EMERSON COSTA DOS SANTOS

PRODUCTIVITY, FISCAL AND MONETARY SHOCKS: A REAL BUSINESS CYCLE APPROACH TO THE BRAZILIAN CASE

Tese apresentada à Universidade Federal de Viçosa, como parte das exigências do Programa de Pós-Graduação em Economia Aplicada, para obtenção do título de *Doctor Scientiae*.

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To my father and my mother.

"Tudo me é permitido, mas nem tudo me convém." (I Cor 6, 12)

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Resumo

SANTOS, Emerson Costa, D.Sc., Universidade Federal de Viçosa, dezembro de 2016. **Choques de produtividade, monetário e fiscal: Um modelo de ciclos reais de negócios para o caso brasileiro.** Orientador: Wilson da Cruz Vieira.

Os estudos sobre interação das políticas fiscais e monetárias, por muito tempo, estiveram em segundo lugar no debate sobre a política macroeconômica. Era prática comum na literatura ignorar a política fiscal. Implicitamente, os modelos assumiam que o orçamento fiscal era equilibrado em todos os momentos, por meio de taxas do tipo *"lump sum"*. Supunha-se a existência de um regime Ricardiano, em que o orçamento do governo sempre estaria equilibrado. Nos últimos anos têm surgido uma maneira diferente de pensar sobre essas questões. Consequentemente, uma nova visão da interação da política monetária e fiscal é importante. Agora, as interações das políticas fiscal e monetária são importantes para explicar os movimentos das variáveis macroeconômicas. Os desenvolvimentos recentes, tanto na prática como na teoria, têm mostrado crescente relação entre a política fiscal e monetária. Deste modo, o tema é bastante relevante, sobretudo, questões como a sustentabilidade da dívida pública, altas taxas de juros e o controle da inflação, as quais estão no centro das discussões sobre a política macroeconômica, especialmente no Brasil. Neste contexto, este estudo examina a interação entre as políticas monetárias e fiscais em um modelo de equilíbrio geral dinâmico estocástico (DSGE), no âmbito de modelos de ciclos reais de negócios (RBC), com uma restrição de *"cash in advance"* (CIA) para o caso da economia brasileira entre os anos de 2000 e 2013. Tradicionalmente, a literatura RBC coloca os choques tecnológicos como a principal causa das flutuações econômicas. No entanto, a relevância e o papel de diferentes choques como tecnológico, fiscal e monetário simultâneos, é uma questão que pode contribuir para a literatura existente sobre o assunto. Para isso, várias economias artificiais foram simuladas a fim de obter os segundos momentos (desvio padrão e coeficiente de correlação) e as funções de resposta para as principais variáveis. Apesar dos choques de política monetária e fiscal e a interação entre ambas as políticas serem o principal objetivo para este trabalho, foram utilizados os choques de produtividade para testar o modelo e comparar a simulação com os segundos momentos dos dados brasileiros configurando a economia artificial calibrada com dados da economia real. A partir daí, várias economias artificiais foram simuladas, e foram analisados como as variáveis respondem aos choques de produtividade, fiscal e monetário isoladamente, e por fim, simultaneamente. Comparando as estatísticas de dados brasileiros com as estatísticas de economia artificial, o modelo forneceu um ajuste muito bom para a variabilidade real, com a exceção das horas trabalhadas. A simulação com choques monetários e produtividade antecipados sugere uma maior volatilidade das variáveis nominais. Ao contrário da economia artificial, as variáveis nominais desempenham um papel importante no modelo com choques monetários. Outra simulação foi com choques de produtividade e choques fiscais. Em comparação com a economia artificial, todas as variabbles mostraram maior volatilidade, com exceção da simulação da série do produto. As simulações mostraram um aumento de 50% da volatilidade da inflação em comparação com a economia artificial. A última simulação é com os choques de produtividade, fiscal e monetário simultâneos no modelo de equilíbrio geral. O resultado da simulação foi capaz de captar em torno de 32% da variabilidade da inflação brasileira. Além disso, os resultados da simulação sugerem que a política monetária restringe a política fiscal, afetando as taxas de juros nominais, sugerindo que a política monetária é a política ativa, enquanto a política fiscal é a política passiva nas flutuações de curto prazo no Brasil.

Abstract

SANTOS, Emerson Costa, D.Sc., Universidade Federal de Viçosa, December, 2016. **Productivity, monetary and fiscal shocks: a real business cycle aproach to the brazilian case.** Advisor: Wilson da Cruz Vieira.

The studies of the interaction about the fiscal and monetary policies, for a long time, were in second place in the debate on macroeconomic policy. It was standard practice in the literature to ignore fiscal policy. Implicitly, the models assumed that fiscal budget was balanced at all times by means of lump-sum taxations. It was assumed the existence of a Ricardian regime, in which the government budget was always balanced. The last years have emerged a different way of thinking about this issue. Consequently, a new view of the interaction of monetary and fiscal policy is important. Now, fiscal and monetary policies interactions are important to explain the movements of the macroeconomic variables. The recent developments, both in practice and in theory, have shown growing relationship between fiscal and monetary policy. Thus, the theme is very relevant, especially issues such as sustainability of public debt, high interest rates and inflation control at the center of discussions on macroeconomic policy, especially in Brazil. In this context, this study examine the interaction between monetary and fiscal policies in a dynamic model stochastic general equilibrium (DSGE), in the framework of models of real business cycles (RBC), with a cash in advance restriction (CIA) to the Brazilian case between 2000 and 2013. Traditionally, the RBC literature puts the technology shocks as the main cause of economic fluctuations. However, the relevance and the role of different simultaneous shocks such as technological, fiscal and monetary, is an issue that can contribute to existing literature on the subject. For this, various artificial economies were simulated in other to get the second moments (standard deviation and correlation coefficient) and the response functions for the main variables. Although monetary and fiscal policy shocks and the interaction between both policies are the main goal for this work, it was used the productivity shocks to test the model and compare the simulation with the Brazilian data second moments. From this, various artificial economies were simulated and was analyzed how the variables respond to productivity, fiscal and monetary shocks, isolated and then, simultaneously. Comparing Brazilian data statistics to the artificial economy statistics, we claim that the model provides a very good fit for the real variability, with exception the hours worked. The simulation with unanticipated monetary and productivity shocks suggest a higher volatility of nominal variables. Contrary to the artificial economy, nominal variables play an important role in the model with monetary shocks. The other simulation it was the productivity and fiscal shocks. In comparison with the artificial economy, all variables show higher volatility, with exception the output. This simulation shows an increase of

50% of the volatility on inflation in comparison with the artificial economy. The last simulation is about the productivity, fiscal and monetary shocks simultaneous in general equilibrium models. The result of simulation was able to capture approximately 32% of Brazilian inflation variability. Furthermore, the results of simulation suggest that the monetary policy constraints fiscal policy by affecting nominal interest rates, suggesting that monetary policy is the active policy, while fiscal policy is the passive policy in the short-run fluctuations in Brazil.

1 Introduction

1.1 Initial Remarks

The studies of the interaction about the fiscal and monetary policies, for a long time, were in second place in the debate on macroeconomic policy. Both the monetarist theory, which suggests a lower government intervention and is against discretionary economic policies, as the Keynesians, who are more intrusive and seek to establish optimal rules for monetary and fiscal policy, tended to separate the debate between fiscal and monetary policy. Thus, work on the conduction of monetary policy became more restricted to issues related to rules versus discretionary behavior, leaving aside the possible influences that fiscal policy could play in determining the price level.

From the 1970s, when the inflation was high, there was a movement in the Organization for economic Cooperation and Development countries towards central banks with political independence and with the objective of price stability. But the focus was on macroeconomic performance more generally, and not just price stability. The interaction between monetary and fiscal policy was often modeled as a noncooperative game between a central bank and its government, each having its own priorities over inflation, output, and so forth (CANZONERI ROBERT CUMBY, 2011).

As in Schimitt-Grohe & Uribe (2006), an important dimension along which the existing studies from reality is the assumed fiscal regime. It was standard practice in the literature to ignore fiscal policy. Implicitly, these models assumed that the fiscal budget was balanced at all times by means of lump-sum taxations. Fiscal policy was left to the second plan, in a monetarist model that assumed the existence of a Ricardian regime, in which the government budget was always balanced. If there was problem, the balances correction would be corrected through taxes or inflation tax without influence on the agent's decisions. With this assumption, the tax authority behaves with discipline, so that government tax cuts financed by increases in debt would be offset by higher taxes in the future to ensure that the debt is solvent. In this context, the discussion of coordination between fiscal and monetary policies did not make sense (NUNES; PORTUGAL, 2009).

The last years have emerged a different way of thinking about these issues. Many works have treated this question and in part due to central banker's tendency to choose an interest rate as instrument of monetary policy, the form of stable price paths has become an issue again, and fiscal policy has now a fundamental role in price determination and control. Consequently, a new view of the interaction of monetary and fiscal policy is important.

The determination of price has always been at the central point of monetary economics. And indeed, traditional discussions of price determination made it sound as if fiscal policy played little or no role. Friedman (1963) says that "inflation is always and everywhere a monetary phenomenon." The monetarist theory was reduced to the quantitative theory of prices. With velocity constant, and output was exogenously given, the price level is completely determined by the money supply, and price stability is responsibility of the central bank. The conclusion was that there appeared to be no need to coordinate monetary and fiscal policy in a control prices. But this has challenged, suggesting that fiscal policy might even play the dominant role in certain circumstances.

Following Sargent & Wallace (1981), if seigniorage is exploited as a source of revenue for the government, then the monetary and fiscal policies should be coordinated. In this sense, the stabilization of the level price depends on the questions: Who acts first, the fiscal or the monetary authority? Or, who imposes discipline on whom? This is denominated as the unpleasant monetarist arithmetic by the authors in which the fiscal policy dominates monetary policy and the monetary authority confronts itself with restrictions imposed by the demand of government bonds. This is a possible case of active fiscal policy and passive monetary behavior. For the author, the relationship between fiscal deficit and inflation is dynamic. Under an independently set fiscal policy, deficits determine the present value of the necessary money creation to finance them, but do not necessarily determine current seigniorage and, hence, current inflation. In this sense, the fiscal policy can create inflation due the increase the money supply by the central bank.

In Sargent & Wallace (1981), the price stability problem has to do with which government agent has to see that the consolidated government present value budget constraint is satisfied. In his model is assumed that government bonds are real. In this formulation, the real value or the inherited government debt is fixed at the beginning of the period, and it has to be financed by the central bank's collection of seigniorage, and/or government's of taxation. Furthermore, is characterized the interaction between monetary and fiscal policy in terms of game theory and leadership, or who gets to go first. The new element in monetarist arithmetic is the possibility that the government gets to go first, and the central bank must, sooner or later, deliver the seigniorage.

As evidenced out in the framework by Sargent & Wallace (1981), monetary policy may not be able to control the stability of the price level if the fiscal agent does not have the budget constraint balanced. This mean that, in any macroeconomic model, the government's budget constraint entails contributions of both monetary and fiscal policy, and the control of the level price.

The monetary policy is the main determinant of the equilibrium price level. This monetarist position implies that control of fiscal policy is unnecessary in order to achieve price stability, as long as fiscal policy can be prevented from having any effect upon monetary policy. Monetarists recognize that fiscal problems are often the root cause of high inflations, with rapid money growth in create revenues for the governments through seigniorage. But it is often doubted that such concerns have much effect upon monetary policy in low-inflation countries where seigniorage revenues remain a small fraction of the government budget, unless a truly radical change in policy occurs (WOODFORD, 1996). However, Sargent & Wallace (1981) argue that the monetary authority's control over inflation is limited. They believe that, if the fiscal authority dominates the monetary authority, then the fiscal authority independently announces all current and future deficits, such that the monetary authority is constrained by the demand of government bonds and monetizes the deficit. Accordingly, the government rurns persistent deficits with seigniorage and produces inflation, and fiscal deficits and inflation are dynamically correlated.

A well-established theory in macroeconomics is that fiscally dominant governments running persistent deficits have sooner or later to finance those deficits through money creation (seigniorage), thus producing inflation (SARGENT; WALLACE, 1981). While this theory does not rule out the importance of other mechanism through which inflation can be fueled and become persistent, fiscal imbalances have remained central to most models.

Recently emerged, the Fiscal theory of level prices (FTPL) was developed primarily by Leeper (1991), Woodford (1994), Sims (1994), and others. A basic principle of the FTLP is that monetary policy by itself does not provide the nominal anchor for an economy. Instead, it is the pairing of a particular monetary policy with a particular fiscal policy that determines the path of the price level. For the price determination is necessary a good coordination of monetary and fiscal policy.

