influenced me not to give up. She helped me financially with everything. My father, Antonio Hilário Gomes, likewise always be with me at a distance helping as much as I could. To friends Pablo, Raya, Yuri, Luis, Erica, Maicker, Bira, Renan, Jackson, Mateus and all the teachers who accompanied me and others who were not mentioned.

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" And not only so, but we also rejoice in our tribulations: knowing that tribulation worketh stedfastness; and stedfastness, approvedness; and approvedness, hope".

(Romans 5:3,4 - Bible)

ABSTRACT

GOMES, Sepúlveda Fabrício, M.Sc., Universidade Federal de Viçosa, April, 2020. **the impact of climate change on the economy of the Parnaíba River Basin**. Adviser: Ian Michael Trotter.

The present work used a deterministic growth model made by Li and Swain to simulate the economic variables of the Parnaíba River Basin in different scenarios of water availability. The region in which the Parnbaíba River Basin is not considered a developed region and current management models do not make long run forecasts for the hydrographic demands of the basin. In the scenario with a slight reduction (10% of reduction) in water availability, no significant difference was found in relation to the standard scenario in the region. With more severe reductions about 38,5%, which is considered as a reduced inflow in dry times, the well-being and growth of the region is compromised.

Keywords: Growth model. Water. Long run.

RESUMO

GOMES, Sepúlveda Fabrício, M.Sc., Universidade Federal de Viçosa, abril de 2020. **The impact of climate change of the Parnaíba River Basin**. Orientador: Ian Michael Trotter.

O presente trabalho utilizou um modelo determinístico de crescimento feito por Li e Swain para simular as variáveis econômicas da Bacia do Rio Parnaíba em diferentes cenários de disponibilidade de água. A região em que a Bacia do Rio Parnbaíba não é considerada uma região desenvolvida e os atuais modelos de gestão não fazem previsões de longo prazo para as demandas hidrográficas da bacia. No cenário com ligeira redução (10% de redução) da disponibilidade hídrica, não foi encontrada diferença significativa em relação ao cenário padrão da região. Com reduções mais severas em torno de 38,5%, o que é considerado uma afluência reduzida em épocas de seca, o bem-estar e o crescimento da região ficam comprometidos.

Palavras-chave: Modelo de crescimento. Àgua. Longo prazo.

LIST OF ILLUSTRATIONS

Figure 1 – Projected Drought Frequency (today - 2050) 12
Figure 2 – Number of cities in state of public calamity or state of emergency . . . 12
Figure 3 – Water consumption and GDP 13
Figure 4 – Number of cities in state of public calamity or state of emergency . . . 14
Figure 5 – Parnaíba Hydrographic Region 15
Figure 6 – Historical Rainfall Series 16
Figure 7 – Jenipapo Den Historical Series 16
Figure 8 – Increase in Average Annual temperature 17
Figure 9 – Equilibrium Path of Main Variables 32

LIST OF TABLES

Table 1 – GDP (R$), Capital Stock (R$), Consumption (thousands R$) and Population for state for year 2010. 27
Table 2 – Water Demand by Sector in the Parnaíba River Basin (m^3/s) 28
Table 3 – Model Parameters 28
Table 4 – Deterministic Models to be Analyzed 30
Table 5 – Steady State Comparison 30
Table 6 – Steady State Shadow Prices 31

SUMMARY

1 INTRODUCTION . . . 11
1.1 Topic and Context . . . 11
1.2 Focus and Scope . . . 13
1.3 Relevance and Importance . . . 16
1.4 Questions and Objectives . . . 19
1.5 Overview of the Structure . . . 20
2 LITERATURE REVIEW . . . 20
3 METHODOLOGY . . . 23
3.1 Data Base And Parameter Selection . . . 27
3.2 Scenarios . . . 28
4 RESULTS . . . 30
5 DISCUSSION . . . 32
6 CONCLUSION . . . 34

BIBLIOGRAPHY . . . 36

APPENDIX . . . 41

APPENDIX A – SOLVING THE MAXIMIZATION PROBLEM . . . 42

APPENDIX B – EQUILIBRIUM CONDITIONS . . . 47

APPENDIX C – STEADY STATE . . . 48

APPENDIX D – PYTHON CODE FOR CALCULATING THE STEADY STATE . . . 50

APPENDIX E – DYNARE MODEL CODE . . . 53

1 INTRODUCTION

1.1 Topic and Context

Attitudes related to climate change have been changing as the impacts on the economy become clearer. Recently, one of the largest asset companies in the world, BlackRock[1] wrote a letter to their clients reporting on their concern that climate risks are also risks for their investments (FINK, 2020). In addition to extreme impacts, such as long droughts or floods, climatic impacts can directly affect the reserve of natural resources. The reduce in the water availability may not attend the needs of crop irrigation due to a increase in evaporation rates, which can lead to a lower production and financial returns below expected (DÖLL; HAUSCHIL, 2002).

In Brazil, global warming can lead to extreme precipitations or extreme droughts, which in turn cause destruction of human, physical and natural capital. Yet, the proportion of people affected by drought are far higher than floods (88,2% people suffering from droughts in years 2013 to 2016) (ANA; MMA, 2015). In Brazil, floods occur mainly in South (Rio Grande do Sul) and Northwest (Amazonas and Acre) regions with higher precipitations and water flows from rivers. In developed countries there is a growing concerning about what investments are being made to prevent those hazards and this is reflected in the decisions of investment banks. Developing countries may be more at risk because lower savings and less infrastructure. Climatic risks are seen by financial markets and investors and this can trigger capital relocation and assets repricing, leading the underdeveloped areas to be forgotten (WOETZEL et al., 2020).

A projection made by Woetzel et al. (2020) depicts three scenarios. The first is the frequencies of droughts today and the next two are the projected droughts for 2030 and 2050 respectively. The Figure 1 shows three images of projected droughts according to Intergovernmental Panel on Climate Change, the darker the area greater is the risk of drought. At the year 2050 there is a prediction of an increase up to 80% of probability of drought in southern Africa and South America meaning that every ten years, eight will be facing droughts conditions. In the year 2015 Southern Africa droughts caused a loss in 15% in the agriculture outputs, and in 2013, the Australian heat wave implied in $6 billion in loss Woetzel et al. (2020). From extreme events (excess or lack of water) in Brazil, as shown by ANA and MMA (2015), from 1991 to 2014 there were 39 thousand natural disasters and 127 million affected people with R$ 9 billion annually economic losses. Furthermore, the most affected people are from extreme droughts events and the main part is from northeast. Through the years the number of cities who declared state of calamity or state of emergency has grown as shown in Figure 4.

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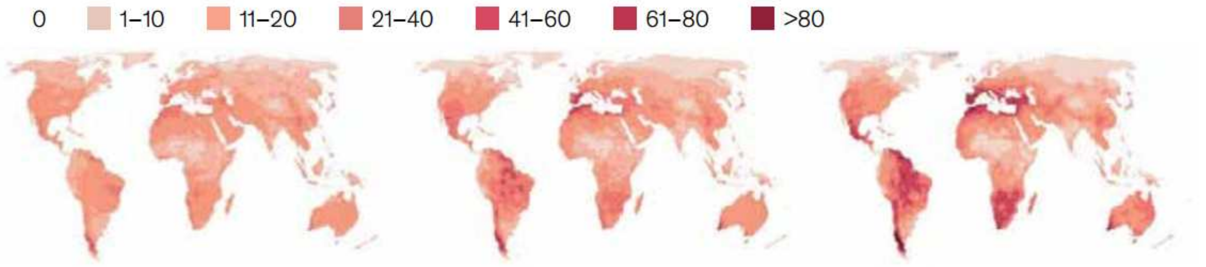
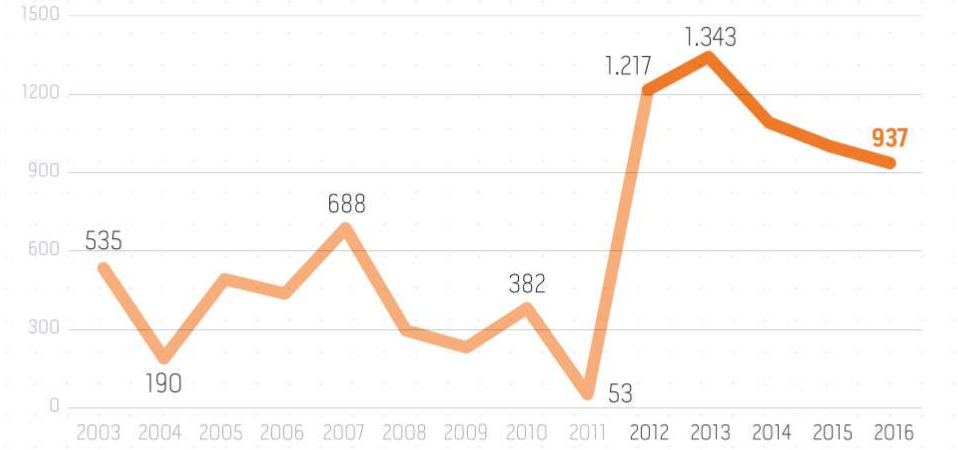
[1] Black Rock is an American global investment management corporation and is the world's largest asset manager. It operates in 30 countries and currently possess assets in the value of US$ 6,288 billion.

Figure 1 – Projected Drought Frequency (today - 2050)



Source : Woetzel et al. (2020)

Figure 2 – Number of cities in state of public calamity or state of emergency



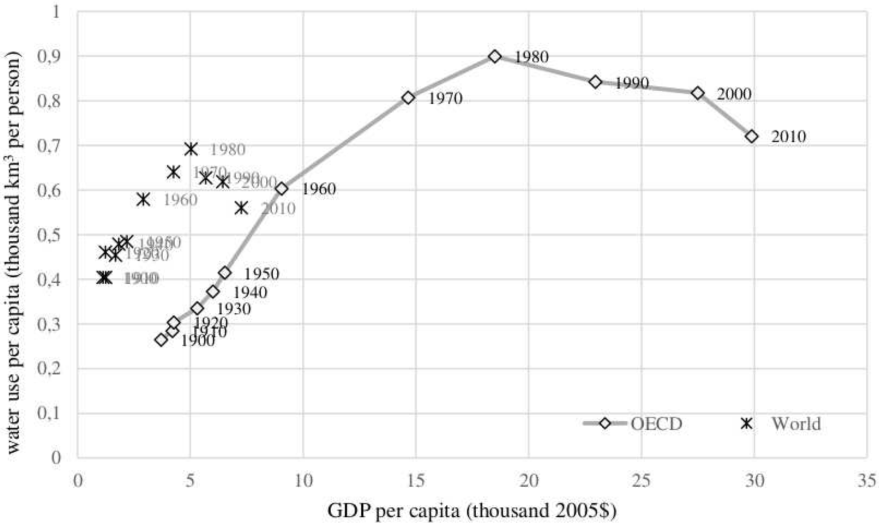
Source : ANA and MMA (2015)

Water is one of the most abundant resources of our planet and it is fundamental for our development and environment, though not everyone accesses it equally. It is estimated that more than 2 billion people around the world do not have access to safely managed drinking water and in the future years, water availability is becoming less predictable (UNITED NATIONS, 2018). For that reason, it is important to manage this resource in order to maintain the same or equal utility (since water is considered an input for production and a good for utility function of agents) for the future generations. When there is abundant free input, economic agents tend to deplete it very quickly if there are no rules to protect it, leading to the tragedy of the commons (OSTROM, 2005). Concerning its economic value, Rogers, Silva and Bhatia (2002) consider water as universally below its full cost which can lead to inefficiencies in water allocation. Now and then, some cities provide water to their inhabitants almost free of charge because water is considered a necessity.