Woodford (1994) and Sims (1994) emphasized the role of the government budget constraint in price determination. If primary surpluses are determined independently of the level of debt, then the path of the money supply and the price level must satisfy the needs of fiscal solvency. This is called a fiscal dominant regime. If, on the other hand, primary surpluses respond to the level of debt in a way that assures fiscal solvency, then money and prices can be determined by supply and demand for money. This is called a monetary dominant regime.

Now, fiscal policy is important in determining the path of prices in an economy and many authors emphasize the effects of fiscal policy on the price level. If in one hand, the Monetarist theory assumes that the money stock is the most important determinant of the price level in an economy, on the other hand, the FTLP requires not only an appropriate monetary policy, but also an appropriate fiscal policy. When it is the non-Ricardian view, it is assumed that the primary surplus is adjusted by the government to guarantee solvency for any price level. To the FTLP, if governments adjust primary surplus independently of government debt, the presence of significant effects of fiscal shocks on the price level may be expected and, hence, FTPL suggests the possibility that the primary surplus can be set independently from government. For this reason, the price level will adjust to make government's intertemporal budget constraint hold at any point of time (CEVIK; DIBOOGLU; KUTAN, 2014). These two causes of fiscal authority behavior are dubbed "Ricardian" and "non-Ricardian" in Woodford (1995), or "passive" and "active" into the terminology of Leeper (1991), where the fiscal authority sets primary surpluses due to government debt in the passive ("Ricardian") fiscal policy and the active or "non-Ricardian" fiscal policy refers to the other case. It should be noted that intertemporal budget constraint may hold in equilibrium in both cases.

The principal point of FTPL is that the present value government budget constraint and the fiscal policy are important factors in determination of level prices. The FTPL can be understood as an application of one of the aspects discussed by Sargent & Wallace (1981), where the conduct of fiscal policy catch up. As argues Sala (2004), the distinction between the classical theory and the FTPL is the interpretation of the present value of the government budget constraint. The monetarist tradition, the government intertemporal constraint is provided for any price level. As argue the FTPL, the intertemporal budget constraint is a condition of equilibrium, and as such, selects the level of equilibrium price.

Based on the monetarist tradition, a good prescription monetary policy is necessary and sufficient condition to guarantee low inflation. An independent central bank with a strong institutional commitment to ensure price stability must compel the tax authority to adopt a correct and responsible fiscal policy. On the other hand, to the FTLP, a good prescription of monetary policy is not a sufficient condition to guarantee low inflation, unless additional measures are takent to restrict the action of fiscal authority.

1.2 The problem and its importance

It is a wide part of the literature that has examined the interaction between fiscal and monetary policies. At a theoretical level, Sargent & Wallace (1981) showed that under certain conditions, if the time paths of government spending and taxes are exogenous, bond-financed deficits are nonsustainable, and the central bank should monetize the deficit. This will increase the money supply and inflation in the long run. Sargent & Wallace (1981) and Aiyagari & Gertler (1985) explained that theories that ignored fiscal policy were incomplete and that ignoring fiscal-monetary interactions could lead to policy errors. Dixit & Lambertini (2003) considered monetary and fiscal interactions when the monetary authority is more conservative than the fiscal. The results showed that, when both policies are discretionary, Nash equilibrium yields lower output and higher price than the ideal points of both authorities. With fiscal discretion, monetary commitment yields the same outcome as discretionary monetary leadership for all realizations of shocks. Kirsanova, Stehn & Vines (2005) studied the interactions of fiscal and monetary policy when they stabilize a single economy against shocks in a dynamic setting. They built a five-equation model of an economy, in which fiscal policy has a potential role in assisting monetary policy to stabilize against shocks. They also studied the interactions on various schemes as: Nash game, lead in a Stackelberg game and whether fiscal authorities and monetary authorities are benevolent and cooperate with each other in the setting of their macroeconomic policy instruments.

Both in academic debate and in discussion of the anti-inflationary policy, there has been a modest of theoretical and empirical analysis to assess with precision the results that can wait for inflation varies when the level of the deficit. In Sargent & Wallace (1981), was introduced the notion of monetary dominance and fiscal dominance regimes. In fiscal dominance schemes, the tax authority establishes whether your budget, announcing present and future deficits, and thus the amount of resources to be collected by seigniorage or the sale of public securities.

In the case of funding by seigniorage, the expansion of the monetary base can results in an immediate increase of the price level. The government's position to finance their spending continuously by selling bonds implies high public debt and consequent increasing costs. However, funding for the expansion of the debt has a limit expressed by the relative size of the economy and perception of economic agents as the government's debt solvency. Once this limit is reached, the government can use the expansion of the monetary base to honor its commitments, generating inflationary effects in the future or even the present at the time of issuance of the securities. If the fiscal dominance regime, the monetary authority loses efficiency in combating inflation, as the government's fiscal policy is an additional source of instability on the price level and acts independently.

The link among budget deficits, inflation and money supply always remained a critical issue. Budget deficits and inflation nexus studied from several perspectives and different views exist about this link. First argument is based on rationale that part of budget deficits financed by borrowing from central bank leads to increase in money stock and higher money stock causes inflation. Conversely, in an unutilized resource economy, high level of deficits may lead to higher demand which raises output. The second argument embark that higher demand due to large fiscal deficits will raise prices where output of essential commodities cannot be increased (ISHAQ; MOSHIN, 2015).

It has grown the literature that researches empirical basis for inflationary impact of budget deficits in the last decades. Karras (1994) investigates the effects of budget deficits on money growth, inflation, investment and real output growth using annual data from a sample of 32 countries. The findings are that deficits generally do not monetized and, therefore, they do not produce inflation via monetary expansions. However, according Catao & Torrones (2005), macroeconomic theory postulates that persistent fiscal deficits are inflationary. For Catao & Torrones (2005), yet empirical research has had limited success in uncovering this relationship. In order to examines the issue, they made the inflation as non-linearly related to fiscal deficits through the inflation tax base and estimate this relationship as intrinsically dynamic, using panel techniques that explicitly distinguish between short- and long-run effects of fiscal deficits to 107 countries over 1960–2001. The results showed a strong positive association between deficits and inflation among high-inflation and developing country groups, but not among low-inflation advanced economies.

Neyapti (2003) investigates the relationship between budget deficits and inflation with the view that the nature of it depends on the characteristics of monetary and financial institutions. The main hypothesis is that budget deficits are especially inflationary when both the central bank is not independent and the financial market is not developed enough to contain inflationary expectations. The empirical analysis using a panel data that comprises 54 developed and less developed countries, each with 10 to 20 years. Estimation results showed that budget deficits have a significant positive effect on inflation.

Lin & Chu (2013) examined the relationship between deficits and inflation on an extensive panel data set. The panel data spanned 91 countries over the 1960–2006 period. The empirical results showed that the dynamic panel quantile regression (DPQR) estimates of the impact of deficits on inflation vary across different quantiles of inflation. Fiscal deficits are inflationary in high-inflation and middle-inflation episodes, and they are weakly or insignificantly related to inflation in low-inflation episodes. Furthermore, they showed that in high-inflation episodes, the inflationary effect of deficits is higher because of faster money creation.

The analysis of the importance of fiscal policy and monetary policy limits is worked in Blanchard (2004), which discusses the regime of inflation targeting in Brazil under a fiscal dominance regime. The author discusses a way in which fiscal policy practiced by the government could affect price stability in 2002 and 2003. This consisted mainly the effects of the exchange rate on the solvency of public debt. The hypothesis is that increasing in real interest rates as a means to combat inflation can be perverse if the rise in real interest compromising the sustainability of public debt. Higher probability of default, depending on the risk aversion of foreign investors, causes the reduction of capital flows and the depreciation of the exchange, with a negative impact on inflation. His conclusion is that raising interest rates to curb inflation, may result in more inflation. In an open economy with monetary policy based on interest rates, other limits may be imposed on the monetary authority, since the standard proposition about a contractionary monetary policy is that the increase in the interest rate makes it more attractive public debt, leading to a capital in owe and, consequently, in appreciation of the exchange rate. This mechanism contributes to combating inflation. However, the rise in interest rates causes an increase in the probability of default of debt, increasing the country's insolvency risk, government debt becomes less attractive, which depreciates the exchange rate via output capital, and contributes to the rise in inflation, achieving the opposite effect desired. Lopreato (2006) argues that the increase in interest rates undermine the sustainability of the debt, the alternative is to increase the fiscal effort necessary to intertemporal adjustment of public accounts as a central element of economic policy.

Favero & Giavazzi (2004) present an empirical model in which the economy is a good balance for a bad balance when the debt/GDP ratio exceeds certain baseline. In bad balance, there is a vicious circle in which the increasing in the Selic rate increases the probability of non-payment of debt, due to the increase in public debt burdens. As there is high correlation between the exchange rate and probability of default, there would be a depreciation rather than appreciation. To reduce inflation arising from this exchange rate depreciation, the Central Bank would have to raise further the interest rate, creating a vicious circle in the conduct of monetary policy.

Favero & Giavazzi (2004) and Blanchard (2004) suggest that, in the specific case of the Brazilian economy, the increase in the debt/GDP ratio, due to the increase in interest rates would tend to increase the risk perception of agents and consequently, the probability of non-payment of debt. In this case, there would be a situation of fiscal dominance in that monetary policy would be ineffective to control inflation as a result of imbalances in fiscal policy.

Pastore (1994) argues that the analysis of the relationship between public deficit and inflation in Brazil is influenced by the demonstration in Sargent & Wallace (1981), that just monetary policy, does not avoid the inflationary effects of fiscal policy persistently expansionist. When the primary deficits persist, the government budget constraint is violated, and the simple expectation that the money supply will grow, can produce increase in current inflation rates.

Gadelha & Divino (2008) aimed to verify the existence of a fiscal or monetary dominance regime in the Brazilian economy in the Real Plan period through the Granger causality test. The results suggest that the Brazilian economy is under monetary dominance regime, according to the definitions proposed by Sargent & Wallace (1981). In other words, monetary policy is not affected by the dynamics of public debt, featuring a monetary dominance regime.

Moreira, Soares & Sachsida (2011) tested empirically whether the Brazilian fiscal policy for the period between 1995 to 2008 was active or passive. To analyze fiscal policy transmission mechanisms, they estimated functions by which the public debt/GDP ratio affects investment, primary surplus, output gap and the demand for money. They concludes that the Brazilian regime was non-Ricardian in the context of fiscal dominance. Furthermore, the results show that public debt plays a key role in determining variables such as the real demand for money, the ratio of investment to GDP and the output gap. For them, there are strong empirical evidences that the fiscal policy was active and the monetary policy was passive based on Leeper (1991) model. They observed that the public debt negatively affected the level of investment and the output gap.

Until recently, studies on the analysis of macroeconomic dynamics strongly emphasized the effects of monetary policy on macroeconomic variables, especially on price level, sometimes neglecting the role of fiscal policy. However, recent developments, both empirical and theoretical, have shown an increasing relationship between fiscal and monetary policy to guarantee the economic equilibrium. The motivation for this work is to analyze the impact of unexpected fiscal and monetary policies on Brazilian macroeconomic variables. The theme is opportune, in recent years, issues such as sustainability of public debt, high interest rates and inflation control are at the center of discussions on macroeconomic policy, especially in Brazil. For that, it seeks to analyze a dynamic stochastic general equilibrium (DSGE) model, in the framework of models of real business cycles (RBC), with special attention on the effects of interaction between monetary and fiscal policies on the price level. Traditionally, the RBC literature puts technology shocks as the main cause of economic fluctuations. However, the relevance and the role of different shocks, such as technological, fiscal and monetary together, are issues that can contribute to the existing literature on the subject.

Models of Real Business Cycle have been widely used to quantitatively analyze the business cycle. In fact, the article by Kydland & Prescott (1982) introduced a new methodology of macroeconomic analysis that has become one of the main tools of study of the business cycle and the simulation of artificial economies and its comparison with the actual data (MUSSOLINI; TELES, 2012). Several modifications have been made in order to improve their results and assist in the understanding of the business cycle.

In this study, it will be explored the impact of fiscal and monetary policies on the macroeconomic variables under a cash-in-advance (CIA) model. The basic (CIA) assumption is based on Walsh (2010), in a business real cycle model with technological shocks.This setting is similar Cooley & Hansen (1989), where money is held due to a CIA constraint. (RBC) models research has been focused on a class of models in which fluctuations associated with the business cycle are the equilibrium outcome of competitive economies that are subject to exogenous technology shocks (COOLEY; HANSEN, 1989). This model is important because the holding money yields utility. So, the level prices can change the intertemporal decision of agents.

Government budget constraint achieves the goal that changes the stock of money and affects the prices level by enforcing fiscal and monetary policies on the level prices. While the incorporation of persistent shocks can complicate the characterization of equilibrium, the model remains tractable. Equilibrium fiscal policies converge to a stochastic steady state in which they can vary predictably over the business cycle. Upon entering a boom or recession, the deficit public can vary, and will be necessary a primary surplus or seigniorage to make that budget constraint be balanced, given that the tax is fixed.