The link between economic growth and water scarcity is shown in some recent articles such as the work of Distefano and Kelly (2017). In this work, the demand for water is related to income and economic growth. The water saved by the technological increase may not be enough to sustain the growing demand. The work of Duarte, Pinilla and

Serrano (2019) reveals growth trends by comparing groups of developed and developing countries and this can be seen in the figure 3. This comparison led to a kuznetz curve for water consumption, revealing that the demand for water grows with income, but after a moment it starts to decrease. Alcamo, Flörke and Märker (2007) concludes that the main cause of the decrease in water stress is the increase in the amount of rain but warns that this analysis is geographically dependent. Which leads us to question how climatically and economically disadvantaged regions will develop in the future.The work of Shiklomanov (2000) predicts that the water withdraw will increase from 10 to 12 percent at every 10 years meaning a increase from observed value of 3,788m^3/year in 1995 to a predicted value of 5,235 km^3/year at 2025.

Figure 3 – Water consumption and GDP

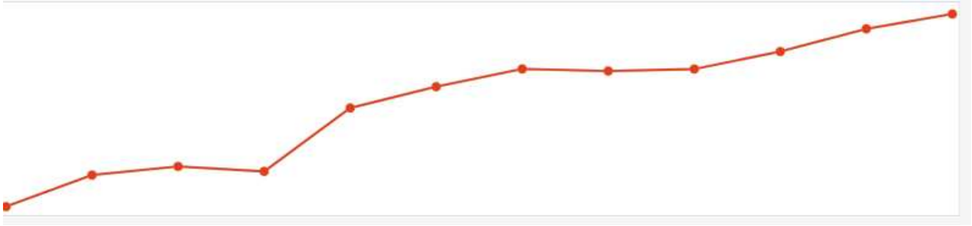


Source : Duarte, Pinilla and Serrano (2019)

1.2 Focus and Scope

Northwestern Brazil has a semi-arid climate, which means this region has seasonal rains and high rates of evapotranspiration, higher temperatures, strong solar incidence, high evapotranspiration rates and low pluviometric indexes. Since 2012, the drastic reduction in rain in the Northeast region it is seen as an anomalie caused by climate change and this causing negative impact on recharge rate of aquifers, rivers and level of water in dams (ANA, MMA, 2015). The region consists the poorer states of Brazil and its main activity is agriculture, a sector with high water usage. There is a considerable rural population on the river basin whose main income sources are from agriculture and government transfers (ANA; MMA, 2015). The demands are met by rainwater, rivers, lakes, ponds and groundwater. These elements are part of the hydrographic basin, which is a geographical area demarcated by topographic divisions limited as areas of land that are drained by a main river, its affluents and sub-affluents (ARAI; PEREIRA; GONÇALVES, 2012).

Figure 4 – Number of cities in state of public calamity or state of emergency



Source : ANA and MMA (2015)

Water as a productive input and a consumption good, can lead to conflicts about its usage. With population growth, the water demand can increase at same time as the water usage in agriculture, and the degradation in water sheds and rives caused by erosion and pesticides can lead to worsen situation(MMA, SRH, 2006a; MONTGOMERY, 2007; WANTZEN; MOL, 2013). Also, the production of energy can be compromised forcing the use of thermoeletrics stations rising up the price of electric energy, and at some points the river transport can be stopped.

Water is a public, vulnerable and economically valuable good and legislation in Brazil searches for its maintenance for future generations. A great challenge of the present day is the internalization of the concept that freshwater is scarce if you are looking in the future in a region subject to desertification. One of the main basins of Northwest of Brazil is the Parnaíba Basin and MMA(Ministry of Environment) considers it almost under desertification (MMA, SRH, 2006a).

The Parnaíba Hydrographic Region is one of the most important in the Northeast of Brazil, and is occupied by the states of Ceará, Piauí and Maranhão, between latitude 02○21'S and 11○06'S and 47○21'W and 39○44'W longitude, occupying an area of 331.441km^2, of which 249,497km^2 in Piauí, 65,492km^2 in Maranhão, 13,690km^2 in Ceará and 2,762km^2 in litigation area between Piauí and Ceará[2]. Its waters traverse different biomes, such as the Cerrado in the Upper Parnaíba, the Caatinga in the Middle and Lower Parnaíba, and the Coastal in the Lower Parnaíba, making the hydrological characteristics of each of these regions different. Most of the State of Piauí (99%) is located in the Parnaíba Basin, and only the city of Luiz Correia is not within the basin. In all, there are 220 municipalities. The main urban centers are Teresina, with more than 655 thousand inhabitants (about 25% of the state population); Parnaiba, with 131 thousand; Picos, with 65 thousand; Piripiri with 60 thousand; and Floriano with 53,000 (MMA, SRH, 2006b).

Agricultural activity is predominant in most of the Parnaíba river basin, represented by 5, and it is an activity with great potential for growth (MMA, SRH, 2006a). This

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[2] It is an area of territoriality that does not have a definition as to whether the municipalities found there belong to Piauí or Ceará (CIDADE VERDE, 2009).

requires the use of large amounts of water to optimize and increase production. Inadequate land management practices already show signs of impact on the region's watercourses, evidenced by the increasing siltation of the Parnaiba, Canindé, Poti and Longá rivers. Agricultural inputs such as fertilizers and agrochemicals can become major polluters of water stored in lakes and reservoirs in the region if there is no control and supervision of its use. There are projects aimed at changing the situation and provide more clean water and sanitation, but the resources allocated to it are low (MMA, SRH, 2006a).

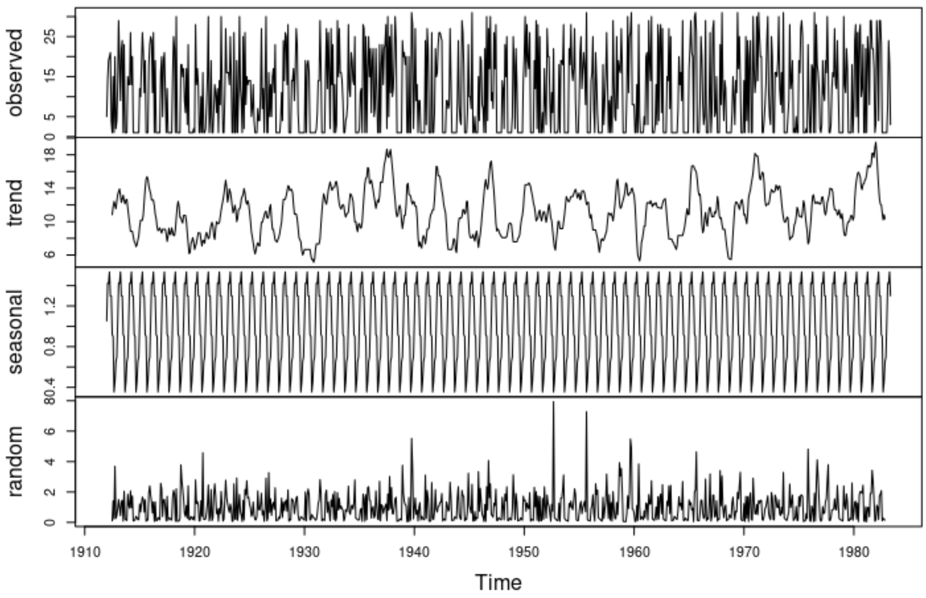
Northeast Brazil is marked by droughts and a strong irregularity of rains as seen in the figure 6. This climatic variability can lead to what the literature calls edaphic drought, which is the insufficiency or irregular distribution of rains, strongly impacting the agricultural production (CAMPOS; STUDART, 2001). Hydrological drought, on the other hand, can be considered as insufficient water in reservoirs for whatever reasons. The figure 7 represents the oscillation of the largest water reservoir in the region in one year.

Figure 5 – Parnaíba Hydrographic Region

Source : MMA, SRH (2006a)

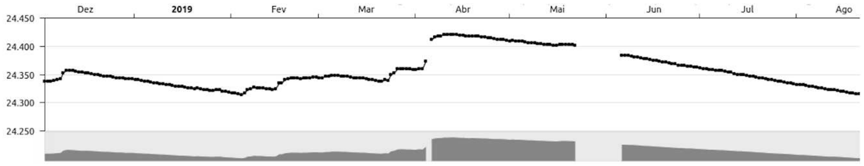
There are plenty of pollutants to water and it's hard to determine the most damaging, instead, some indicators are used to characterize water quality. These include the Basic Oxygen Demand, which indicates the amount of oxygen consumed in biological processes of degradation of organic matter in the aquatic environment. This pollution comes mainly from urban centers through sewage disposal and from factories whose effluents have organic loads. Phosphorus found far from urban environments may indicate improper soil and fertilizer management. Turbidity may be associated with various factors such as industrial pollution and erosive processes in the watershed arising from irregular occupation. A Water quality indicator is formed by analyzing the following factors: water temperature, pH, Dissolved Oxygen, Basic Oxygen Demand, thermotolerant coliforms,

Figure 6 – Historical Rainfall Series



Source : ANA

Figure 7 – Jenipapo Den Historical Series



Source : ANA

total nitrogen, total phosphorus, total solids, and turbidity. Despise of that, the water quality in river basin in most of it has good quality (ANA; MMA, 2015).

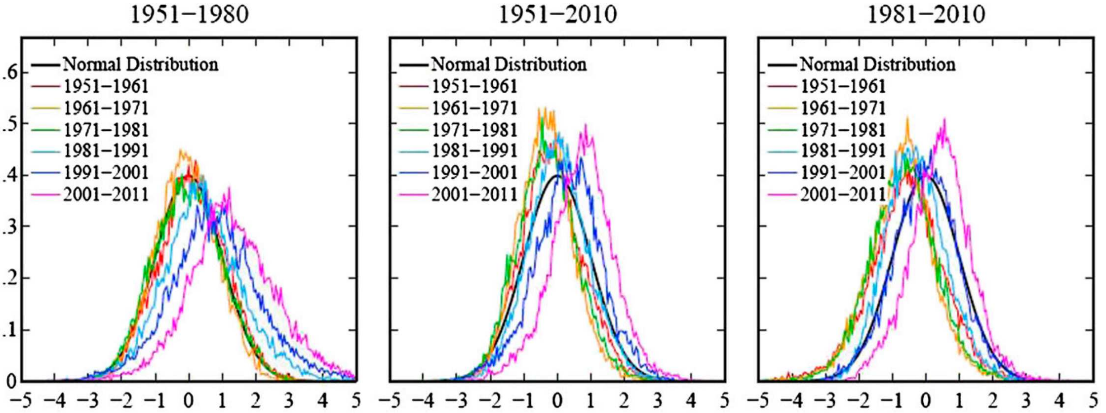
This study investigates the impact of water scarcity on the dynamics of economic growth in the Parnaíba River Basin. The importance of this is to know how water stock will affect the economic growth in a region prone to drought. To this end, a deterministic general equilibrium model made by Li and Swain (2016) will be used. The work will evaluate how a reduction on water availability within a particular region will impact economic variables like GDP, consumption and capital stock.