In order to develop several of important extensions of the (RBC) model with money, following Walsh (2010), Cooley & Hansen (1989), Lucas & Stokey (1987) and Lucas (1982) studies about fiscal and monetary policies, the model that will be studied here differs in order to analyze the impact of discretionary fiscal and monetary policies at the same time on macroeconomics variables.

More recently, empirical and theoretical work investigating the impact of monetary and fiscal policy has relied on estimated dynamic stochastic general equilibrium (DSGE) models: Bhattarai, Lee & Park (2014), Chen, Kirsanova & Leith (2013), Nunes & Portugal (2009), and others. These models combine rational expectations with a microeconomic foundation in which households and firms are assumed to behave optimally, given their objectives (utility maximization, profit maximization) and the constraints they face.

DSGE models have become a standard tool in various fields of economics, most notably in macroeconomics. Nowadays, they are the main tool for analyzing various questions in business cycle theory, monetary and fiscal policy problems, growth or other fields in macroeconomics, or international macroeconomics. They become attractive because they specify explicitly the objectives and constraints faced by households and firms, and then determine the prices and allocations that result from their market interaction on uncertain environment.

Solving DSGE models remains a wide area of research. Both theoretical and computational issues are explored when setting up a DSGE model and applying it for macroeconomic analysis. In short, the simulation will follow the strategy:1) To find all the First order necessary conditions; 2) To calculate the economy steady state; 3) Log-linearize the model around the steady state; 4) To solve for the recursive law of motion; 5) To calculate the impulse response function (IRF) in response to different shocks; 6) To calculate the moments: correlations, and standard deviations for the different variables; 7) To compare how well the model economy matches the actual economy's characteristics.

Unlike studies that take into account only monetary shocks or productivity shocks, this work will introduce simultaneously technological shocks, monetary and fiscal shocks as a source of changes in the dynamic behavior of the economy. This class of models, guided by the optimizer behavior of rational agents in a dynamic environment, and stochastic general equilibrium adds specific features, such as monetary and fiscal shocks to the basic structure of the models real business cycles. This approach is characterized by integrating aspects of the theory of business cycles models, allowing analyze the dynamic behavior of the real economy in the presence of monetary and fiscal shocks, and inflation.

Val & Ferreira (2001) used a model of cash in advance with distorcionary taxation, but did not address fiscal issues. On the other hand, Mussolini & Teles (2012) dealt with fiscal issues, but did not address monetary issues. Filho et al. (2012) used a model of money-in-the-utility function, but did not address fiscal issues, and Magalhaes (2005) used a model with fiscal policy and external shocks, but did not address monetary issues. This work for the Brazilian economy provides an interesting contribution in the methodological field when taking into account the inclusion of political, fiscal and monetary shocks together in a model of real business cycles. In economic theory field, the analysis of monetary and fiscal policies interaction on macroeconomic variables provides an interesting contribution by characterizing, which short-run policy is active or passive.

The study is organized as follows. Chapter two presents a theoretical discussion about real business cycle with fiscal and monetary policies. In addition, it will be made a discussion on the treatment of inclusion of money on models of real business cycle. Chapter three shows the proposed model and the axioms, besides the DSGE approach and strategies of calibration and treatment of data. On the chapter four, there are the computational results and simulations. The chapter five is the conclusion remarks.

1.3 Hypotheses

The hypotheses to be tested are:

1) Interaction between fiscal and monetary policies smooths the volatility of macroeconomic variables;

2) Monetary policy constraints fiscal policy getting a more active role in the economy.

1.4 Objectives

1.4.1 General Objective

This work has as main objective to examine the relationship of monetary and fiscal policies on the main macroeconomic variables in Brazilian cycles.

1.4.2 Specific Objectives

(i) Build a theoretical model with flexible prices in a dynamic general equilibrium setup in which both monetary and fiscal policies influence real and nominal macroeconomic variables;

(ii) Calibrate the model using Brazilian time series from 2000 to 2013;

(iii) Examine the impact of unexpected fiscal policy in Brazilian cycles;

(iv) Examine the impact of unexpected monetary policy in Brazilian cycles;

(v) Analyze the interaction between fiscal and monetary policies in Brazilian cycles;

(vi) Determine which policy, fiscal or monetary, is the active policy during fluctuations in Brazil.

2 Real Business Cycle Theory and Government

2.1 Real Business Cycle theory

The research related to real business cycles models showed a big growth ever since its emergence in early 1980. The initial models were based on the gender of simple economies without imperfections, seeking to explain short run fluctuations of the economy from a Walrasian general equilibrium framework. To King, Ploser & Rebelo (1988), the RBC model cycle analysis investigates the role of neoclassical factors in shaping the character of economic fluctuations.

In a seminal article "Time to Build and Aggregate Fluctuations", Kydland & Prescott (1982) propose a theory of actual business. In this article, the authors have integrated the analysis of long run growth with macroeconomics fluctuations in short run. They demonstrated that many characteristics of the business cycles, such as variables movements and their relative variability can be generated by a model based on technological shocks.

Kydland & Prescott (1990) and Lucas (1977), define the business cycles as the deviations of aggregate real output from trend. Furthermore, the business cycle is an explicit procedure for calculating a time series trend that successfully mimics the smooth curves most business cycle researchers would draw through plots of the data. The business cycle fact as statistical properties of the co-movements shows deviations from trend of various economic aggregates with those of real output. This means that business cycles are recurring fluctuations of various macroeconomic series around an average or constant. The trend can be seen as the growth of the economy at steady state in which the economic activity is characterized by a long-term sustainable growth, i.e, balanced growth path. In this sense, the steady state can be characterized by variables such as product, consumption, investment, capital stock and real wages growing at the same rate.

Acoording to Rebelo (2005), Finn Kydland and Edward Prescott introduced three revolutionary ideas in their 1982 paper. The first idea, which builds on prior work by Lucas & Prescott (1971), is that business cycles can be studied using dynamic general equilibrium models. These models feature atomistic agents who operate in competitive markets and form rational expectations about the future. The second idea is that it is possible to unify business cycle and growth theory by insisting that business cycle models must be consistent with the empirical regularities of long run growth. The third idea is that we can go way beyond the qualitative comparison of model properties with stylized facts that dominated theoretical work on macroeconomics until 1982. We can calibrate models with parameters drawn, to the extent possible, from microeconomic studies and long run properties of the economy, and we can use these calibrated models to generate artificial data that we can compare with actual data.

The nature of this model's name is because of their emphasis on the role of all shocks, particularly technology shocks, in driving business fluctuations. In addition, this model came to be widely used as laboratories for policy analysis in general and for the study of optimal fiscal and monetary policy (REBELO, 2005). Lucas (1980), argued that:

> One of the functions of theoretical economics is to provide fully articulated, artificial economic systems that can serve as laboratories in which policies, that would be prohibitively expensive to experiment with in actual economies, can be tested out at much lower cost. To serve this function well, it is essential that artificial "model" economy be distinguished as sharply as possible in discussion from actual economies. (...) Though, we are interested in models because we believe they may help us to understand matters about which we are currently ignorant, we need to test them as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies, or parts of economies, would react. (LUCAS, 1980)

Following Plosser (1989), Real business cycle (RBC) models view aggregate economic variables as the outcomes of the decisions made by many individual agents acting to maximize their utility subject to production possibilities and resource constraints. As such, the models have an explicit and firm foundation in microeconomics. More explicitly, real business cycle models ask the question: How do rational maximizing individuals respond over time to changes in the economic environment and what implications do those responses have for the equilibrium outcomes of aggregate variables?

The study of business cycles, frequently take about notions of persistence or serial correlation in economic aggregates; co-movement among economic activities; leading or lagging variables relative to output; and different amplitudes or volatilities of various series. The RBC model must be dynamic to show the fluctuation, and the objective is to generate an understanding of how and why the fluctuations arise.

Real business cycle research has focused almost exclusively on models with no role for money in the initial researches. For some economists. Any works have explored methods of incorporating money and investigating its implications in a real business cycle model. Without an understanding of the real fluctuations inherent in the basic neoclassical model without money, it will be difficult if is not impossible to measure the quantitative importance of money in actual business fluctuations. The nature and magnitude, however, of the fluctuations and responses in the real neoclassical model means that real business research poses a challenge to conventional views regarding the relative importance of money. This is particularly true given the difficulties that economists have faced in developing a convincing and coherent explanation of the monetary transmission mechanism (PLOSSER, 1989).

2.2 Real Business Cycle model and money

Money has very important functions in a market economy, and improves economic efficiency by economizing on the costs of information. Nevertheless, a significant portion of the research in monetary economics over the past years has been devoted to developing and exploring a monetary theory of the business cycle (PLOSER, 1990)

The neoclassical growth model due to Ramsey (1928) and Solow (1956) provides the basic framework for much of modern macroeconomics. However, these models are nonmonetary economies. Solow's growth model with three key ingredients: a production function allowing for smooth substitutability between labor and capital in the production of productivity, a capital accumulation and labor supply process. As a result, for Solow, the economy would converge to a steady-state growth path along which output, the capital stock, and effective supply of labor (WALSH, 2010). To explain about monetary policy is necessary to introduce a monetary instrument.

As Walsh (2010) argued, the neoclassical growth model is a model of nonmonetary economy. It is important to exist a medium of exchange that is the money, able to make the transactions. He continue saying that, to employ the neoclassical framework to analyze monetary issues, the money must be especified, so that the agents will wish to hold positive quantities of money. A positive demand for it is necessary if, in equilibrium, money has a positive value.

For Gali (2008), the attempts by Cooley & Hansen (1989) and any others to introduce a monetary sector in an otherwise conventional RBC model, were not perceived as yielding a framework that was relevant for policy analysis. The resulting framework, which is referred to as the classical monetary model, generally predicts neutrality of monetary policy with respect to real variables. That finding is at odds with the widely held belief (certainly among central bankers) in the power of that policy to influence output and employment developments, at least in the short run [1].

Following Walsh (2010), there are three general approaches to include money into a general equilibrium model. First, assume that money yields direct utility by incorporating money balances into the utility functions of the agents of the model, that was developed by Sidrauski (1967). Second, impose transaction costs of some form that give rise to a demand for money, by making asset exchanges costly, developed by Baumol (1952) and tobin (1956). Furthermore, requiring that money be used for certain types of transactions, by Clower (1967). Also, assuming that time and money can be combined to produce

[1] View Friedman & Schwartz (1963).

transaction services that are necessary for obtaining consumption goods (Brock (1974); McCallum & Goodfriend (1987)). Finally, third, treat money like any other asset used to transfer resources intertemporally, by Saumelson (1958).

The first of the three approaches by incorporating into the basic neoclassical model agents, whose utility depends directly on their consumption of goods and their holdings of money. Explain how can we introduce money - money is necessary to make transactions - basically we have: Money-in-the-Utility Function (MIU) Sidrauski (1967), CIA Clower (1967), money as asset allowing to shift resources intertemporally Saumelson (1958).

In Sidrauski (1967), model of money in the utility function in a neoclassical growth model is quite popular. The superneutrality result, that steady state real output is independent of the growth rate of money supply, is the common artificial in studies of monetary policy. This model is part of those that take into account the importance of currency within the general equilibrium models. It is assumed that the money generates direct utility by incorporating real money balances in the utility function of economic agents.

Sidrauski (1967), the MIU approach assuming that real money balances can be directly included in the agents' utility function. The MIU model is a useful framework to study the interactions in monetary economics, relationship among money, prices and inflation, effects of monetary policy on the economy in balance.

In this model the utility depends directly on the consumer goods and also maintaining currency. On balance, agents choose positive quantities coin so that the coin will have positive values

In summary, in the Model Money-in-the-Utility Function (MIUF), money is regarded as an ordinary commodity from which an individual derives its usefulness. The MIU approach helps to model the cash liquidity services, although it is not able to provide an answer to how the agents use exactly your money.

Farmer (1997) constructed a real business cycle model in which real money balances yield utility. He calibrated the model for the U.S data and simulated a set of impulse response functions that are generated by the model for GDP, the rate of interest, money growth and real balances. The model corresponded in to capturing the dynamic interactions of money and real variables in U.S data.

Clower (1967) suggests the cash-in-advance constraint approach, assuming that agents need money to purchase goods, money being the only means of payment. In this model, money is considered to play a medium of exchange role.

The CIA approach requires that purchases of goods must be paid by currency held from a preceding period. In addition, to standard budget constraint individuals face cash-in-advance constraint, in order to support future spending in period t, agents must hold enough cash for them from period t-1, that is $c_t \leq \frac{Mt-1}{P_t}$. In others words, the real spending on consumptions in a current period cannot exceed the amount of real money balances carried from the preceding period.

In this model, every period a consumer has to chose their consumption, their money balances and their saving. However, all consumption goods have to be paid for by cash, with respect the constraint that the consumer faces, $c_t \leq \frac{Mt-1}{P_t}$. The velocity of money (V) is defined by the identity MV=PY, where M is the money supply, P is the price level and Y is the volume of transactions in the economy.

The basic cash-in-advance model is due to Svensson (1985). His main concern is how to price assets when you have cash in advance constraint. It is assumed that consumers have to choose how much cash to hold before they know the current state of the world. As a result of this uncertainty the velocity of money is no longer constant. Agents will prefer choose to hold $m > c$ for precautionary reasons. For simplicity, when there is no uncertainty, the agents know exactly how many money balances to keep out of the previous period for consumption in the current period. Every period a consumer observes a state of the world and makes a decision over the amount of consumption, money balances and saving assets.

Another version of the CIA mode can be view as the cash-credit model developed by Lucas & Stokey (1987)[2]. In this model agents gain utility from two goods,c_1 and c_2, where c_1 can only be purchased using cash, and c_2 can be purchased on credit. The timing of the model is as follows is that, the agents observe the state of the world, decide on c_1 and c_2 and m, they then go and purchase cash goods paying for them with their money balances and also purchase credit goods, and then at the end of the period all credit bills are settled. This is another way of making the velocity of money variable. In this model, agents get utility from two goods, but on one good they have to pay cash and so lose interests on any assets held in the form of cash. Therefore, when the interest rate is high they will tend to lower c_1 and increase c_2 to compensate, because they consume less of the cash good they also hold fewer money. Therefore the velocity of money ($(c_1 + c_2)$/m) varies positively with the interest rate - the higher the interest rate, the lower are money balances.

A shopping-time model is a class of models that modeling the role of money in facilitating transactions is to assume that the purchase of goods requires the input of transaction services. The central assumption is that purchasing goods requires the input of transaction services. In this model is assumed that time and money produce transaction services. So, the consumer must balance the opportunity cost of holding money against the value of leisure in deciding how to combine time and money to purchase consumption goods ((WALSH, 2010); (ANTIPINA, 2013)).

[2] Another version can be view in Cooley & Hansen (1998), Cooley & Hansen (1995).

This model suggests that higher levels of money holdings reduce the time needed for shopping, thereby increasing the individual agent's leisure. There is a trade off between the opportunity cost of holding money and leisure value. The consumer must choose how to combine time and money to facilitate transactions, taking into account the fact that the more money he has, the less time it fails to produce transaction services. Demand for money depends on the cost of transaction services, in terms of time spent in shopping and be time left for leisure. According to their level of currency holdings and their level of consumption, the consumer decides how much time to spend shopping (ANTIPINA, 2013). It is straightforward to show that shopping-time models and MIU models are equivalent.

2.3 Theories of fiscal and monetary policies on the price level

Monetarist theory monetarist orientation describes a monetary authority that sought primarily control the price level, assuming that the government debt would always be solvent. The use of monetary policy discretion could only smooth business cycles. In addition, the rules would be subject to dynamic inconsistency and called the Lucas's critique. In this theoretical line we highlight the work of Kydland & Prescott (1977), Lucas (1983), Sargent & Wallace (1981) and Sargent (1982). On the other hand, the more interventionist orientation of literature, or Keynesian, highlights the importance of discretionary policies. However, to overcome the criticism of the monetarist discretionary policies, Keynesian models were refined by introducing microeconomic foundations, such as rational expectations and price rigidity. In this line of thought, the work of Leeper (1991), Taylor (1993), Sims (1994) and Woodford (1995), among others, were the precursors. However, fiscal policy was relegated to background, it was assumed a Ricardian regime, in which the government budget was always in balance, subject to only cyclically variations, and any imbalance would be corrected via taxes or inflation tax. With the assumption behind Ricardian equivalence, in which the tax authority is well disciplined, that is, cutting taxes for the financial increases in government debt should be offset by higher taxes in the future to ensure that the debt is solvent. In this context, the discussion of coordination between fiscal and monetary policies makes no sense.

Inflation is kind of tax. And as a tax, they both generate revenue for the government and distorts private sector. In the Sidrauski model, inflation distorts the demand for money, thereby generating welfare effects because real money holdings directly yield utility. In cash-in-advance model, inflation serves as an implicit tax on consumption, so a higher inflation rate generates a substitution toward leisure, leading to lower labor supply, output, and consumption (WALSH, 2010).

Following Walsh (2010) for government to obtain goods and services, it is necessary to generate revenue. And one way that it can obtain goods and services is to print money that is then used to purchase resources from the private sector. The government's budget constraint is:

$$G_t + i_{t-1}B_{t-1}^T = T_t + (B_t^T - B_{t-1}^T) + RCB_t, \forall t \tag{2.1}$$

Where variables are in nominal terms. The left side consists of government expenditures on goods, services, and transfers G_t plus interest payments on the outstanding debt $i_{t-1}B_{t-1}^T$ (the superscript T denoting total debt, assumed to be one period in maturity, where debt issued in period t-1 earns the nominal interest rate i_{t-1} . The right side consists of lump-sum T_t, plus new issues of interest-bearing debt $B_t^T - B_{t-1}^T$, plus any direct receipts from the central bank (RCB_t). This equation is referred to as the *treasury's budget constraint*.

The monetary authority, or central bank, also has budget identity thas links changes in its assets and liabilities. Therefore,

$$(B_t^M - B_{t-1}^M) + RCB_t = i_{t-1}B_{t-1}^M + (H_t - H_{t-1}), \tag{2.2}$$

Where $B_t^M - B_{t-1}^M$ is equal to the central bank's purchases of government debt, $i_{t-1}B_{t-1}^M$ is equal central bank's receipt of interest payments from the Treasury, and $H_t - H_{t-1}$ is the change in the central bank's own liabilities. These liabilities are the monetary base because they form the stock of currency held by the nonbank public plus bank reserves, and they represent the reserves private bank can use to back deposits.

Letting $B = B^T - B^M$ be the stock of government interest-bearing debt held by the public, the budget identities of the Treasury and the central bank can be combined to produce the consolidated government sector budget identity:

$$G_t + i_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}). \tag{2.3}$$

This mean that, the government purchases G_t, plus payment of interest on outstanding privately held debt $i_{t-1}B_{t-1}$, must be funded by revenue that can be obtained from one of three alternative sources. First, T_t is the revenue generated by taxes. Second, borrowing from the private sector (equal $B_t - B_{t-1}$). And finally, the government can print currency to pay for its expenditures, and is represented by the change in the outstanding stock of non-interest-bearing debt ($H_t - H_{t-1}$).

The last equation can be divided by the price level P_t to obtain

$$\frac{G_t}{P_t} + i_{t-1}(\frac{B_{t-1}}{P_t}) = \frac{T_t}{P_t} + \frac{B_t - B_{t-1}}{P_t} + \frac{H_t - H_{t-1}}{P_t} \tag{2.4}$$

The government's budget identity in real terms is

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + h_t - \frac{h_{t-1}}{(1+\pi_t)} \tag{2.5}$$

Where $r_{t-1} = [(1 + i_{t-1})/(1 + \pi_t)] - 1$ is the ex post real return from $t - 1$ to t.

The last term represents seignorage, the revenue from money creation. Seignorage can be written as

$$s_t \equiv \frac{H_t - H_{t-1}}{P_t} = (h_t - h_{t-1}) + (\frac{\pi_t}{1 + \pi_t})h_{t-1} \tag{2.6}$$

Seigniorage arises from two sources. First, $h_t - h_{t-1}$ is the change in real high-powered money holding. An increase in the amount of high-powered money that private sector is willing to hold allows the government to obtain real resources in return. In a steady-state equilibrium, h is constant and seigniorage is zero. In Walsh (2010), the second term in equation can be nonzero even in the steady state. To maintain a constant level of real money holdings, the private sector needs to increase its nominal holdings of money at the rate π to offset the effects of inflation on real holdings. By supplying money to meet this demand, the government is able to obtain goods and services or reduce other taxes.

If θ is the growth rate of the nominal monetary base H, the growth rate of h will equal $(\theta - \pi)/(1 + \pi) \approx \theta - \pi$[3]. In a steady state, h will be constant, implying that $\pi = \theta$. In this case, the seigniorage will equal $(\frac{\pi}{1+\pi})h = (\frac{\theta}{1+\theta})h$. For small values of the rate of inflation, is approximately equal to π, so s can be thought of as the product of a tax rate of π, the rate of inflation, and a tax base of h, the real stock of base money. Base money does not pay interest; its real value is depreciated by inflation.

For a given level of the government's total real liabilities $d = b + h$, interest costs will be a decreasing function of the fraction of this total that consists of h. A shift from interest-bearing to non-interest-bearing debt would allow the government to reduce total tax revenues or increase transfer or purchases. Using (2.5) and (2.6) to consider the government's budget constraint expressed in terms of the total liabilities of the government is [4]:

$$g_t + r_{t-1}d_{t-1} = t_t + (d_t - d_{t-1}) + (\frac{i_{t-1}}{1 + \pi_t})h_{t-1} \tag{2.7}$$

The seigniorage is the last term $\bar{s} = (\frac{i}{1+\pi})h$

The relevant tax rate on high-powered money depends directly on the nominal rate of interest. Thus, under the Friedman rule for the optimal rate of inflation, which calls for setting the nominal rate of interest equal to zero, the govern collects no revenue from seigniorage. Reducing the nominal interest rate to zero implies that the lost revenue must be replaced by an increase in other taxes, real borrowing that increases the government's net indebtedness, or reductions in expenditures.

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3 See Walsh (2010).

4 Add $r_{t-1}h_{t-1}$ on both sides and use the Fisher equation: $(1 + r_{t-1}) = (1 + i_{t-1})/(1 + \pi_t)$.

The budget relationship links the government's choices concerning expenditures, taxes, debt, and seigniorage at each point in time. If governments, like individuals, are constrained in their ability to borrow, then this constraint limits to choice. Ignoring the effect of surprise inflation, the single-period budget identity of the government is given by:

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + s_t \tag{2.8}$$

Assuming the interest factor r is a constant and positive, this equation can be solved forward to obtain:

$$(1+r)b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i}}{(1+r)^i} = \sum_{i=0}^{\infty} \frac{t_{t+i}}{(1+r)^i} + \sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} + \lim_{i\to\infty} \frac{b_{t+i}}{(1+r)^i} \tag{2.9}$$

The government's expenditure and tax plans satisfy the requirement of intertemporal budget balance (the *no Ponzi game condition*) if the last term is zero. The right side becomes the present discounted value of all current and future tax and seigniorage revenues, and this is equal to the left side, which is the present discounted value of all current and future expenditures plus current outstanding debt. Letting $\triangle = g - t - s$, intertemporal budget balances implies:

$$(1+r)b_{t-1} = -\sum_{i=0}^{\infty} \frac{\triangle_{t+i}}{(1+r)^i} \tag{2.10}$$

Thus, when $(b_{t-1} > 0$, the present value of future primary deficits must be negative (i.e., the government must run a primary surplus in present value. The surplus can be generated through adjustments in expenditures, taxes or seigniorage.

The nominal money supply could change because of a shift from tax-financed government expenditures to seigniorage-financed expenditures or as the result of an open market operation in which the central bank purchases interest-bearing debt, financing the purchase by an increase in non-interest-bearing debt, holding other taxes constant. Because these two means of increasing, the money stock have implications for taxes and stock of interest bearing government debt, they may lead to different effects on prices and/ or interest rates. The government budget constraint linked monetary and fiscal policies in ways that can matter for determining how a change in the money stock affects the equilibrium price level.

In most traditional analyzes about the relationship between monetary and fiscal policies, the last is assumed to adjust to ensure that the government's intertemporal budget is always in balance while monetary policy is free to set the nominal money stock or the nominal rate of interest.

If fiscal policy affects the real rate of interest, then the price is not independent of fiscal policy, even under regimes of monetary dominance. A balanced budget increase in expenditures that raises the real interest rate raises the nominal interest rate and lowers the real demand for money. Given an exogenous path for the nominal money supply, the price level must jump to reduce the real supply of money. Another case is one in which the fiscal authority sets its expenditure and taxes without regard to any requirement of intertemporal budget balance. If the present discounted value of these taxes were not sufficient to finance expenditures, seignorage must adjust to ensure that the government's intertemporal constraint is satisfied. This regime is call *fiscal dominance* and passive monetary, as monetary policy must adjust to deliver the level of seigniorage required to balance the government's budget. Inflation and prices are affected by changes in fiscal policy because these fiscal changes, if they require a change in seigniorage, alter the current and/ or future money supply. It is called a *Ricardian regime* when taxes and/ or seigniorage adjust to ensure the government's intertemporal budget constraint.