1.3 Relevance and Importance

It is believed the global warming will continue to increase temperatures on the south hemisphere, which can affect precipitation and water evaporation leading to a reduction in water. In a work made by Hansen, Sato and Ruedy (2012) it is shown the frequently hot summers affect the mean of the temperatures for highers values in the recent years

(1981-2010) depicted in 8. This graph shows the increase in the temperature through the years with anomalies and extreme temperatures for high and low. Each of the three graphs represents a base period in wich standards deviations (x axis) from the global mean temperature (y axis) are represented. In the first graph of the figure the temperature anomaly distribution with standard deviation based on 1951–1980 shows a large deviation from the normal bell curve. The increase in the mean of temperature are most likely to affect the water cycle and increase the chance of extreme droughts[3](HANSEN; SATO; RUEDY, 2012).

Figure 8 – Increase in Average Annual temperature



Source : Hansen, Sato and Ruedy (2012)

According to Ostrom (2005), if a natural resource, such as water, is plentiful, users will not observe a need for their conservation and investment. Only after a shortage, will this resource be valued and the consequences of the price increase may be catastrophic. Despite the semiarid climate and irregular rainfall, the Brazilian northwest has a comfortable water availability (1.700m^3/consumer/year), with the exception of two sub-basins, Parnaíba 06 and Parnaíba 07, both with less than $500m^3/consumer/year$ (MMA, SRH, 2006a). However, the river basin, presents contrasting situations of abundance and water scarcity, which requires governments, users and civil society to take special care in the organization and planning of its use. According to ANA, in 1931 the usage of water corresponded only to a 6,3% of current usage. The total withdraw of water today is 2082.7m^3/s and its is predicted to increase up to 24% until 2030 (ANA, 2019).

In many parts of the world, water remains virtually free. In Brazil currently, water treatment and distribution services are paid. That is, consumers pay the utilities for the service rendered and not for the water itself. The Water Law 9.433/97 (Presidência da República do Brasil, 1997) , grants the rights of use and abstraction of water resources. The current methodology of calculating water tariffs does not consider the water level in the basin, it only accounts for consumption and pollution (ELISABETE, 2015). In the second article of the Water Law is considered an objective of "assuring to the current and

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[3] Drought is considered a large period with little rainfall and dry environment (GIRARDI, 2000).

future generations the necessary availability of water, in quality standards appropriate to the respective uses". The "necessary availability of water" is understood as the water level capable of generating the same utility for this and the next generations in all its uses, consumption and production. This work aims to investigate if this objective will be attained, in other words, if the welfare can be sustained at the same or higher levels in the long run.

The purpose of introducing natural resources into economic models is to define a better allocation of resources that the market cannot reach. One reason that there are no markets for natural goods is the high costs of trading and monitoring. In some situations, negotiation is prevented because the future generation does not exist at the same time as the present generation to agree on prices for the natural good (DASGUPTA et al., 2001). In most Brazilian states, especially in the northeast, water as a common natural resource plays a key role in the growth and development of the region (MMA, SRH, 2006a). In these states, agricultural activity is the main economic activity and a reduction in this resource combined with misuse can have serious consequences for the local economy.

Normally water isn't consumed fully. Most of the urban and industrial effluents go back to the river basin, decreasing water quality, implying that the cost of reusing this water is higher. But, most studies assume no effluents. To increase the water rates, new investments or new technologies must be implemented. Some accessible technologies are restricted to places, like water desalination in Florida (US), which has low seawater salinity. Other water sources like the recycling of sewage water, deep aquifer development, and long water transfers are capital intensive and energy intensive (ROGERS; SILVA; BHATIA, 2002).

The work will focus on a deterministic growth model with water being a state variable. Its basis is the classic model of Ramsey (1927) in which economic aggregates are determined by decisions at the microeconomic level of the agents that will maximize their intertemporal utility. In this sense, the work differs from the current literature that uses multisectoral input-output models for an analysis of water demands (DISTEFANO; KELLY, 2017; ANTONELLI; ROSON; SARTORI, 2012). These latest works focus on the amount of water involved in production and how the exchange of these produced goods affects the demands and quantities of water between regions. This work, however, has a regional focus based on self-sufficiency issues.

We hoped the study can help the government to take some actions towards to reach a sustainable level of water consumption. Revenue of water can be used not only for the sustainability of the resource but in Water Sanitation Services, which rise the life expectancy. An increase in health leads to less missing days of work and a rise in a lifetime (ZON; MUYSKEN, 2001). In 2008 more than 300 thousand workers missed work and 700 thousand pupils missed school due to a problem related to water: diarrhea (IBGE, 2013). Hutton and WHO (2012) claim that global return on water spending is US$2.0 per US dollar invested, but previous studies on this subject had problems "addressing" local issues such as varying population densities. The welfare increment can be seen as the improvement in WSS (Water and Sanitation Service) and maintenance of these services as well as the maintenance of water level in the river basin. Regulations and incentives related to the irrigation and pesticides can achieve positive adaptation and mitigation outcomes.

1.4 Questions and Objectives

From the development of water scenarios for 2050, a global drop in freshwater supply is expected to have a major effect on Africa and parts of South America, particularly in the northeast of Brazil(ARNELL, 1999). Given rising water consumption, worsening climate change and watershed recharge, is well-being sustainable in the Parnaíba River Basin? A study by Li and Swain (2016) investigate these effects on Africa, a region similar to the northeast of Brazil. By applying and adapting the dynamic stochastic general equilibrium made by then, this work will analyze the dynamic welfare in the Parnaíba River basin.

The main hypothesis of this work is that when water supply is reduced, economic growth will be decreased. With less water availability some cultures like beans, maize and rice are less productive. Döll and Hauschil (2002) in a study of the expanse of irrigated areas on Brazil, predicted that there will not be sufficient water for irrigation in dry years. In one study by Borghetti et al. (2017), it is shown a great number of areas are suitable for irrigated agriculture, but the challenge lies in the dispute about domestic use or irrigation. The study suggests a prioritization of more profitable cultures and improvement in technology and modern methods of sustainable agriculture. One argument to implant agriculture in one region is the water availability. Because of that reasons with the increasing demand for water, population growth, adverse weather conditions, and discount rate, the welfare of the Parnaíba River basin region may be not sustainable in the future.

The intention of this work is to analyze the role of water in the economic growth in the Parnaíba River Basin. For that purpose, the model presented by Li and Swain (2016) will be used to model the economy of the Parnaíba River Basin. First, the scenario for the year 2010 will be simulated in the model and the path of the variables will be taken as a benchmark for comparison with simulations with less available water. This scenario already occurs and the values represented are the observed average values that are already effects of climate change (MMA, SRH, 2006a). The second scenario will have a small reduction in water availability and a third scenario will simulate a situation of extreme drought. The fourth scenario will simulate the change in the preferences of the agents increasing present consumption to the detriment of future consumption.

The second scenario simulate the predicted reduction of 10% in the annual runoff by Arnell (1999). Extreme droughts depicted in the third scenario are characterized as a monthly runoff below the tenth percentile as in the work of Espinoza et al. (2011). This value varies throughout the year and the average runoff in drought was used as in reports by MMA, SRH (2006a) which is about 38,5% of the regular water inflow. This low amount of available water already occurs in the driest months of the year, but in the case of extreme events this situation can persist for more than a year. According to Marengo (2010), these prolonged droughts have occurred about 12 times since 1710 and in less aggravating ways in the years 2003 and 2004 for the northeast of Brazil.

According to Dasgupta et al. (2001) it is common to have a sensitivity analysis with the discounting factor varying between 4 and 10 per cent. A study by Fraiture et al. (2007) states that the demand for food will continue to increase even with reduced population growth. In the same way, the competition between the use of water for food production and other sectors will intensify, since greater urbanization will demand more water for families and industrial sectors, which should grow by a factor of 2.2 by 2050. And so the unfavorable climate change will put further pressure on water resource management. This sensitivity analysis is made in the fourth scenario where the discounting factor decreases.

1.5 Overview of the Structure

In addition to this introduction, the work consists of five more chapters. In chapter 2 there will be a brief review of the literature commenting on recent work on the topic. In the third chapter, it will be commented on the methodology used in the work, as well as details of the simulated scenarios and data used. The following chapters will present the results of the research, its discussion and conclusion.

2 LITERATURE REVIEW

A recurrent concept in the current literature to investigate the scarcity of water in an economy is calculating its shadow price. Ziolkowska (2015) defines the shadow price of water in agriculture as "the ratio between the production net returns and the total amount of water used for irrigating." As the net returns increase, the shadow value increases, and this is the same when available water decreases. To have an effective economic policy, the marginal costs of using water must be equal to its overall social benefits.

In one paper, Do (2019) synthesize some studies on game theory to address local water issues. The main findings are about that the predicted agent behavior is not always harmonious. When the game is not cooperative, some results suggest that higher monitoring and sanctioning are required to implement water allocation, and overall coordinated management is better.

A number of econometric studies estimate the price elasticity of water demand, for instance the study by Bailey and Buckley (2005). Analyzing water tariffs, they utilize Ramsey principle of taxation[4] and find that this increase in tariffs can provide sufficient revenue for water services providers and conservation of water resources. Another study by Baerenklau and Pérez-Urdiales (2018) investigates a system of increasing block rates[5]for the state of California. The results show that increasing tarrifs by income block tends to be welfare-preferred than uniform price for water conspumption.

In the CGE category, there are contributions like one made by Juana, Strzepek and Kirsten (2010), which analyses, using a computable general equilibrium model utilizing a social accounting matrix, the reallocation of water from agricultural sector to industrial sector, to see the gains in GDP. The main findings are that reallocation can increase industrial output by the cost of loss in agricultural profit, but beyond the market scope, the allocation can decrease the welfare of the poorest families in the Africa economy, who are dependent on agriculture. A similar study by Seung et al. (2000) inquires about the reallocation water from the agricultural sector to recreational use. They conclude that the gains from the recreational sector do not compensate the loss in the agricultural sector.

In one particular article, Ferrarini et al. (2019) analyzes an expansion in irrigated agriculture using a particular computable general equilibrium model, TERM-BR, which is "an interregional, bottom-up, dynamic computable general equilibrium model (CGE) of Brazil, based on the theoretical structure of the Australian TERM[6] model, calibrated for 2005". Focusing on the Northeast of Brazil, the results show the three states Bahia, Piauí and Maranhão have a lot of potential for expansion in the agricultural sector, thus increasing its GDP. However, because of the increasing water demand, ay not be consistent with water supplies. In its simulation of the water levels, the Parnaíba River Basin showed a considerable difference when compared with data from the National Water Agency (ANA). They conclude that the study does not include climate change effects, and highlight the importance of analysing water demand at the basin level instead of an aggregate perspective.