The intertemporal budget constraint implies that any government with a current outstanding debt must run, in present value terms, future surpluses. One way to generate a surplus is to increase revenues from seignorage, and this has implications of budget deficits for future money growth. The problem that can emerge is impact of prices when the monetary authority must act to ensure that government's intertemporal budget is balanced. This interpretation views fiscal policy as set independently, so that the monetary authority is forced to generate enough seignorage to satisfy the intertemporal budget balance condition. From (2.10), the government's intertemporal budget constraint is given by:

$$b_{t-1} = -R^{-1} \sum_{i=0}^{\infty} R^{-i} (g_{t+i} - t_{t+i} - s_{t+i}) \tag{2.11}$$

Where $R \equiv 1 + r$ are the real interest rate, $g_t - t_t - s_t$ is the primary deficit, and s_t is the real seignorage revenue. Let $s_t^f \equiv t_t - g_t$ be the primary fiscal surplus. Then the government's budget constraint can be written as:

$$b_{t-1} = R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i}^f + R^{-1} \sum_{i=0}^{\infty} R^{-i} s_{t+i} \tag{2.12}$$

In that the government debt is financed, in present value terms, by either a fiscal primary surplus or seigniorage. It is what Sargent & Wallace (1981) showed as "unpleasant monetarist arithmetic" in a regime of fiscal dominance. When the present value of the fiscal primary surplus is reduced, the present value of seigniorage must rise to maintain the debt controlled. If the fiscal authority does not adjust, the monetary authority will be forced eventually to produce higher inflation. In a regime of monetary dominance, the monetary authority can determine inflation and seigniorage, the fiscal authority must then adjust either taxes or spending to ensure that 2.12 is satisfied.

Fiscal considerations determine money supply, but the traditional quantity theory holds that the price level is proportional to the nominal quantity of money. Even that the initial nominal stock of money is set exogenously by monetary authority, the fiscal policy can affect the initial equilibrium price level when the initial nominal quantity of money is given and the government's intertemporal budget constraint must be satisfied at all price levels.

2.4 Ricardian Equivalence

The causes and consequences of rising budget deficits have received increased attention in both developed and developing countries. The funding deficit theory has been widely discussed especially in the context of the Ricardian equivalence theorem. One of the recurring themes in economic research that arouse curiosity are the impacts of government interventions in the economy. It has being understood that fiscal policy interventions - that is, related to changes in tax debt and government spending - are related to these economic questions: how changes in the public deficit (or debt) are able to generate change the real variables of the economy? And if so, what should be the magnitude and direction? (SACHSIDA; CARLUCCI, 2010).

The debate about the importance of fiscal policy and its economic effects has been recurrent in recent years. Increased deficits and public debt has been a problem, particularly in developing economies. The way how this deficit is financed has no effect, according to the theory of Ricardian equivalence.

The idea that agents are indifferent to the way the government finances its spending is through the issuance of bonds or taxation, and it is not new in economics. It was first proposed by the British economist David Ricardo in the nineteenth century. The central argument of Ricardian equivalence approach is that the financing of public spending on the debt issue has the same effect on economic activity when financing through taxes.

Given the occurrence of government deficits and therefore increased public debt, rational agents anticipate the need for future taxes increases in order to give conditions to the government to meet the financial burden of debt. Thus, the increase in disposable income and consumption capacity in this will be offset by a further reduction. Under these conditions, rational agents would tend to retain the current increase in income in the form of savings, which will finance the future increase in tax expense. Therefore, the public deficit would not bring any benefit in terms of economic growth and, conversely, a negative impact on the well being of society, represented by the debt burden to be paid by future generations. Hence, the need for a fiscal policy with a balanced budget.

In 1974, Barro questioned whether government securities represented net wealth. Since then, this issue has to be explored in the literature. The research seeks to understand whether consumers considered a tax in the short term, when kept government spending, as an increase in taxes in the future, so that taxes would be discounted in the present, so that the level of private consumption does not suffer changes.

The assumption is that the government bonds are perceived as net wealth by the private sector plays an important role in theoretical analyses of monetary and fiscal effects. It is important in demonstrating the real effects of a shift in the stock of public debt, and in establishing nonneutrality of changes in the stock of money.

Barro (1974) begins with the effect of government bond issue on the calculus of individual wealth in an overlapping-generations economy with physical capital where individuals have finite lives, and no elements of capital market imperfections are introduced into his model.

Barro (1974) uses a version of the Samuelson-Diamond overlapping-generations model with physical capital. The individual lives two periods: young and old and, each generation have the same number of people, N, and all individuals are identical in terms of tastes and productivity. Generations are numbered beginning with the generation which is currently old (subscript 1) and currently young (subscript 2). The young agent works and receive an amount of wage w. There is no uncertainty on future wages or income. Asset holdings (A) take the form of equity capital (k). Government bonds are introduced as an additional form in which assets can be held. The rate of return on assets is r and will be paid out once per period. Expectations on r of future periods are assumed to be static. This way, a member of the ith generation holds the amount of assets A_i^y while young and the amount A_i^o while old. The asset holding while old constitutes the provision of bequest, wish is assumed to go the immediate descendant, a member of generation $i+1$. For simplifications, the government neither demands commodities nor provides public services.

In model, the letter c denote consumption, and is assumed that consumption and receipt of interest income both occur at the start of the period, the budget equation for a member of generation 1, who is currently old, is

$$A_1^y + A_0^o = c_1^o + (1-r)A_1^o \tag{2.13}$$

The resources available are the assets while young, A_1^y, plus the bequest from the previous generation, A_0^o. The total expenditure is consumption while old, c_1^o, plus the bequest provision, A_1^o, less interest earnings at rate r on this asset holding.

To generation 2, is assumed that wage payments occur at the start of the young period:

$$w = c_2^y + (1-r)A_2^y \tag{2.14}$$

and, for the old period:

$$A_2^y + A_2^o = c_2^o + (1-r)A_2^o \tag{2.15}$$

A portion of the lifetime resources of a member of generation i goes to a bequest provision, A_1^o, wish the author assume motivated by a concern for a member of generation i+1.

Is assumed that the utility of a member of generation i depend solely on own two-period consumption, c_i^y and c_i^o, and on the attainable utility of his immediate descendant, U_{i+1}^*[5].

The utility function for a member of the ith generation:

$$U_i = U_i(c_i^y, c_i^o, U_{i+1}^*) \tag{2.16}$$

Each member of generation 1 determines his allocation of resources to maximize U_1, subject to equations (2.13)-(2.16) and the inequality conditions, $(c_i^y, c_i^o, A_i^o) \geq 0$ for all i. The key restriction is that the bequest to the member of the next generation cannot be negative. The solution to this problem will take the general form;

$$\begin{aligned} c_1^o &= c_1^o(A_1^y + A_0^o, w, r), \\ A_1^o &= \frac{1}{1-r}(A_1^y + A_0^o - c_1^o) = A_1^o(A_1^y + A_0^o, w, r). \end{aligned} \tag{2.17}$$

For the generation 2, and $i \geq 2$:

$$\begin{aligned} c_2^y &= c_2^y(A_1^o, w, r), \\ A_2^y &= \frac{1}{1-r}(w - c_2^y) = A_2^y(A_1^o, w, r), \\ c_2^o &= c_2^o(A_2^y + A_1^o, w, r), \\ A_2^o &= \frac{1}{1-r}(A_2^y + A_1^o - c_2^o) = A_2^o(A_2^y + A_1^o, w, r). \end{aligned} \tag{2.18}$$

The model can be closed by specifying a constant-returns-to-scale production function that depends on the amounts of capital and labor input, and equating the

[5] The * denotes the maximum value of utility, conditional on given values of endowment and prices.

marginal products of capital and labor, r and w, respectively. The value of r for the current period would then determine in order to equate the supply of assets to demand:

$$K(r,w) = A_1^o + A_2^y \tag{2.19}$$

With the marginal product of labor equated to w and with constant returns to scale, output is given by:

$$y = rK + w \tag{2.20}$$

The equations (2.14), (2.15), (2.19), and 2.20, imply a commodity market clearing condition:

$$c_1^o + c_2^y + \Delta K = y \tag{2.21}$$

In equation, ΔK is the change in capital stock from the previous to the current period. In steady state the value of ΔK would be zero.

2.4.1 Government Debt

The model of Barro (1974) is supposed that the government issues an amount of debt, B, which can be thought of as taking the form on-period, real-valued bonds. These bonds pay the specified amount of real interest, rB, in the current period and the specified real principal, B, in the next period. It s suppose that asset holders regard equity and government bonds as perfect substitutes. For the households, it could be assumed that the bonds were sold on a competitive capital market, with the proceeds from this sale used to effect a lump-sum transfer payment to generation 1 households. Allowing some portion of the proceeds to go to generation 2 households would not alter any the basic conclusions.

The future interest payments will be financed in some manner. Interest payments will be financed by a lump-sum tax levy on generation 2 households (while young), and that the principal is paid off at the beginning of the next period by an additional lump-sum tax levy on generation 2 households (while old). In this setup there is no direct effect of the government debt issue and its financing on generation 3 later generation.

The generation 1 budget constraint is:

$$A_1^y + A_0^o + B = c_1^o + (1-r)A_1^o \tag{2.22}$$

where B is the lump-sum transferred payment in the beginning of the period. For the generation 2, the budget restriction constraint becomes:

$$w = c_2^y + (1-r)A_2^y + rB \tag{2.23}$$

And for the next period's budget constraint for generation 2 is:

$$A_2^y + A_1^o = c_2^o + (1-r)A_2^o + B \tag{2.24}$$

After combined the two generation into a single period budget equation, becomes:

$$w + (1-r)A_1^o - B = c_2^y + (1-r)c_2^o + (1-r)^2 A_2^o \tag{2.25}$$

The Utility function for the generation 2 can be written in the indirect form:

$$U*_2 = f_2^*[(1-r)A_1^o - B, w, r] \tag{2.26}$$

The "net bequest", $(1-r)A_1^o - B$, determines the "endowment" for members of generation 2.

From equation (2.22), c_1^o varies inversely with $(1-r)A_1^o - B$ for a giver value of $A_1^y + A_0^o$. For Barro (1974), given the pre-determined value of c_1^y and using equations (2.16), (2.22), and (2.26), U_1 can be rewritten:

$$U_1 = U_1(c_1^y, c_1^o, U_2^*) = f_1[(1-r)A_1^o - B; c_1^y, A_1^y + A_o^o, w, r] \tag{2.27}$$

For given values of $c_1^y, A_1^y + A_0^o$, w, and r, the choice problem for members of generation 1 amounts to the optimal selection of the net bequest, $(1-r)A_1^o - B$, subject to the constraint that the gross bequest, A_1^o, positive. Any marginal change in B would be met solely by a change in A_1^o that maintains the value of the net bequest, $(1-r)A_1^o - B$. This response in A_1^o will keep unchanged the values of c_1^o, c_2^y, c_2^o, and A_2^o. Hence, the utility levels attained by members of generatinos1,2, etc., will be unaffected by the shift in B.

In terms of the effect on r, the equation (2.19) is modified to:

$$K(r, w) + B = A_1^o + A_2^y. \tag{2.28}$$

The increase in B implies a one-to-one increase in the asset supply. A_1^o rises by $\frac{1}{1-r}B$ in order to maintain the size of the net bequest, $(1-r)A_1^o - B$. With c_2^y fixed, the increase in rB (taxes) implies that A_2^y falls by $\frac{r}{1-r}B$. On net, total asset demand on the right-hand side of equation (2.28) rises one-to-one with B, so that on change in r is required to clear the asset market. Equivalently, the commodity market clearing condition continues to hold at the initial value of r because the bond issue has no impact on aggregate demand (BARRO, 1974).

For a positive value of B, financed by a tax levy on the next generation, enables a member of the old generation to leave the debt for his descendant. However, if the member of the old generation had already selected a positive bequest, this individual already had an option of shifting resources from his descendant to himself. Since the change in B does not alter the relevant opportunity set in this sense, the values of current and future consumption and attained utility will be unaffected.

On the other hand, if the bequest were zero or negative, $(A_1^o = 0, or A_1^o < 0)$, then an increase in B creates a relevant new opportunity. Generation 1 household would react by increasing c_1^o. The upward shift in B would then correspond to an excess of earning-asset supply over demand, which would tend to raise the value of r. This increase in r would induce a drop in capital formation, which constitutes the real effect of government debt. This happens when there is a non-operative bequest motive. The main point is an operative intergenerational transfer [6]

According Barro (1974), changes in government debt would not induce any alteration in consumption plans even a model where (1) the present generation have finite lives, (2) the present generation may, in some sense, give lesser weight to the consumption or utility of future generations that they give to own consumption, and (3) the present generation may give direct weight at all to consumption or utility of generation beyond their immediate descendants. A sufficient condition for changes in government debt to have no impact on consumption plans and no effect on aggregate demand and interest rates is that the solution for current generation's inheritances also be interior. The result will hold as long as current generations are connected to all future generations. This condition can be a difficult problem and require some specialization of the form of the utility function in order to make any headway.