The decrease in natural resources may occur due to population growth factors combined with industrial and income growth. The work of Vörösmarty et al. (2000) made a connection between water demand and supply and its passage through river networks.

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[4] Optimal taxes are inversely related to the compensated elasticity of demand and supply : $t/p = 1/elasticity\ of\ demand + 1/elasticity\ of\ supply$ (STIGLITZ, 2015).

[5] Under a block tariff scheme, users pay different amounts for different consumption levels, in this work they used three blocks.

[6] TERM (The Enormous Regional Model) is a "bottom-up" CGE model which treats each region as a separate economy. It has represented 182 sectors in 205 statistical sub-divisions (WITTWER, 2012).

In particular, they define how the typology of rivers define the sustainable supply of water and its uses. The flow of the river is the sustainable supply of water, thus, the main finding of the article is that Relative Water Demand (RWD)[7] is negatively affected by population growth and economic growth. To do a water study, it is necessary to have some estimates of population growth, capital, and water use in addition to how the water supply will be in the future. That said, the model can simulate these factors by its formulas.

Due to the growth of international trade in commodities whose production is intensive in water, freshwater is becoming a global resource. The water footprint concept developed by Hoekstra et al. (2011) takes into account all the water used in the production chain in addition to extracting water from its source. The concept of water footprint (WF) is divided into three and is related to its origin [8] serving as a tool for analyzing how human activities are related to water scarcity and pollution.

A work by Konar et al. (2016) takes into account the WF methodology and investigates human adaptation to climate change with a focus on basic plantations. Through two models, one being Global Trade Analysis Project (GTAP) and the other a global hydrological model (H08), a static analysis is carried out through four scenarios: one is the standard of analysis, benchmark; the second scenario, such as the tariff release for the plantation trade; the third with the climatic impact; and finally, the fourth scenario with tariff release and climatic impacts. The general results indicate that a tariff release reduces water stress even taking into account climatic worsening, however, there is still a need for more focused studies due to the heterogeneity of countries and within countries.

Another work that uses the concept of water footprint is that of Mekonnen and Hoekstra (2016) and it concludes that the reduction of consumption by hydrographic basin combined with the increase in the efficiency of water use and a better sharing of freshwater resources can reduce the threat of water scarcity for biodiversity and well-being. The work mentioned above also took into account spatial and temporal variations and how they affect the demand and availability of water for production on a global scale. This is interesting because there are months of greater demand for water due to the climate, and shows how the demand for water from nearby places is spatially dependent. The study indicates that the greatest water scarcity is related to irrigated agriculture and a higher population density. Since the water consumption of a river is accelerated at certain times of the year, its flow may be interrupted before reaching the end, thus leading to the disappearance of lakes in other regions, the authors exemplify the Lake Chad in Africa.

In the work of Murray, Foster and Prentice (2012) a global dynamic model that takes vegetation into account includes the biospheric character related to water projections.

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[7] Is defined as the ratio between water use and discharge.

[8] Rainwater is said to be "green", surface and subsoil water are "blue" and the water needed to balance pollutants of rivers due to pollution is "ash", (HOEKSTRA et al., 2011).

In their scenarios, an increase in temperature can lead to an increase in precipitation, which would contribute to reducing water stress in some regions. However, vegetation is a limiting factor in the biological question, since vegetation is subject to decreasing returns from water use. Although their results point to an increase in runoff (given an increase in temperature of 2 degrees Celsius) with a possible cause as the decrease in fractional vegetation coverage [9], climate change combined with population growth is still the biggest factor of water stress.

Using structural decomposition analysis (SDA) together with network theory, Distefano, Riccaboni and Marin (2018) points to a tradeoff between systemic vulnerability and increased virtual water trade. Structural decomposition analysis is based on the input-output tables (IO) and through these tables, it is possible to know the direct and indirect uses of water in a production chain. Network Theory, on the other hand, allows the analysis of bilateral flows and interdependence between nodes.

Li and Swain (2016) developed a dynamic stochastic general equilibrium (DSGE) for analyzing how water resilience affects economic growth and dynamic welfare in South Africa. They use rainfall variability as a stochastic disturbance affecting the groundwater recharge and surface water flow. What makes the work different from others, is the fact it incorporates water stocks, being the first work of this kind. They first analyze the effects of discount rates and precipitation using a deterministic setup. The main finding is that with sufficient capital accumulation, development may be sustainable, compensating for future water shortages. When stochastic perturbations included, they find that the disturbance decreases the intertemporal welfare, but when there is positive correlation between disturbances, individuals are better off than in the case with pure randomness.

3 METHODOLOGY

Model developed by Li and Swain (2016) will be used as a basis for analysing the sustainability of the well-being of the region supplied by the Parnaíba River Basin. We will adapt the model to take into account" instead of "For this purpose, an adaptation will be made taking into account the local population, the capital stock and water level. After determining the model parameters - such as population growth rate, relative labor, capital and water share in the production function, and the discount rate - the deterministic model will be solved numerically using the Dynare software (ADJEMIAN et al., 2011), which is executed using a computational language for numerical calculations called Octave (EATON et al., 2017). With the deterministic model, one can modify the parameters variables and simulate new steady states, in which the development of the well-being of

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[9] Fractional vegetation cover is the "ratio of the vertical projection area of above ground to the total vegetation area." And it is used as an parameter for environmental management and shows the interaction between hydrosphere and biosphere (LIANG; WANG, 2019, p. 478).

the population will be analyzed. After incorporating the climate disturbance that will change aquifer recharge, one can see how economic variables will adjust to it and in which scenarios the welfare will be sustainable.

The DSGE model has three stock variables: population, capital stock, and water stock. Originally, Li and Swain (2016) used groundwater stock but since Parnaíba River Basin has more superficial water proportional to groundwater than South Africa, the study takes into count the total water reserves, with groundwater included. The N_t variable will be the population size, K_t will be the physical capital, and X_t will be the water level over the period ranging from $t = 0, 1, 2, 3, ...\infty$. A Cobb-Douglas production function is considered for the economy:

$$Y_t = AK_t^{\gamma_1} N_t^{\gamma_2} W_t^{\gamma_3}, \tag{1}$$

where A is the total factor productivity, W_t the total freshwater for production and $\gamma_1, \gamma_2, \gamma_3$ are positive parameters. Technological progress is not explicitly modeled, and the population and workers are not differentiated.

There are two water sources in the economy: the extracted groundwater Z_t, and extracted surface water S_t. Both can be consumed by households (H_t) or used in production (W_t). If total water extraction is $S_t + Z_t$, then the water use in the production is : $W_t = S_t + Z_t - H_t$. Surface water depends on how climate-related variables behave. However, groundwater is stored and is less subject to climate changes since it doesn't have losses from evaporation. The water stock will be X_t and the next period water stock is represented by X_{t+1}. There is a natural recharge rate m_t that fills the aquifer, and the ground water extraction rate Z_t affects X_{t+1}.

$$\begin{aligned} K_{t+1} &= Y_t + (1-\delta)K_t - C_t - \phi(X_t)S_t, \\ N_{t+1} &= b_1 N_t - b_2 N_t^{;} \\ X_{t+1} &= X_t - Z_t + m_t + \varepsilon_t, \end{aligned} \tag{2}$$

where $\delta \in (0, 1)$ is the capital depreciation rate, C_t is the total consumption, K_t is the capital stock available for use in production in period t, and K_{t+1} the capital stock in the next period. In the next period the capital will be the depreciated capital plus the investment. The investment is : $I = Y_t - C_t - \phi(X_t)S_t$, where $\phi(X_t)S_t$ is the payment for surfice water extracted. The second equation of (2) represent the assymptotic function of population dynamics, where N_t denotes the population size in period t and N_{t+1} that in period $t + 1$. The parameter b_1 is the intrinsic growth rate with $(b_1 - 1/b_2)$ as the asymptotic population size[10]. The last equation represents the dynamics of the water

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[10] For this model a population size remains constant in the long run therefore, when $t \to \infty$ the population

stock, where X_t is the water stock in period t and X_{t+1} is the water stock in the next period. Water recovers with the ${m_t}^{11}$, and is subject to a stochastic disturbance ε_t. Society derives instantaneous utility $U(C_t, H_t, N_t)$ from consumption of C_t and household water consumption H_t:

$$U(C_t, H_t, N_t) = N_t \left[\ \ln(\tfrac{C_t}{N_t}) + \alpha \frac{(\frac{H_t}{N_t})^{1-\theta}}{1-\theta} \ \right]^{12}, \tag{3}$$

which is the aggregate utility of individuals. The α parameter is the relative weight of the welfare to utility of residential water services. This means the higher the parameter the higher is the utility from water servicies. The parameter θ denotes consumer risk aversion, and the smaller is this parameter, the more families tend to shift the consumption from t to $t + 1$ and the ratio of consumption will depend only on relative prices rather than income level since income elasticities are equal to one (ROMER, 2012). Just like the basic model presented by Torres (2016), 3 this is a constant relative risk aversion (CRRA) function. The logaritmic part of (3) is a special case of CRRA when $\theta \to 1$ [13]. Because of its homothetic properties this kind of formulation is interesting because allows application to any size of economy (SIMON; BLUME et al., 1994). The objective of the households is to maximize the discounted sum of instantaneous utilities:

$$V_0 = maxE_0 \sum_{t=0}^{\infty} \beta U(C_t, H_t, N_t), \tag{4}$$

where $\beta \in (0, 1)$ is the discount factor, and E_0 is the expectations operator conditional on the information available at the initial time period. The maximization problem can be rewritten as:

$$\begin{aligned} V(K, N, X) &= \max_{C,Z,H} U(C, H, N) + \beta \mathbb{E}[V(K', N', X')] \\ \text{s.t.} \quad K' &= Y + (1 - \delta)K - C - \phi(X)Z \\ X' &= X - Z + m + \varepsilon \\ W &= S + Z - H \\ Y &= AK^{\gamma_1} N^{\gamma_2} W^{\gamma_3} \\ U(C, H, N) &= N \left[\ ln\left(\tfrac{C}{N}\right) + \alpha \tfrac{(H/N)^{1-\theta}}{1-\theta} \ \right], \end{aligned} \tag{5}$$

N_{t+1} will be a constant. Another way to avoid populations change is consider the well-being per capita and sum this result for all generation using a discount rate (DASGUPTA et al., 2001).

[11] New inflows of water in the basin each new period.

[12] The utility function can be a combination of a logaritmic funcition and and CRRA(Constant Relative Risk Aversion) (TORRES, 2016).

[13] If one take the limit of this utility function with $\theta \to 1$ then apply the l'Hopital rule the result will be lnc_t.

here, and in the next set of equations, the variables with no apostrophe refers to the current period and with apostrophe refers to the following period. The first-order conditions of this maximization problem are:

$$\begin{aligned}
\left(\frac{C}{N}\right)^{-1} - \beta\mathbb{E}\left[-\frac{\partial V(K', N', X')}{\partial K'}\right] &= 0 \\
\beta\mathbb{E}\left[\frac{\partial V(K', N', X')}{\partial K'}\left(\gamma_3\frac{Y}{W} - \phi(X)\right) - \frac{\partial V(K', N', X')}{\partial X'}\right] &= 0 \\
\alpha\left(\frac{H}{N}\right)^{-\theta} + \beta\mathbb{E}\left[\frac{\partial V(K', N', X')}{\partial K'}\left(-\gamma_3\frac{Y}{W}\right)\right] &= 0
\end{aligned} \tag{6}$$

The first FOC represents that the marginal utility of consumption should be equal to the present value of utility if an additional unit of capital would be invested. The second FOC is interpreted as the marginal productivity of water in utility units, which should, in the optimum, be equal to the expected discounted shadow price of water in present value. And finally, the third FOC shows the equality between the marginal utility of water for residential consumption and the utility from from the industrial use.

In the steady state the variables in the next period are the same as the current period. Because there is no next period the expectations can be removed. After replace all current and next-period variable with their steady-state, in the following six steady state are found:

$$\begin{aligned}
\alpha\left(\frac{\overline{H}}{\overline{N}}\right)^{-\theta} &= \left(\frac{\overline{C}}{\overline{N}}\right)^{-1}\left(\gamma_3\frac{\overline{Y}}{\overline{W}}\right) \\
\overline{K} &= \frac{\beta\gamma_1}{1-\beta(1-\delta)}\overline{Y} \\
(\beta - 1)\left(\gamma_3\frac{\overline{Y}}{\overline{W}} - \phi(\overline{X})\right) &= \beta\frac{\partial\phi(\overline{X})}{\partial\overline{X}}\overline{Z} \\
\overline{Y} - \delta\overline{K} - \overline{C} - \phi(\overline{X})\overline{Z} &= 0 \\
\overline{W} &= S + \overline{Z} - \overline{H} \\
\overline{Y} &= A\overline{K}^{\gamma_1}\overline{N}^{\gamma_2}\overline{W}^{\gamma_3}
\end{aligned} \tag{7}$$

The model represents a Walrasian economy meaning the markets are competitive, complete, free of externalities, and with complete information (ROMER, 2012). Conceptually, the price of water should be charged with an externality to reflect the price of removal from the environment. But the model focuses only on its impact on growth and how the decrease in water availability can impact economic growth. Thus, the external cost of water is not taken into account.. Because of this one, can study how changes in restrictions regarding state variables affects the overall conditions of an economy. This could give us an idea how a natural resource can affect the welfare of the households. With less natural resources, the utility will decrease because the productive sector will produce less and the households will have less access to water. The model may be simplistic but the lack of others perturbations help to focus in one problem of water. In this way one can investigate how the drop on water supply reduces the economic growing.

3.1 Data Base And Parameter Selection

The river basin is divided between three estates, $4,5\%$ belongs to Ceará, Maranhão has $19,8\%$ of it, and $75,7\%$ is under Piauí[14]. The parameters for asymptotic population growth were found by a logistic regression for population growth in $t+1$ and t, with no constant from IPEA (Research Institute Applied Economics[15]) data. Also from IPEA, the production (Y) and population (N) for the three estates in the year 2010 were found. For the capital and consumption (C and K), the work of Cavalcanti and Vereda (2014) was used as a basis for model calibration. In that work, the authors create a DSGE model for analyzing the public spending multiplier calibrated for Brazilian economy. In their calibration, in the steady state, they find the ratio between consumption and GDP to be $((\overline{C}+\overline{G})/\overline{Y} = 0,82)$ and capital and GDP to be $(\overline{K}/\overline{Y} = 2.28)$.The study weighted the variables for each state by the participation in the river basin, and are resumed in Table (1).

Table 1 – GDP (R$), Capital Stock (R$), Consumption (thousands R$) and Population for state for year 2010.

	GDP	Capital	Consumption	Population
PI	16,699,542	38,074,956	13,693,624	2,416,831
CE	3,503,944	7,988,992	2,873,234	388,069
MA	8,960,677	20,430,343	7,347,755	1,328,771
Total	29,164,162	66,494,290	23,914,613	4,133,671

Source : IBGE (2019)

The demands for water are shown in Table (2). More than half of cities are supplied by underground water in Brazil (FERREIRA et al., 2007). In Piauí and Maranhão these values are about 75% for household usage, and this proportion was applied to the water demand (ZOBY; MATOS, 2002). The recharge rate m_t is considered to be 20% of total underground water level, X_t. This parameter is taken from Li and Swain (2016) and we

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[14] There is a litigation area between Piauí and Ceará corresponding to $0,8\%$ of the river basin. It was divided equally $0,5\%$ for it for the analysis.

[15] *Instituto de Pesquisa Economica Aplicada.*

believe that it is well suited since both countries, South Africa and Brazil, have the same bioma[16], savannah[17] (COUTINHO, 2006).

Table 2 – Water Demand by Sector in the Parnaíba River Basin (m^3/s)

Urban	6.695	Household
Rural	1.387	
Animal	2.673	Productive Sector
Industrial	0.638	
Irrigation	9.225	
Total	21.617	

Source : MMA, SRH (2006a)

The costs of the water extraction will be estimated using the price of water use, a and d, provided by Law Decree Nº32.159 from Ceará state. The base cost a will be a mean of all eight sectors included in the law. In face of a public calamity or emergency situation[18] (such as extended droughts)it is assumed the cost d will be four times greater than the normal cost a. The reaming parameters are taken from standard DSGE models from literature (TORRES, 2016). The model parameters are summarized in Table 3.

Table 3 – Model Parameters

W_0	N_0	Z_0	C_0	X_0	H_0
12.535 (m^3/s)	4.133 (mil)	8.066 (m^3/s)	23.914 (bi R$)	135.1 (m^3/s)	8.082 (m^3/s)
Y_0	K_0	b_1	b_2	α	β
29.164 (bi R$)	66.494 (bi R$)	1.00439	0.00127	0.0765	0.98
δ	γ_1	γ_2	γ_3	θ	m
0.05	0.41	0.45	0.14	0.5	27.0200
a	d	r	S	A	σ
0.3375	1.35	0.1	12.55 (m^3/s)	2.1	0.1

Source : research results

3.2 Scenarios

To simulate the scenarios, the very first thing to do is to develop a benchmark for comparison, and this is made by solving the model with help of a software plataform,

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[16] A uniform environment area belonging sharing similar ambiental characteristics like climate, fauna, ground, which is defined according to the climatic zone (COUTINHO, 2006).

[17] Grassland containg isolated trees. A gradient between grasslands and tropical forest (DYKSTERHUIS, 1957).

[18] The Emergency Situation or State of Public Calamity depends on legal recognition by the government of abnormal situations caused by one or more disasters. The Emergency Situation is when the damage are bearable and overcome by the community. State of Public Calamity is when an event cause serious damage, puting the lives on danger and preventing the movement of people and goods (ANA; MMA, 2015).

Dynare (ADJEMIAN et al., 2011). In this scenario, there is no climate change, and one can observe the equilibrium path of economic variables and understand their behavior. The second scenario simulates a light climatic effect. It is expected, by the year 2050 in the Amazon River Basin , a decrease in runoff by 40% (compared to 1990) , about 20% increase in evaporation, and a decrease in precipitation off about 10% (ARNELL, 1999). In the Parnaíba River Basin, however, there is already a high evaporation rate (94% MMA, SRH (2006a)), and light climatic change will be considered as a decrease in 10% in recharge rate and water extracted.

The average annual flow of the Parnaíba River Basin is 763m^3/s and when there is drought this value drops in average to 294m^3/s, meaning a 38,5% loss in the water flows. The third scenario simulates a persistent drought period[19] which has a probability of occurrence about 15% to 33% (ANA; MMA, 2015). And the fourth scenario depicts the same as the third scenario but with a heavier discounting rate. Because the climate is change not only the available water will change, but it is expected the demand for water increases. For example, a rise in temperature also increases the consumption of water by people, animals, and plants. It means the water shortage could increase other conflicts about its usage.

The increase in deforestation in some riverside areas for rice planting can lead to a reduction in water withdraw or hamper fishing activity. When households appreciate more the present than the future, their rate for pure time preference increases, which reduces the discount factor [20]. Over the years the conflicts about water usage increased on Brazil mainly in Northwest region. Those conflicts are related with the constructions of dams and unequal distribution of water(PEREIRA; CUELLAR, 2015). To simulate the concerning about the present,a reduction in β from 0.98 to 0.95 will be simulated in scenario 4.

The table (4) summarizes the differences between the simulated scenarios. The difference between the first three scenarios was the aquifer recharge rate and the amount of water available on the surface for consumption and production. In the fourth scenario, in addition to the decrease in these rates, there is a reduction in the discount rate for families, symbolizing a greater preference for consumption of goods today.

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[19] Campos and Studart (2001) points out the time period of a drought can last for years before water availability back to normal rates. But in a large time interval can occour serveral droughts with differents levels of intensity. In the interval of 2012 to 2014, in Brazilian semi-arid, an extremly crital drought occurred with probability of occour again in 100 years ANA, MMA (2015).

[20] Discount factor is defined by $\beta = \frac{1}{1+r}$ where r is the pure time preference value (TORRES, 2016).

Table 4 – Deterministic Models to be Analyzed

scenario 1	Light Discounting	$\beta = 0.98$
	Without Climate Change Effect	$m = 27.02$, $S = 12.55$
scenario 2	Light Discounting	$\beta = 0.98$
	With Light Climate Change Effect	$m = 24.318$, $S = 11.295$
scenario 3	Light Discounting	$\beta = 0.98$
	With Heavy Climate Change Effect	$m = 16.60$, $S = 7.71$
scenario 4	Heavy Discounting	$\beta = 0.95$
	With Heavy Climate Change Effect	$m = 16.60$, $S = 7.71$

Source : Research Results

4 RESULTS

Four scenarios were simulated with different amounts of water available to the population and the productive sector. The deterministic model allowed to find the steady state variables summarized in the Table 5. Scenario 1 is the benchmark for comparison as it represents the current economy without major changes.

Table 5 – Steady State Comparison

Variables	**Benchmark**	**LCC**	**HCC**	**HD**
Consumption (mil R$)	36.40	36.44	36.04	30.17
Water Reserves (m^3/s)	101.79	88.12	70.54	73.43
Household water use (m^3/s)	15.52	13.60	8.23	9.03
GDP (mil R$)	65.38	64.03	59.43	45.19
Capital Stock (mil R$)	380.75	372.86	346.10	180.55

LCC = Light Climatic Change; HCC = Heavy Climatic Change; HD = Heavy Discounting.

Source – Research Results.

The simulations show a decrease in steady state GDP, consumption and capital as as water availability decreases. The same trend is seen in water reserves, except for in scenario with heavy discounting, in which it is slightly higher than in scenario 3. The level of water reserves in the steady state in the benchmark scenario is 101.79m^3/s meaning a difference of 33.31m^3/s below from the value in 2010 (135,1m^3/s). On the other hand, there is a very large increase in capital stock compared to the initial year, an increase of almost six times. However, the same is not observed for the variables of GDP, consumption and water used in production. Water consumption by families increases substantially. In terms of *per capita* this value is greater since the asymptotic population is less than the initial population.