If the budget deficit contributes to aggregate demand, on should also observe that the price level ultimately rises when the budget deficit rises. Ricardian equivalence provides a coherent theoretical explanation. According to this explanation, governments must service their debts with future taxes, households anticipate these future tax liabilities, and they internalize the choices of future generations. As a result, households do not raise consumption when their government's budget deficit increases. Therefore, aggregate

[6] View note 7 and 8 in Barro (1974), page 1104.

demand, nominal and real interest rates, nominal and real exchange rates and the price level are unaffected Evans (1987).

In Barro (1974) version, the Ricardian equivalence proposes that when an increase in the public deficit, government saving is exactly offset by an increase in private saving, so that national saving (or domestic) does not change. Thus, the public deficit has no effect on economic activity and, therefore, on the real output of the economy. This neutrality can be associated with the fact that a tax in this period, in case of a possible increase in public debt, would mean higher taxes in the future, so that economic agents will not alter their consumption patterns, and thus policy tax would not change the trajectory of the real macroeconomic variables.

The notion of Ricardian equivalence has come to play an important role in modern economic theory, due in large part to the work of (BARRO, 1974). In evaluating of the Ricardian equivalence, it is essential to distinguish between the short-run effects and the long-run effects. The short-run effects refer, primarily, to the potential for stimulating aggregate demand, and its implications for macroeconomic stabilization policy. On the other hand, the long-run effects are, primarily, about the potential for depressing capital accumulation (BERNHEIM, 1987).

In according Gali, Vallés & López-Salido (2007), the model of real business cycle pattern Business generally provides a decline in consumption in response to an increase in government spending on goods and services. In contrast, the IS-LM model predicts consumption growth, amplifying the effects of the expansion of government spending on product. The reason for the different impacts between these two models is in how consumers behave in each case. The model RBC presents families living infinitely (Ricardian), where consumption decisions at any point in time are based on an intertemporal budget constraint. Ceteris paribus, an increase in government spending reduces the present value of post-tax income, generating a negative wealth effect that induces a cut in consumption. On the other hand, consumers IS-LM behave non-Ricardian way in which consumption is a function of their current disposable income and not its resources throughout life. Thus, the effect of an increase in government spending will depend on how the latter is supported with the multiplier increases with the extent of financing.

For a given path of government spending, a deficit-financed cut in current taxes leads to higher future taxes that have the same present value as the initial cut. This result follows from the government's budget constraint, which equates total expenditure for each period to revenues from taxation or other sources and the net issue of interest-bearing public debt. In the perspective The Ricardian view proposes that the substitution of a budget deficit for current taxes, or an alternative temporal arrangement, has an equal effect on aggregate demand. Therefore, the substitution of a budget deficit for current taxes has no impact on the aggregate demand for goods. In this sense, budget deficit and taxation have equivalent effects on the economy (BARRO, 1989).

The Ricardian equivalence proposition is based on some strong assumptions: (i) individuals bahave as if they had an infinite horizon since the care about their offspring; (ii) the capital market is perfect; (iii) consumers are rational and farsighted: (iv) future incomes and taxes are certain; and (v) taxes are non-distortionary (KHALID, 1996)

Such an "equivalence" arises because economic agents, being fully aware of the path of future fiscal policies, consider today's deficit financing as tomorrow's tax liabilities. (a) If capital markets are perfect and consumers do not face any borrowing constraints; (b) both private and public sectors have the same planning horizons; and (c) taxes are non-distortionary, then Ricardian Equivalence theorem holds (GHATAK; GHATAK, 1996).

3 The Model

3.1 The model of Cash In Advance Constraint (CIA)

The model of economy will be a basic real business cycle model with money introduced by imposing a cash-in-advance constraint (CIA) like (COOLEY; HANSEN, 1989) in that money is incorporated into a real business cycle model using a CIA constraint. However, in each period, agents must first go to the money market and obtain cash for transactions in both the consumption and capital goods markets. In addition, this model will be used to analyze monetary and fiscal polices in control of level prices. This economy consists of a continuous of infinitely lived households with identifical preferences and initial endowments. The timing is as follows, the agent begins period t with money M_{t-1} and receives the lump-sum nominal transfer T_t. Prices are subsequently determined and the goods market opens.

3.1.1 Consumer's preferences

The representative household maximizes the following lifetime utility function:

$$U = E_0 \sum_{t=o}^{\infty} \beta^t u(c_t, 1 - n_t) \tag{3.1}$$

In that $0 < \beta < 1$ is the subjective discount factor, n_t is the labor expressed as a fraction of the time available, and $1 - n_t$ is leisure time. E_0 is the expectation. The utility function $u(.)$ is bounded, continuosly differentiable, strictly increasing, and strictly concave. The agent enters the period with money holdings M_{t-1} and receives a lump-sum transfer T_t (in nominal terms). The household then goes to the asset market. The model of Lucas (1982) assumed that the asset market opens before, and individuals can engage in asset transaction at the start of each period before the goods market has opened. This means that the agent enters period t with wealth that can be used to purchase nominal bonds B_t or carried as cash into the goods market to purchase consumption goods. In contrast, here the good market open first[1]. Therefore, the timeline for this model is demonstrated in Figure 1:

The shopper takes the money and buys goods c_t subject to cash in advance constraint. If the goods markets open first, the agent enters the period with money

[1] If assets market open first, the CIA constraint becomes $c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau - b_t$. In this case, individuals can engage in asset transactions at the start of each period before the goods market has opened.

Figure 1 – Timing of the Model.

holdings M_{t-1} and receives a lump-sum transfer T_t (in nominal terms), the CIA constraint takes the form:

$$(1+\tau_t^c)P_t c_t \leq M_{t-1} + T_t \tag{3.2}$$

where c_t is real consumption, P_t is the aggregate price level, and T_t is the nominal lump-sum transfer. τ_t^c is the consumption's tax. In real terms,

$$(1+\tau_t^c)c_t \leq \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} = \frac{m_{t-1}}{1+\pi_t} + \tau_t \tag{3.3}$$

where $m_{t-1} = \frac{M_{t-1}}{P_{t-1}}, \pi_t = (\frac{P_t}{P_{t-1}}) - 1$ is the inflation rate in period t, and $\tau_t = \frac{T_t}{P_t}$. M_{t-1} refers to nominal money balances chosen by agent in period $t-1$ and carried into period t. The real value of these balances is determined by the period t price level P_t. The household is able to adjust its portfolio between money and bonds before entering the goods market to purchase consumption goods because this reflects our timing convention that goods and money market transactions occur sequentially in the same period. The implications of this alternative timing, suppose there is a positive opportunity cost of holding money when the interest rate be positive.

The representative agent's resource constraint for the asset market shows that right-side is endowments of agent from the labor income, rental from capital, capital, bond holding and money from period $t-1$ to t.

$$\begin{aligned}(1+\tau_t^c)c_t + k_t + m_t + b_t \leq (1-\tau_t^w)w_t n_t + (1-\tau_t^k)r_t k_{t-1} + (1-\delta)k_{t-1} + \\ +(1+i_{t-1})\frac{b_{t-1}}{1+\pi_t} + \frac{m_{t-1}}{1+\pi_t}\end{aligned} \tag{3.4}$$

households supply labor and rent capital to firms that produce goods, where w_t and r_t are real wage and real interest rate, respectively. $0 < \delta < 1$ is the depreciation rate. b_{t-1} and m_{t-1} is bond holding and money balance, respectively. The $\tau_t^c, \tau_t^w, \tau_t^k$ are consumption's, labor's and capital's taxes, respectively.

Assume that the agent's objective is:

$$E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, 1 - n_{t+i}) = E_0 \sum_{i=0}^{\infty} \beta^i \left[\frac{c_{t+i}^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_{t+i})^{1-\eta}}{1-\eta} \right] \tag{3.5}$$

The parameters Φ, Ψ and η are restricted to be positive.

The household problem is to choose consumption, labor, real holdings, capital to rent and real money balances to satisfy:

$$\max_{c_t, n_t, b_t, k_t, m_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t) \tag{3.6}$$

such that

$$(1 + \tau_t^c) c_t \leq \frac{m_{t-1}}{1 + \pi_t} + \tau_t \tag{3.7}$$

$$\begin{aligned} (1 + \tau_t^c) c_t + k_t + m_t + b_t \leq (1 - \tau_t^w) w_t n_t + (1 - \tau_t^k) r_t k_{t-1} + (1 - \delta) k_{t-1} + \\ + (1 + i_{t-1}) \frac{b_{t-1}}{1 + \pi_t} + \frac{m_{t-1}}{1 + \pi_t} \end{aligned} \tag{3.8}$$

Solving the lagrangian:

$$\begin{aligned} \mathcal{L} = E_0 \sum_{i=0}^{\infty} \beta^t \{ \left[\frac{c_{t+i}^{1-\Phi}}{1-\Phi} + \Psi \frac{(1-n_{t+i})^{1-\eta}}{1-\eta} \right] + \\ + \lambda_t [(1 - \tau_t^w) w_t n_t + (1 - \tau_t^k) r_t k_{t-1} + (1 - \delta) k_{t-1} + \\ + (1 + i_{t-1}) \frac{b_{t-1}}{1 + \pi_t} + \frac{m_{t-1}}{1 + \pi_t} - (1 + \tau_t^c) c_t - k_t - m_t - b_t] \\ + \mu_t [(\frac{m_{t-1}}{1 + \pi_t}) + \tau_t - (1 + \tau_t^c) c_t] \} \end{aligned} \tag{3.9}$$

Letting λ_t denote the lagrangian multiplier associated with the budget constraint and μ_t to be the lagrangian multiplier associated to the CIA constraint. The first-order necessary conditions (FONC) for the agent's choice of consumption, labor, bond, money holdings and capital take the form:

The FONC's are, $\forall \quad t$:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Leftrightarrow \quad c_t^{-\Phi} - \lambda_t (1 + \tau_t^c) - \mu_t (1 + \tau_t^c) = 0 \tag{3.10}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \Leftrightarrow \quad -\Psi (1 - n)^{-\eta} + \lambda_t (1 - \tau_t^w) w_t = 0 \tag{3.11}$$

$$\frac{\partial \mathcal{L}}{\partial b_t} = 0 \Leftrightarrow \quad -\lambda_t + \beta E_t [\lambda_{t+1} (\frac{1 + i_t}{1 + \pi_{t+1}})] = 0 \tag{3.12}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = 0 \Leftrightarrow \quad -\lambda_t + \beta E_t [(\frac{\lambda_{t+1}}{1 + \pi_{t+1}} + \frac{\mu_{t+1}}{1 + \pi_{t+1}})] = 0 \tag{3.13}$$

$$\frac{\partial \mathcal{L}}{\partial k_t} = 0 \Leftrightarrow \quad \beta E_t [\lambda_{t+1} (1 - \tau_{t+1}^k) r_{t+1} + (1 - \delta)] - \lambda_t = 0 \tag{3.14}$$

Before, the Fisher[2] relationship is: $(1 + r_t) = (\frac{1+i_t}{1+\pi_{t+1}})$, so the Euler equation is identical to the FONC (3.12), that is a standard asset pricing equation and is a familiar condition from problems involving intertemporal optimization according Walsh (2010).