Scenario with light climate change effects is similar to benchmark in the dynamics of economic variables. A drop of 10% in water recharge rate from groundwater stock, and a reduction of the same value for surface water, decreased water reserves and residential water consumption in 13.42% and 12.37%, respectively.In scenario with heavy climate change, the decrease in GDP, consumption and capital are smaller than 10 % in comparison to benchmark. However, when household water consumption is considered,its steady state value decrease of around 50%. The values for capital stock and GDP in scenario with heavy discounting are very low compared to the others scenarios,although the household water use is similar to scenario with heavy climate change. The water reserve, however, is higher than scenario with heavy climate change. This is because lower production uses less water and thus, water reserves are slightly increased in the steady state for this scenario. The impact of the decrease in the discount factor resulted significant changes when comparing scenario 3 with scenario 4, mainly in GDP and capital stock. The change in consumer behavior towards a greater appreciation of the present in relation to the future implied lower capital accumulation and a greater use of water for households in scenario 4. Figure 9 shows the equilibrium path of main variables of the scenarios simulated.

The results are therefore sensitive to the change in the rate of impatience as simulated in scenario 4. Attempting to forecast certain scenarios can lead companies and governments to plan better in the face of resource constraints. Also, the result of this work showed that under certain conditions (scenario 04) there may be a worse situation for future generations. One way to handle less water availability is to increase productivity and in conjunction with inspection, soil salinization can be avoided. Failure to overuse the resource beyond its limits can guarantee the supply of future generations.

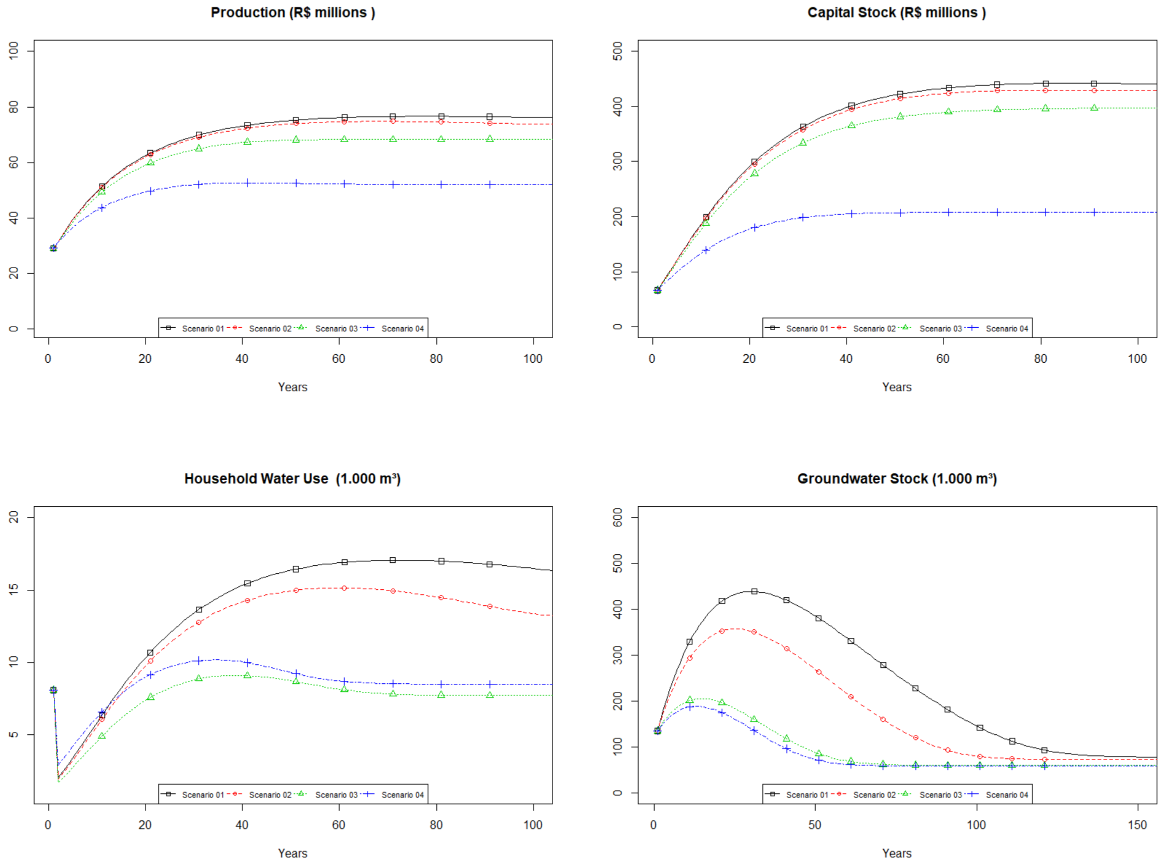
Figure 9 shows the similarity between scenarios, and on which variable the climate change has more impact. Table 6, we show the shadow prices for each scenario. The less water in aquifer, the higher is the shadow price of underground water, and the higher is the GDP, the higher is the shadow price for extracted water. The shadow price of water extracted reflect the gain per unit of water in the utility of households that could come from consumption of water itself or incorporated in productions. The shadow price of water stock correspond to a gain in social welfare from an increase in the stock in the scenario, representing a resilience stock (WALKER et al., 2010).

Table 6 – Steady State Shadow Prices

Shadow Prices	**Scenario 1**	**Scenario 2**	**Scenario 3**	**Scenario 4**
Water Stock	0.01	0.03	0.14	0.04
Extracted Water	1.71	1.60	1.23	1.08

Source – Research Results.

Figure 9 – Equilibrium Path of Main Variables



Source - Research results.

5 DISCUSSION

The results show that climate change may have a direct impact mainly on household water consumption and on the water stock. When a permanent reduction in the water supply is simulated, the effect is a sharp reduction in the steady state values of these variables. However, the steady states of scenarios 3 and 4 showed only a small change compared to the benchmark consumption values. Comparing scenario 2 to the benchmark case, an unexpected result is the small increase in household consumption. This is because the increase in the price of water does not occur in a linear manner. The decrease in water availability was not enough to increase the price in a significant quantity. The amount of water extracted from the underground, in steady state, is equivalent to the amount of water from the aquifer recharge for all scenarios. This is important because an extraction rate higher than recharge rate could deplete all the water reserves.As some studies show that there is a water price elasticity(BAILEY; BUCKLEY, 2005; JUANA et al., 2008), tariffs can be raised according to income to promote greater revenue for water distribution infrastructure works.

The results are consistent with the conclusions of Ziolkowska (2015), where the shadow prices of extracted water for agricultural activities in Texas fell, due to a extreme drought in 2011. The calculation of the shadow value of extracted water, following the strategy of Li and Swain (2016), depends not only on the GDP but on the water consumed in the households, and for that reason the drop in this price can be higher than in studies that focus only in the net returns of economic activities. Along the lines of the hypothesis of Mäler and Li (2010), the shadow price of underground water can be seen as a resilience value, meaning the increase in one unit of water results in an increase in welfare. Therefore, the scenario that would gain more by increasing this value is scenario 3, where the water reserves are lower. In the view of Koundouri (2000), those scarcity values can be inserted in the pricing system of water to improve the natural resource management.

The results complement conclusions of Ferrarini et al. (2019), since they do not consider the long run and do not account for climatic changes. Despite the model simulated in present study not working with a specific sector, it also predicts a great increase in GDP. In the work of Ferrarini et al. (2019), Piauí and Maranhão has more potential for growth in relation to Ceará and since the results were weighted by the proportion of each state in the basin, this seems to be consistent with the literature. And even in effects of a persistent drought, if the economic agents do not change their preferences between present and future, growth in the region can still be achieved.

Water use by households is one of the most worrying factors. In 2000, IBGE estimated that less than 60% of families in the region had access to piped water and public sanitation services. Health problems such as cholera, typhoid, amoebic and bacillary dysentery, and other diarrhea diseases can lead to reduced productivity and decrease human capital (HUTTON; WHO, 2012). There is a great increase in the use of water by families in scenarios 1 and 2, however in scenario 3 and 4 this value has little increase. This result indicates that greater attention should be given to expanding water and sewage services.

In addition to suggesting the expansion of water and sewage services, the results show a great distance between current capital stock and the capital stock at the steady state for all scenarios. This call attention to the installed infrastructure. In the work of Borghetti et al. (2017), one of the criteria for excluding priority areas for the expansion of sustainable irrigated agriculture was the existence of electricity distribution lines. According to this study, only four municipalities in Maranhão and one in Piauí were classified as a priority area for expansion. Because of that, the growth may not be an automatic process and public spending may be a key to achieve that.

The main limitations of the work are the estimates about the capital stock and the production, in relation to the amount of water used. The production function uses only the amount of water, but the quality of the water interferes in the production processes as well. In addition, the data does not reflect the risk of a qualitative change in water. The reduction in the amount of water in the aquifer and the increase in waste can interfere with water quality and in turn with production and household consumption.

Future studies should take into account distinct productive sectors to estimate different shadow prices of water. There is also the uncertainty of climatic change that could affect the decisions of agents. The study by Hansen, Sato and Ruedy (2012) and ANA and MMA (2015) as well as others, suggest a great uncertainty about temperatures and according to Li and Swain (2016) when there is a positive correlation between climatic disturbances, despite a persistent drought, the welfare could be large in comparison to the pure randomness. This happens because the social planning has more information and can optimize the intertemporal choice.

6 CONCLUSION

The main hypothesis of the article was that a reduction in the availability of water caused by climatic changes would lead to reduced GDP and lower welfare of the Parnaíba River Basin. The model simulations supported this hypothesis. This effect, however, only becomes significant when faced with a permanent strong reduction of water in the river basin, in which water consumption and capital accumulation are greatly reduced. The study contributed to a better understanding of the behavior of the economy in the face of a restriction of natural resources with regard to long-term behavior. It reiterates that the possibilities for growth in the region are conditioned by investment in capital, which in turn is linked to infrastructure works under the government's responsibility.

The work is limited with regard to the estimates of capital stock and its productivity in relation to water input. A refinement of the model with the distinction of productive sectors may indicate a shadow price for each of these, indicating different tariffs according to the return of water. This can help to allocate water in the best possible way and increase government revenue for water-related infrastructures. The work of Mekonnen and Hoekstra (2016) points out that assessment only on a hydrographic basin scale can hide a water shortage spatially and temporally. Still, in the same study, the sensitivity of water availability is low, which means that an increase in water does not have the same impact on the reduction of people suffering from water shortages. However, the work does not take into account climate change and suggests that future work will also take into account water reservoirs and flows between hydrographic basins.

Although the model does not capture the random climate effect, an investment in water stocks such as weirs or dams can lessen the impact of adverse climatic changes on the economy since this increase has a similar effect to increasing aquifer recharge or relaxing a water restriction in the model studied. ANA's reports also point to a lower number of households with access to water, with their highest percentage concentrated only in large urban centers. Even though the model doesn't distinguish the workforce from population, expansion of this service is extremely important for the development of the region's potential, as can improve the health of workers. Because agricultural activity is the main source of income in the region, it is necessary to amplify the inspection of this activity in order to avoid desertification problems compromising the quantity and quality of water.

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APPENDIX

APPENDIX A – Solving the Maximization Problem

The maximization problem :

$$\begin{gathered}
V(K,N,X) = \max_{C,Z,H} U(C,H,N) + \beta \mathbb{E}[V(K',N',X')] \\
s.t. \quad K' = Y + (1-\delta)K - C - \phi(X)Z \\
X' = X - Z + m + \varepsilon \\
W = S + Z - H \\
Y = AK^{\gamma_1} N^{\gamma_2} W^{\gamma_3} \\
U(C,H,N) = N \left[\; ln\left(\tfrac{C}{N}\right) + \alpha \tfrac{(H/N)^{1-\theta}}{1-\theta} \; \right]
\end{gathered}$$

We solve it using substitution, so we avoid the Lagrange multiplier method. The first-order conditions are as follows :

$$\begin{aligned}
\frac{\partial V(K,N,X)}{\partial C} &= \frac{\partial U(C,H,N)}{\partial C} + \beta \mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial C}\right] = 0 \\
\frac{\partial V(K,N,X)}{\partial Z} &= \frac{\partial U(C,H,N)}{\partial Z} + \beta \mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial Z}\right] = 0 \\
\frac{\partial V(K,N,X)}{\partial H} &= \frac{\partial U(C,H,N)}{\partial H} + \beta \mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial H}\right] = 0
\end{aligned}$$

For the first term of each of these three first-order conditions, we will need the partial derivative of the utility function with respect to each of the choice variables (C,Z and H) :

$$\begin{gathered}
\frac{\partial U(C,H,N)}{\partial C} = \left(\frac{C}{N}\right)^{-1} \\
\frac{\partial U(C,H,N)}{\partial Z} = 0 \\
\frac{\partial U(C,H,N)}{\partial H} = \alpha \left(\frac{H}{N}\right)^{-\theta}
\end{gathered}$$

For the second term of each of the fist-order conditions, we must recall the chain rule for each of them:

$$\begin{aligned}
\frac{\partial V(K',N',X')}{\partial C} &= \frac{\partial V}{\partial K'}\frac{\partial K'}{\partial C} + \frac{\partial V}{\partial N'}\frac{\partial KN'}{\partial C} + \frac{\partial V}{\partial X'}\frac{\partial X'}{\partial C} \\
\frac{\partial V(K',N',X')}{\partial Z} &= \frac{\partial V}{\partial K'}\frac{\partial K'}{\partial Z} + \frac{\partial V}{\partial N'}\frac{\partial KN'}{\partial Z} + \frac{\partial V}{\partial X'}\frac{\partial X'}{\partial Z} \\
\frac{\partial V(K',N',X')}{\partial H} &= \frac{\partial V}{\partial K'}\frac{\partial K'}{\partial H} + \frac{\partial V}{\partial N'}\frac{\partial KN'}{\partial H} + \frac{\partial V}{\partial X'}\frac{\partial X'}{\partial H}
\end{aligned}$$

Therefore, we will need to calculate the partial derivative of the transition equation of each state variable (K',N' and X') with respect to each choice variable (C,Z and H) :

$$\begin{gathered}
\frac{\partial K'}{\partial C} = -1 \\
\frac{\partial N'}{\partial C} = 0 \\
\frac{\partial X'}{\partial C} = 0 \\
\frac{\partial K'}{\partial Z} = \gamma_3\frac{Y}{W} - \phi(X) \\
\frac{\partial N'}{\partial Z} = 0 \\
\frac{\partial X'}{\partial Z} = -1 \\
\frac{\partial K'}{\partial H} = -\gamma_3\frac{Y}{W} \\
\frac{\partial N'}{\partial H} = 0 \\
\frac{\partial X'}{\partial H} = 0
\end{gathered}$$

Based on these partial derivatives and the chain rules, we can rewrite the first-order conditions :

$$\begin{gathered}
\left(\frac{C}{N}\right)^{-1} - \beta\mathbb{E}\left[-\frac{\partial V(K',N',X')}{\partial K'}\right] = 0 \\
\beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K'}\left(\gamma_3\frac{Y}{W} - \phi(X)\right) - \frac{\partial V(K',N',X')}{\partial X'}\right] = 0 \\
\alpha\left(\frac{H}{N}\right)^{-\theta} + \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K'}\left(-\gamma_3\frac{Y}{W}\right)\right] = 0
\end{gathered}$$

Rewritting those conditions a little clearer :

$$\begin{aligned}
\left(\frac{C}{N}\right)^{-1} &= \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K'}\right] \\
\beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial X'}\right] &= \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K'}\right]\left(\gamma_3\frac{Y}{W}-\phi(X)\right) \\
\alpha\left(\frac{H}{N}\right)^{-\theta} &= \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K'}\right]\left(\gamma_3\frac{Y}{W}\right)
\end{aligned}$$

If we combine the first and the third condition, we can eliminate the value function and get the following equilibrium condition :

$$\alpha\left(\frac{H}{N}\right)^{-\theta} = \left(\frac{C}{N}\right)^{-1}\left(\gamma_3\frac{Y}{W}\right)$$

To remove the value function from the first condition we use the Benvenist-Scheinkman. That is, we calculate the partial derivative of the value function with respect to K :

$$\frac{\partial(K,N,X}{\partial K} = \frac{\partial U(C,H,N)}{\partial K} + \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K}\right]$$

Notice that the first therm is zero. This requires the use of a partial derivatives chain rule for calculating the partial derivative :

$$\frac{\partial V(K',N',X')}{\partial K} = \frac{\partial V(K',N',X')}{\partial K'}\frac{\partial K'}{\partial K} + \frac{\partial V(K',N',X')}{\partial N'}\frac{\partial N'}{\partial K} + \frac{\partial V(K',N',X'}{\partial X'}\frac{\partial X'}{\partial K}$$

We need to calculate the partial derivatives of each state variable with respect to K:

$$\begin{aligned}
\frac{\partial K'}{\partial K} &= \gamma_1\frac{Y}{K} + (1-\delta) \\
\frac{\partial N'}{\partial K} &= 0 \\
\frac{\partial X'}{\partial K} &= 0
\end{aligned}$$

Using the chain rule and the partial derivatives above, we get the following BS condition :

$$\frac{\partial V(K,N,X)}{\partial K} = \left(\gamma 1\frac{Y}{K} + (1-\delta)\right)\beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K'}\right]$$

Inserting what we know from the first first-order condition, we get:

$$\frac{\partial V(K,N,X)}{\partial k} = \left(\gamma_1\frac{Y}{K} + (1-\delta)\right)\left(\frac{C}{N}\right)^{-1}$$

And we recognize that this relation also holds true for the following period, such that :

$$\frac{\partial V(K',N',X')}{\partial k'} = \left(\gamma_1\frac{Y'}{K'} + (1-\delta)\right)\left(\frac{C'}{N'}\right)^{-1}$$

Inserting this back into the first first-order condition, we find the following Euler equation, which is our second equilibrium condition :

$$\left(\frac{C}{N}\right)^{-1} = \beta\mathbb{E}\left[\left(\gamma_1\frac{Y'}{K'} + ('-\delta)\right)\left(\frac{C'}{N'}\right)^{-1}\right]$$

If we insert the first first-order condition into the second first-order condition, we find:

$$\beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial X'}\right] = \left(\frac{C}{N}\right)^{-1}\left(\gamma_3\frac{Y}{W} - \phi(X)\right)$$

We will need another Benveniste-Scheinkman condition to replace the remaining value function in this expression :

$$\frac{\partial V(K,N,X)}{\partial X} = \frac{\partial U(C,H,N)}{\partial X} + \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial X}\right]$$

As earlier, the utility function does not include X, so the first term is zero. the second term requires the use of a partial derivatives chain rule for calculating the partial derivative in the expectation:

$$\frac{\partial V(K',N',X')}{\partial X} = \frac{\partial V(K',N',X')}{\partial K'}\frac{\partial K'}{\partial X} + \frac{\partial V(K',N',X')}{\partial N'}\frac{\partial N'}{\partial X} + \frac{\partial V(K',N',X'}{\partial X'}\frac{\partial X'}{\partial X}$$

We will therefore need to calculate the partial derivatives of each state variable with respect to X:

$$\frac{\partial K'}{\partial X} = -\frac{\partial\phi(X)}{\partial X}Z$$
$$\frac{\partial N'}{\partial X} = 0$$
$$\frac{\partial X'}{\partial X} = 1$$

Inserting this into the expression for the partial derivative of the value function with respect to X, we get :

$$\frac{\partial V(K,N,X)}{\partial X} = \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial K'}\left(-\frac{\partial\phi(X)}{\partial X}Z\right) + \frac{\partial V(K',N',X')}{\partial X'}\right]$$

We can use the first frist-order condition to replace the partial derivative of thenext-period value function with respect to the next-period capital :

$$\frac{\partial(K,N,X)}{\partial X} = \left(\frac{C}{N}\right)^{-1}\left(-\frac{\partial\phi(X)}{\partial X}Z\right) + \beta\mathbb{E}\left[\frac{\partial V(K',N',X')}{\partial X'}\right]$$

We also have an expression for the partial derivative of the next-period value function with respect to next-period X, so we can put that in too:

$$\frac{\partial V(K,N,X)}{\partial X} = \left(\frac{C}{N}\right)^{-1}\left(-\frac{\partial\phi(X)}{\partial X}Z\right) + \left(\frac{C}{N}\right)^{-1}\left(\gamma_3\frac{Y}{W} - \phi(X)\right)$$
$$\frac{\partial V(K,N,X)}{\partial X} = \left(\frac{C}{N}\right)^{-1}\left(\gamma_3\frac{Y}{W} - \phi(X) - \frac{\partial\phi(X)}{\partial X}Z\right)$$

This relation will hold true for the following period too, so:

$$\frac{\partial V(K',N',X')}{\partial X'} = \left(\frac{C'}{N'}\right)^{-1}\left(\gamma_3\frac{Y'}{W'} - \phi(X') - \frac{\partial\phi(X')}{\partial X'}Z'\right)$$

Inserting this into the condition again will allow us to eliminate the value function entirelu, and this results in the third equilibrium condition :

$$\beta\mathbb{E}\left[\left(\frac{C'}{N'}\right)^{-1}\left(\gamma_3\frac{Y'}{W'} - \phi(X') - \frac{\partial\phi(X')}{\partial X'}Z'\right)\right] = \left(\frac{C}{N}\right)^{-1}\left(\gamma_3\frac{Y}{W} - \phi(X)\right)$$

APPENDIX B – Equilibrium Conditions

First, note that the model contains eight endogenous variables : C: Consumption
H: Residential water consumption
N: Population
K: Physical capital
X: Groundwater stock
W: Productive water use
Y: Production
Z: Withdraw of groundwater