The FONC (3.13) can be expressed as

$$\lambda_t = \beta E_t(\frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}}) \tag{3.15}$$

This last equation can be interpreted as the price of a unity of money in terms of goods is just $1/P_t$ at time t. Its value in utility terms is λ_t/P_t. Dividing through by P_t, it can be rewritten as $\lambda/P_t = \beta E_t(\lambda_{t+1} + \mu_{t+1})/P_{t+1}$

3.1.2 The Firm

To complete the specification of the model, we assume that the economy's technology is given by: $Y = A_t F(k_t, n_t)$, where A_t is the exogenous productivity shocks. We assume that the firms chooses capital and employment each period to maximize profits. The production function is a Cobb-Douglas, and subject to the capital accumulation equation. The profit's function of the firm can be written as:

$$A_t k_t^{\alpha} n_t^{1-\alpha} - r_t k_t - w_t n_t \tag{3.16}$$

Where k_t and n_t are the demand by firm for capital and labor, respectively. And α is the share of physical capital and human capital. We assume that firms face the aggregate productivity shock A_t that depend on the state of the economy (ε)

The capital accumulation equation is standard:

$$k_t = I_t + (1 - \delta)k_{t-1} \tag{3.17}$$

We describe the behavior of firms who hire workers at the wage rate,w, and rent capital at the rental rate, r. Will be assumed that there is a large number of firms and that market is competitive. In our model, firm's decisions don't have any intertemporal consequences. So, the standard assumption that firms maximize the net-present-value of discounted future profits implies that firms maximize profits period by period. The maximization problem for firm is given by:

[2] The Fisher equation estimates the relationship between nominal and real interest rates inflation. Irving Fisher, who was famous for his works on the theory of interest.

$$\max_{k_t,n_t} A_t k_t^{\alpha} n_t^{1-\alpha} - r_t k_t - w_t n_t = 0 \tag{3.18}$$

The first-order conditions for this problem are as follows:

$$\frac{\partial \mathcal{L}}{\partial k_t} = 0 \Leftrightarrow \quad \alpha A_t k_t^{\alpha-1} n_t^{1-\alpha} - r_t = 0 \tag{3.19}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \Leftrightarrow \quad (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} - w_t = 0 \tag{3.20}$$

The equilibrium real wage rate and rental rate will equal the marginal product of labor and capital, respectively. So, we have wages and the interest's rates:

$$w_t(A_t, k_t, n_t) = (1-\alpha) A_t k_t^{\alpha} n_t^{-\alpha} \tag{3.21}$$

$$r_t(A_t, k_t, n_t) = \alpha A_t k_t^{\alpha-1} n_t^{1-\alpha} \tag{3.22}$$

Therefore, we have:

$$r_t = \frac{\alpha y_t}{k_t} \tag{3.23}$$

$$w_t = \frac{(1-\alpha) y_t}{n_t} \tag{3.24}$$

3.1.3 The Government

The government is an economic agent that demands resources for its expenditures, which will be determinate exogenously, g_t, and for paying the interests of debts issued in the previous period. The government has its revenue by levying taxes on consumer's consumption, labor, net capital stock. The government chooses a state -contingent sequence for one-period nominal debt, money and primary surpluses. The government chooses spending, G_t, exogenously. It finances spending with taxes, bonds and seigniorage, as the equation (2.12). We will impose that the government budget constraint is always held. We presume the constraint is following a Ricardian regime. The left-hand side of government budget constraint shows the debt is issued by government at time t. The right-hand side means the debt is reimbursed by government at time t:

$$b_t + m_t + \tau_t^c c_t + \tau_t^w w_t n_t + \tau_t^k r_t k_{t-1} + \tau = g_t + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{m_{t-1}}{1+\pi_t} \tag{3.25}$$

Where g_t is government expenditure in period t, i_{t-1} is the t-1 period nominal interest rate, b_{t-1} is the bonds holdings from period t-1 to t and, m_{t-1} is the real money holding from period t-1 to t. π_t is the inflation rate in period t. Furthermore, the government has a sequence of distorting taxes, $\tau_t = \{\tau_t^c, \tau_t^w, \tau_t^k\}_{t=0}^{\infty}$.

$$g_t + (\frac{1 + i_{t-1}}{1 + \pi_t})b_{t-1} = T + m_t - \frac{m_{t-1}}{1 + \pi_t} + b_t \tag{3.26}$$

In that $\tau_t^c c_t + \tau_t^w w_t n_t + \tau^k r_t k_{t-1} + \tau = T$.

$$(\frac{1 + i_{t-1}}{1 + \pi_t})b_{t-1} - b_t = (T - g) + (m_t - \frac{m_{t-1}}{1 + \pi_t}) \tag{3.27}$$

The left-hand side shows the debt is issued by government at time t. The right-hand side means the debt is reimbursed by government at time t. It can be writing as:

$$(\frac{1 + i_{t-1}}{1 + \pi_t})b_{t-1} - b_t = s^f + s^m \tag{3.28}$$

In that $s^f = (T - g)$ and $(m_t - \frac{m_{t-1}}{1+\pi_t}) = s^m$.

3.1.4 Shocks

Moreover, we assume that economy is hit by three exogenous process $\begin{bmatrix} z_t & g_t & u_t \end{bmatrix}'$ witch are governed by three AR(1) process, and finally we have three exogenous iid process.

The shock following a stochastic process $\{\varepsilon_n, n = 0, 1, 2, ...\}$ that takes on a finite or countable number of possible values. If $X_n = i$, then the process is said to be in state i at time n. We suppose that whenever the process is in state i, there is a fixed probability P_{ij} that it will be in state j next period; i.e.:

$$Pr(\varepsilon_{n+1} = j | \varepsilon_n = i) = P_{ij} \tag{3.29}$$

The conditional distribution of any future state ε_{n+1} given the history $\varepsilon_0, \varepsilon_1, ..., \varepsilon_{n-1}$ and the present state ε_n is independent of the history and depends only on the present state.

Production in period t is subject to a random shock ε_t with values in the compact set $\varepsilon \subset [\varepsilon_{(good)}, \varepsilon_{(bad)}]$. Production shocks follow a Markov process with time-invariant transition probability Q: ε x $\beta(\varepsilon) \rightarrow [0, 1]$. Given an initial state $\varepsilon_0 \in \varepsilon$, the transition Q permits to construct a probability space.

The technological shock is obtained by estimating the Solow residual (z) or total factor productivity, from the estimation of the production function. The result corresponds to the portion of the product that is not explained by either, physical capital or labor.

The exogenous productivity shock is set from a first order autoregressive process $A = e^z$ assumed to follow an AR(1) process:

$$z_t = \rho_z z_{t-1} + e_t \tag{3.30}$$

where $0 \leq \rho_z \leq 1$. e_t is the innovation and has mean zero and variance σ_e^2

We assume that the quantity of money is set exogenously. Higher inflation is a tax on the holders of money. We are going to write the money rule as an AR(1) in the growth rate with a positive mean.

The specification of the stochastic process for the money supply was determined from the long-term average growth rate of nominal supply. Therefore, the deviation of money growth from steady-state average rate assumes:

$$u_t = \rho_u u_{t-1} + \phi z_{t-1} + \varphi_t \tag{3.31}$$

Where φ is a white noise innovation with variance σ_φ^2 and ρ_u is the persistence parameter of the stochastic process. The parameter ϕ is the technological shock coefficient on money growth. If ϕ assumes a positive value, then the growth rate of nominal money supply responds positively to real productivity shocks.

In addition, we assume that government spending follows a first order autoregressive stochastic process.

let g_t assume the form:

$$g_t = \rho_g g_{t-1} + \rho_\theta u_{t-1} + v z_{t-1} + \epsilon_t \tag{3.32}$$

Where ϵ is a white noise innovation with variance σ_ϵ^2 and ρ_g is the the parameter of persistence of the stochastic process. The parameter v is the technological shock coefficient on money growth. If $v z_{t-1}$ assumes a positive value, then government expending responds positively to real productivity shocks, and if ρ_θ assumes a positive value, the government expending responds positively to money growth rate.

3.2 Dynamic competitive equilibrium and steady state

Aggregate market-clearing requires:

$$y_t = c_t + g_t + I_t \tag{3.33}$$

The competitive equilibrium of this closed economy requires that firms and consumers interact in markets. Both firms and consumers take prices as given when they solve their optimization problems. The government budget constraint will be balanced

The economy is in equilibrium when decisions are consistent and both firms and consumers are satisfied with their supply and demand decisions. That is, in equilibrium labor and goods markets clear and, given equilibrium prices, the consumer maximizes utility and the firms maximizes profit, and the government has the budget balanced

In this complete information version of this economy, the agents observe the initial assets given (M_0, B_0) at the beginning of period t before making any decisions. A competitive equilibrium of the economy is given by a set of decision rules of households $\{c_t, n_t, k_t, m_t, b_t\}_{t=0}^{\infty}$ given the sequences of prices of the economy $\{r_t, w_t, p_t\}_{t=0}^{\infty}$ and output levels $\{y_t\}_{t=0}^{\infty}$ and nominal transfers $\{T_t\}_{t=0}^{\infty}$ that satisfy the budget constraint and CIA constraint with the real wage equaling the marginal product of labor at each point in time.

Firms choose capital and labor given the prices and shocks of the economy $w(A_t, k_t, n_t)$ and $r(A_t, k_t, n_t)$, P_t, $A_t(\varepsilon)$

Given M_0 and $B_0(1 + i_o)$, the government policy and the price vector satisfy the government budget constraints describes in expression (3.25).

So, we have the following vector with nine endogenous variables $\begin{bmatrix} y_t & c_t & n_t & I_t & k_t & r_t & i_t & m_t & b_t \end{bmatrix}'$ and a vector of three exogenous shocks $\begin{bmatrix} z_t & g_t & u_t \end{bmatrix}'$:

Rearranging the CPO's and get the equilibrium conditions, we have

$$c^{-\Phi} = (\lambda_t + \mu_t)(1 + \tau_t^c) \tag{3.34}$$

$$\Psi(1 - n_t)^{-\eta} = (1 - \tau^w)(1 - \alpha)(\frac{y_t}{n_t})\lambda_t \tag{3.35}$$

$$\lambda_t = \beta E_t(1 + r_t)\lambda_{t+1} \tag{3.36}$$

$$\lambda_t = \beta E_t \frac{(c_{t+1}^{-\Phi})}{(1 + \tau^c)(1 + \pi_{t+1})} \tag{3.37}$$

$$(1 + r_t) = \beta[\alpha(\frac{y_t}{k_t})(1 - \tau^k) - \delta + 1] \tag{3.38}$$

The steady state is defined as the values all variables would converge to in the absence of any shocks. Variables without subscript denote steady state values. In steady state, this implies that:

$$(c^{ss})^{-\Phi} = (\lambda^{ss} + \mu^{ss})(1 + \tau_t^c) \tag{3.39}$$

$$\Psi(1 - n^{ss})^{-\eta} = (1 - \tau^w)(1 - \alpha)(\frac{y^{ss}}{n^{ss}})\lambda^{ss} \tag{3.40}$$

$$\lambda^{ss} = \beta(1 + r^{ss})\lambda^{ss} \tag{3.41}$$

$$\lambda^{ss} = \beta\frac{(c^{ss})^{-\Phi}}{(1 + \tau^c)(1 + \pi^{ss})} \tag{3.42}$$

$$(1 + r^{ss}) = \beta[\alpha(\frac{y^{ss}}{k^{ss}})(1 - \tau^k) - \delta + 1] \tag{3.43}$$

$$I = \delta k^{ss} \tag{3.44}$$

The drop time indices ($x_{t-1} = x_t = x^{ss}$). In steady state, from (3.41), implies that $r^{ss} = \frac{1}{\beta} - 1$, wich implies that (3.43) becomes:

$$\frac{y^{ss}}{k^{ss}} = \frac{1}{\alpha}(\frac{\frac{1}{\beta} - 1 + \delta}{(1 - \tau^k)}) \tag{3.45}$$

Net real capital return is equal to inverse of time preference. Equation 3.46 can be use the fact that $r^{ss} = \frac{1}{\beta} - 1$, and we can obtain:

$$\frac{y^{ss}}{k^{ss}} = \frac{1}{\alpha}(\frac{r^{ss} + \delta}{(1 - \tau^k)}) \tag{3.46}$$

From CIA constraint (3.3), $(1 + \tau^c)c^{ss} = \frac{m^{ss}}{(1+\pi^{ss})} + \tau^{ss}$, then both sides are divided by k^{ss}:

$$\frac{c^{ss}}{k^{ss}} = \frac{\frac{1}{1+\pi^{ss}}\frac{m^{ss}}{k^{ss}} + \frac{\tau^{ss}}{k^{ss}}}{(1 + \tau^c)} \tag{3.47}$$

From (3.17) in steady state:

$$I^{ss} = \delta k^{ss} \tag{3.48}$$

The resource constraints, $y^{ss} = c^{ss} + \delta k^{ss} + g$. The both sides can be divided by k^{ss}

$$\frac{c^{ss}}{k^{ss}} = \frac{y^{ss}}{k^{ss}} - \delta - \frac{g}{k^{ss}} \tag{3.49}$$

From the production function, in steady state we get:

$$\frac{y^{ss}}{k^{ss}} = (\frac{n^{ss}}{k^{ss}})^{1-\alpha} \tag{3.50}$$

Or, we can get:

$$(\frac{n^{ss}}{k^{ss}}) = (\frac{y^{ss}}{k^{ss}})^{\frac{1}{1-\alpha}} \tag{3.51}$$

We assume that θ is the growth rate of money:

$$\frac{M_t}{M_{t-1}} = 1 + \theta \tag{3.52}$$

Dividing both sides by p_t and rearranging, we have:

$$m_t = m_{t-1}(\frac{1+\theta}{1+\pi}) \tag{3.53}$$

In steady state $m^{ss} = m^{ss}(\frac{1+\theta}{1+\pi})$, implies that $\theta^{ss} = \pi^{ss}$. This mean that, the inflation tax is equal the growth rate of money in the steady state.