Therefore, we expect eight equilibrium conditions to be fulfilled:

$$\alpha \left(\frac{H}{N}\right)^{-\theta} = \left(\frac{C}{N}\right)^{-1} \left(\gamma_3 \frac{Y}{W}\right)$$

$$\left(\frac{C}{N}\right)^{-1} = \beta \mathbb{E}\left[\left(\gamma_1 \frac{Y'}{K'} + ('-\delta)\right)\left(\frac{C'}{N'}\right)^{-1}\right]$$

$$\beta \mathbb{E}\left[\left(\frac{C'}{N'}\right)^{-1}\left(\gamma_3 \frac{Y'}{W'} - \phi(X') - \frac{\partial \phi(X')}{\partial X'} Z'\right)\right] = \left(\frac{C}{N}\right)^{-1}\left(\gamma_3 \frac{Y}{W} - \phi(X)\right)$$

$$K' = Y + (1-\delta)K - C - \phi(X)Z$$

$$X' = X - Z + m + \varepsilon$$

$$W = S + Z - H$$

$$Y = AK^{\gamma_1} N^{\gamma_2} W^{\gamma_3}$$

The model contains the following additional variables :
S: Stochastic withdraw of surface water
M: Natural recharge rate
ϵ: Stochastic disturbance

APPENDIX C – Steady State

In order to find the steady state, we replace all current and next-period variables with their steady-states in the equilibrium conditions, then solve the system to find the steady state for each of the variables. We can also remove expectation operators. Firstly, we replace current and next-period variables with their steady states:

$$\alpha\left(\frac{\overline{H}}{\overline{N}}\right)^{-\theta}=\left(\frac{\overline{C}}{\overline{N}}\right)^{-1}\left(\gamma_3\frac{\overline{Y}}{\overline{W}}\right)$$

$$\left(\frac{\overline{C}}{\overline{N}}\right)^{-1}=\beta\left(\gamma_1\frac{\overline{Y}}{\overline{K}}+('-\delta)\right)\left(\frac{\overline{C}}{\overline{N}}\right)^{-1}$$

$$\beta\left(\frac{\overline{C}}{\overline{N}}\right)^{-1}\left(\gamma_3\frac{\overline{Y}}{\overline{W}}-\phi(\overline{X})-\frac{\partial\phi(\overline{X})}{\partial\overline{X}}\overline{Z}\right)=\left(\frac{\overline{C}}{\overline{N}}\right)^{-1}\left(\gamma_3\frac{\overline{Y}}{\overline{W}}-\phi(\overline{X})\right)$$

$$\overline{K}=\overline{Y}+(1-\delta)\overline{K}-\overline{C}-\phi(\overline{X})\overline{Z}$$

$$\overline{X}=\overline{X}-\overline{Z}+m+\varepsilon$$

$$\overline{W}=\overline{S}+\overline{Z}-\overline{H}$$

$$\overline{Y}=A\overline{K}^{\gamma_1}\overline{N}^{\gamma_2}\overline{W}^{\gamma_3}$$

Manipulating the fifth and sixth conditions, we find:

$$\overline{N}=\frac{b_1-1}{b_2}$$

$$\overline{Z}=m+\epsilon$$

By canceling some factors in the equations, we are left with the following six equations:

$$\alpha\left(\frac{\overline{H}}{\overline{N}}\right)^{-\theta}=\left(\frac{\overline{C}}{\overline{N}}\right)^{-1}\left(\gamma_3\frac{\overline{Y}}{\overline{W}}\right)$$

$$1=\beta\left(\gamma_1\frac{\overline{Y}}{\overline{K}}+(1-\delta)\right)$$

$$\beta\left(\gamma_3\frac{\overline{Y}}{\overline{W}}-\phi(\overline{X})-\frac{\partial\phi(\overline{X})}{\partial\overline{X}}\overline{Z}\right)=\gamma_3\frac{\overline{Y}}{\overline{W}}-\phi(\overline{X})$$

$$\overline{K}=\overline{Y}+(1-\delta)\overline{K}-\overline{C}-\phi(\overline{X})\overline{Z}$$

$$\overline{W}=S+\overline{Z}-\overline{H}$$

$$\overline{Y}=A\overline{K}^{\gamma_1}\overline{N}^{\gamma_2}\overline{W}^{\gamma_3}$$

Rewritting these equations:

$$
\begin{aligned}
\alpha \left(\frac{\overline{H}}{\overline{N}} \right)^{-\theta} &= \left(\frac{\overline{C}}{\overline{N}} \right)^{-1} \left(\gamma_3 \frac{\overline{Y}}{\overline{W}} \right) \\
\overline{K} &= \frac{\beta \gamma_1}{1 - \beta(1-\delta)} \overline{Y} \\
(\beta - 1) \left(\gamma_3 \frac{\overline{Y}}{\overline{W}} - \phi(\overline{X}) \right) &= \beta \frac{\partial \phi(\overline{X})}{\partial \overline{X}} \overline{Z} \\
\overline{Y} - \delta \overline{K} - \overline{C} - \phi(\overline{X}) \overline{Z} &= 0 \\
\overline{W} &= S + \overline{Z} - \overline{H} \\
\overline{Y} &= A \overline{K}^{\gamma_1} \overline{N}^{\gamma_2} \overline{W}^{\gamma_3}
\end{aligned}
$$

APPENDIX D – Python Code for calculating the Steady State

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Thu Nov 14 14:00:38 2019

@author: fabricio
"""

# #### Solve the final equation of the manual derivation numerically ###
from scipy.optimize import fsolve
import math, sys

# Parameters
b1 = 1.067460;
b2 = 0.00330214;
alpha = 0.0839;
beta = 0.95;
delta = 0.05;
gamma1 = 0.41;
gamma2 = 0.45;
gamma3 = 0.14;
epsilon = 0;
theta = 0.5;
m = 12.3196;
a = 1.37025;
d = 2.067;
r = 0.1;
A = 2.1;
S = 7.83446;
sigma = 0.1;

# Known variables :
N = (b1 -1) /b2
Z = m + epsilon

# Helper variables :
```

```
h1 = beta * gamma1 /(1 - beta *(1 - delta ))
h2 = (A*( h1 ** gamma1 )*(N** gamma2 )) **(1/(1 - gamma1 ))
h4 = (beta -1) /( beta -1- beta *r*Z)
h5 = (1- delta *h1)*h2
h6 = (N**(1 - theta ))* gamma3 *h2/ alpha
h7 = h4* gamma3 *h2
h8 = h7*Z
h9 = h4*a*Z - a*Z

eta = gamma3 /(1 - gamma1 )

def equation (W):
    # print (W)
    return h5*(W** eta ) - h6 *(W **( eta -1)) *
((S+Z-W)** theta ) - h8* gamma3 *(W **( eta -1) ) + h9


# Solve for the different variables and print the solutions
print ("␣Analytical␣Solution␣(␣although␣with␣W␣found␣numerically␣):")

# Find W
W= fsolve ( equation , (1.) )[0]
print ("W␣=", W,";")

# Print N and Z, found earlier
print ("N␣=", N,";")
print ("Z␣=", Z,";")

# Find C
C = h6 *(W **( eta -1)) *((S+Z-W)** theta )
print ("C␣=", C,";")

# Find X
h3 = h7 *(W **( eta -1) )-h4*a
X = (1/ r)*( math . log (d)-math . log(h3))
print ("X␣=", X,";")

# Find H
H = S + Z - W
```

```
print ("H␣=", H,";")

 # Find Y
Y = h2 *(W** eta )
print ("Y␣=", Y,";")

# Find K
K = h1*Y
print ("K␣=", K,";")
```

APPENDIX E – Dynare Model Code

```
%Scenario 1
%Light Discounting beta= 98
%whithout Climate Cnange Effect mt= 32.42
% Define the endogenous variables
var W N Z C X H Y K phi dphi ;

% Define an exogenous variable
varexo epsilon ;

% Define the parameters
parameters b1 b2 alpha beta delta gamma1 gamma2 gamma3 theta m a d r A S;


% Calibration : Assign values to the parameters
b1 = 1.067460;
b2 = 0.00330214;
alpha = 0.0839;
beta = 0.98;
delta = 0.05;
gamma1 = 0.41;
gamma2 = 0.45;
gamma3 = 0.14;
theta = 0.5;
m = 32.42;
a = 0.37025;
d = 2.067;
r = 0.1;
A = 2.1;
S = 20.617;
sigma = 0.1;

% The equilibrium equations of the model
model ;
alpha *(H/N)^(- theta ) = (C/N)^( -1) *( gamma3 *Y/W);
(C/N)^( -1) = beta *( gamma1 *Y (+1) /K + (1- delta ))*(C (+1) /N (+1) )
beta *(C (+1) /N (+1) )^( -1) *( gamma3 *(Y (+1) /W (+1) )-phi (+1) - dp
```

```
- phi );
K = Y + (1- delta )*K( -1) - C - phi *Z;
N = b1*N( -1) - b2*N( -1) ^2;
X = X( -1) - Z + m + epsilon ;
W = S + Z - H;
Y = A*K( -1) ^( gamma1 )*N( -1) ^( gamma2 )*W^( gamma3 );
 phi = a + exp(-r*X( -1));
 dphi = -r* exp(-r*X( -1));
 end ;

 % Provide initial values for the economy , example : 95% of steady state
 initval ;
W = 34.517908;
N = 18.527373;
Z = 24.8013;
C = 119.048845105648;
X = 162.1;
H = 10.900392;
Y = 145.181518421523;
K = 331.013862001;
 phi = a + exp(-r*X);
 dphi = -r* exp(-r*X);
 end ;

 % Provide the terminal values ( steady state )
 endval ;

W = 40.316520748683324 ;
N = 20.429176231171326 ;
Z = 32.42 ;
C = 191.61190245666054 ;
X = 62.671810951823176 ;
H = 12.720479251316682 ;
Y = 287.1853553051873 ;
K = 1672.3344313278863 ;
 phi = a + exp(-r*X);
 dphi = -r* exp(-r*X);
 end ;
```

```
% Check the BK conditions
check ;

% Find the steady state
steady ;

% Simulate ( deterministic ) for a certain number of periods
simul ( periods =250) ;

% Plot the variables
subplot (4 ,3 ,1)
plot ( oo_ . endo_simul (1 ,:))
title ('W')

subplot (4 ,3 ,2)
plot ( oo_ . endo_simul (2 ,:))
title ('N')

subplot (4 ,3 ,3)
plot ( oo_ . endo_simul (3 ,:))
title ('Z')

subplot (4 ,3 ,4)
plot ( oo_ . endo_simul (4 ,:))
title ('C')

subplot (4 ,3 ,5)
plot ( oo_ . endo_simul (5 ,:))
title ('X')

subplot (4 ,3 ,6)
plot ( oo_ . endo_simul (6 ,:))
title ('H')

subplot (4 ,3 ,7)
plot ( oo_ . endo_simul (7 ,:))
title ('Y')

subplot (4 ,3 ,8)
```

```
plot ( oo_ . endo_simul (8  ,:))
title ('K')

subplot (4  ,3  ,9)
plot ( oo_ . endo_simul (9  ,:))
title ('phi')

%subplot (4  ,3  ,10)
%plot ( oo_ . endo_simul (10  ,:) )
%title ('dphi')

print -deps plots . eps
```