From (3.42) and (3.39) and using $\theta^{ss} = \pi^{ss}$, we can to determine the marginal utility of income λ^{ss}:

$$\begin{aligned} \lambda^{ss} &= \beta(\frac{\lambda^{ss}(1+i^{ss})(1+\tau^c)}{1+\theta^{ss}}) \\ 1+\theta^{ss} &= \beta(1+i^{ss})(1+\tau^c) \\ 1+i^{ss} &= \frac{1+\theta^{ss}}{\beta(1+\tau^c)} \end{aligned} \tag{3.54}$$

This is the steady-state version of the Fisher equation. This can take the form:

$$\lambda^{ss} = \frac{\beta(c^{ss})^{-\Phi}}{1+\theta^{ss}} \tag{3.55}$$

Combining this with (3.40), and multiplying and dividing by k^{ss} and n^{ss},

$$(1-n^{ss})^{-\eta}(n^{ss})^{\Phi} = \frac{(1-\alpha)(1-\tau^w)}{\Psi}(\frac{\beta}{1+\theta^{ss}})(\frac{c^{ss}}{k^{ss}})^{-\Phi}(\frac{y^{ss}}{k^{ss}})^{(\frac{\Phi-\alpha}{1-\alpha})} \tag{3.56}$$

Following Walsh (2010), the level of real money balances in the steady state is determined by CIA constraint. Hence $(1+\tau^c)c^{ss} = m^{ss}/(1+\pi^{ss}) + \tau^{ss} = m^{ss}$, so , if dividing the both sides by k^{ss}, we have $m^{ss}/k^{ss} = (1+\tau^c)c^{ss}/k^{ss}$.

3.3 Analytical Results for the Deterministic Model

In the absence of shocks, first order necessary conditions above hold without the expectation operator. Therefore, from the FONC's (3.12) and (3.13) we have the relationship between the nominal interest and the lagrangian multipliers:

$$i_t = \frac{\mu_{t+1}}{\lambda_{t+1}} \tag{3.57}$$

In Walsh (2010) the nominal rate of interest is positive if and only if money yields liquidity services, when $\mu_{t+1} > 0$. If the nominal interest rate is positive, the CIA constraint is binding ($\mu > 0$).

λ_t is the marginal utility of wealth. The FONC (3.10) means that the marginal utility of consumption is the marginal utility of wealth plus the value of liquidity services, μ_t, diminished by tax (τ_t^c). Since λ represents the marginal value of income, the marginal utility of consumption exceeds that of income whenever the nominal interest rate is positive and the tax of the consumption is $0 \leq \tau_t^c \leq 1$.

So, the marginal utility of consumption will be exceed that of income whenever the nominal interest rate is positive and the tax is zero. With tax positive, the marginal utility is affected down even with the positive nominal interest rate. We can use the relationship between the nominal rate of interest and the lagrangian multiplies to rewrite the expression for the marginal utility of consumption:

$$c_t^{-\Phi} = (\lambda_t + \mu_t)(1 + \tau_t^c) \tag{3.58}$$

$$c_t^{-\Phi} = (1 + \frac{\mu_t}{\lambda_t})\lambda_t(1 + \tau_t^c) \tag{3.59}$$

$$c_t^{-\Phi} = [1 + i_t][(1 + \tau_t^c)\lambda_t \tag{3.60}$$

The CIA constraint holds with equality when the nominal rate of interest is positive, $c_t = M_{t-1}/P_t + \tau_t$. Knowing that the lump-sum τ_t is $(M_t - M_{t-1})/P_t$, this implies that $c_t = M_t/P_t = m_t$. This means that the consumption velocity of money is one. But this actual velocity can vary over time [3].

In addition, if suppose that the cash in advance constraint is neglected, which $\mu = 0$ at all times, the third and fourth would be equivalent if and only if $i_t = 0$. With the nominal rate positive, we should be a corner solution. The opportunity cost of holding money is the nominal interest rate. In other words, in the absence of distortionary taxes, if $i_t = 0$, monetary policy leads to an equilibrium which is Pareto optimal, this optimal monetary rule is known as Friedman rule.

On the fiscal side, if the monetary policy is such that nominal interest rate is positive, i. e., the lagrangian multiplier for the CIA constraint is positive, $\mu > 0$, then FOCs (3.10) and (3.11) can be written as:

$$\frac{c_t^{-\Phi}}{(1 + \tau_t^c)} = \frac{-\Psi(1 - n)^{-\eta}}{(1 - \tau_t^w)w_t} + \mu_t \tag{3.61}$$

Equation (3.61) shows that fiscal and monetary policy distorts the relative price between consumption and leisure. Since consumption good must be purchased with money,

[3] See Walsh (2010).

as long as the interest rate is positive, there is a cost of buying consumption. Hence, inflation tax is distortionary and the competitive equilibrium is not Pareto optimal when there is inflation. On the other hand, government might use fiscal policy in order to decrease the distortion caused by monetary policy. To better see that, consider the model with just monetary policy, $\tau_t^c = \tau_t^w = 0$, then equation (3.61) becomes:

$$c_t^{-\Phi} = \frac{-\Psi(1-n)^{-\eta}}{w_t} + \mu_t \tag{3.62}$$

With positive nominal interest rate, $\mu > 0$, marginal utility of consumption is higher than the model with no monetary distortion, therefore, consumption is lower and leisure is higher than the Pareto optimal allocation. By considering fiscal policy together with monetary policy, government can use taxation to get Pareto optimality, in this case, by decreasing taxes on consumption and/or labor income, making consumption cheaper and leisure more expensive.

3.4 Log-linear Approximation

The model above contains a nonlinear system of equations which is quite hard to solve. Therefore, it is necessary to log-linearize the model around the steady state in order to get a system of linear equation which will describe the dynamic behavior of the model for small log-deviation around the steady state. For this, the Taylor approximation around the steady state will be used.

We use linear approximation approach for solving the dynamic system around the steady state. The linear expressions of the percentage deviations around the steady state will be obtained for the economy's production functions, resource constraint, the definition of the marginal product of capital, the budget constraint of government, and the first order consumption, money holdings, and labor supply. The linearized model consists of fourteen equations to determine the exogenous disturbances z_t, u_t, and g_t and eleven endogenous variables $\hat{y}, \hat{k}, \hat{n}, \hat{I}, \hat{c}, \hat{\lambda}, \hat{r}, \hat{i}, \hat{\pi}, \hat{m}, \hat{b}$. This approximation holds for small deviations from the steady state, which highlights that log-linearization is a local approximation method [4].

Formally, the log-linear approximation can be defined as $\hat{x}_t = ln(\frac{x_t}{x^{ss}})$ as log-deviation from steady state. It follows that:

$$x_t = x^{ss}e^{\hat{x}_t} \simeq x^{ss}e^0 + x^{ss}e^0(\hat{x}_t - 0) = x^{ss}(1 + \hat{x}_t) \tag{3.63}$$

For any examples, we have some useful tricks:

[4] See Uhlig (1999).

$$\begin{aligned} x_t y_t &= x^{ss} y^{ss}(1 + \hat{x}_t + \hat{y}_t) && (3.64)\\ \frac{x_t}{y_t} &= \frac{x^{ss}}{y^{ss}}(1 + \hat{x}_t - \hat{y}_t) && (3.65)\\ x_t^a &= (x^{ss})^a(1 + a\hat{x}_t) && (3.66)\\ lnx_t &= lnx + \hat{x}_t && (3.67) \end{aligned}$$

The dynamic implications of the model can be explored by obtaining a first order linear approximation around the steady state.

The marginal utility of consumption (3.10), can be rewrite as:

$$c_t^{-\Phi} = \lambda_t(1 + i_t)(1 + \tau^c) \tag{3.68}$$

So, the approximation is:

$$(c^{ss})^{-\Phi}(1 - \Phi\hat{c}_t) = \lambda^{ss}(1 + \hat{\lambda}_t)i^{ss}(1 + \hat{i}_t)(1 + \tau^c) \tag{3.69}$$

Since the steady state is $(c^{ss})^{-\Phi} = \lambda^{ss}(1 + i^{ss})(1 + \tau^c)$, the final step is to divide the both sides for the steady state. So, we have:

$$-\Phi\hat{c}_t = \hat{\lambda}_t + \hat{i}_t \tag{3.70}$$

From the fist order condition with respect to money holdings (3.13) and the result that $c_t^{-\Phi} = (\lambda_t + \mu)(1 + \tau^c)$, from (3.10), $\lambda_t = \beta E_t(\frac{(c_{t+1}^{-\Phi})/(1+\tau^c)}{(1+\pi_{t+1})})$.

Rewrite this and the making the approximation:

$$\begin{aligned} \lambda^{ss}(1 + \hat{\lambda}_t) &= \beta E_t[\frac{(c^{ss})^{-\Phi}(1 - \Phi\hat{c}_{t+1})/(1 + \tau^c)}{1 + \pi_{t+1}}]\\ \lambda^{ss}(1 + \hat{\lambda}_t) &= \beta\frac{(c^{ss})^{-\Phi}}{(1 + \tau^c)}E_t(1 - \Phi\hat{c}_{t+1} - \pi_{t+1}) && (3.71) \end{aligned}$$

Note that, in steady state we have:

$$\lambda^{ss} = \beta E_t(\frac{(c^{ss})^{-\Phi}/(1+\tau^c)}{(1+\pi^{ss})})$$

So, substituting we have:

$$\begin{aligned}\lambda^{ss}(1+\hat{\lambda}_t) &= \lambda^{ss}(1+\pi^{ss})E_t(1-\Phi\hat{c}_{t+1}-\pi_{t+1})\\ \lambda^{ss}(1+\hat{\lambda}_t) &= \lambda^{ss}E_t(1-\Phi\hat{c}_{t+1}-\pi_{t+1}+\pi^{ss})\end{aligned} \tag{3.72}$$

Then, dividing for the steady state, implies that:

$$\hat{\lambda} = -E_t(\Phi\hat{c}_{t+1}+\pi_{t+1}) \tag{3.73}$$

From the CIA restriction we have that all consumption is purchased with money, so we have:

$$(1+\tau_t^c)c_t = m_t$$

From the CIA restriction we can obtain:

$$(1+\tau_t^c)c_t = m^{ss}(1+\hat{m}_t)$$

Since the steady state is $(1+\tau_t^c)c^{ss} = m^{ss}$. Dividing the both sides for the steady state, we have:

$$\hat{c}_t = (1+\tau_t^c)\hat{m}_t \tag{3.74}$$

From the resource constraint $y = c + I + g$ divided for capital we get:

$$\frac{y_t}{k_t} = \frac{c_t}{k_t} + \frac{I_t}{k_t} + \frac{g_t}{k_t} \tag{3.75}$$

After, the log linearization will take the form:

$$\frac{y^{ss}}{k^{ss}}(1+\hat{y}_t-\hat{k}_t) = \frac{c^{ss}}{k^{ss}}(1+\hat{c}_t-\hat{k}_t) + \frac{I^{ss}}{k^{ss}}(1+\hat{I}_t-\hat{k}_t) + \frac{g^{ss}}{k^{ss}}(1+\hat{g}_t-\hat{k}_t) \tag{3.76}$$

The steady state for the resource constraint is:

$$\frac{y^{ss}}{k^{ss}} = \frac{c^{ss}}{k^{ss}} + \frac{I^{ss}}{k^{ss}} + \frac{g^{ss}}{k^{ss}} \tag{3.77}$$

So, knowing that $I^{ss} = \delta k^{ss}$ and make the division for the steady state, we have:

$$\frac{y^{ss}}{k^{ss}}\hat{y}_t = \frac{c^{ss}}{k^{ss}}\hat{c}_t + \delta\hat{I}_t + \frac{g^{ss}}{k^{ss}}\hat{g}_t \tag{3.78}$$

From the marginal product of capital we have that $r_t = \alpha\frac{y_{t+1}}{k_t}$. This implies that: $r^{ss}(1+\hat{r}_t) = \alpha\frac{y^{ss}}{k^{ss}}(1+E_{t+1}\hat{y}_{t+1} - k_t)$.
Since the steady sate is $r^{ss} = \alpha\frac{y^{ss}}{k^{ss}}$. Next to divide by the steady state, we have:

$$\hat{r}_t = \alpha\frac{y^{ss}}{k^{ss}}(E_t\hat{y}_{t+1} - \hat{k}_t) \tag{3.79}$$

From the FONC of the work we have:

$$\Psi(1-n_t)^{-\eta} = \lambda_t(1-\tau_t^w)(1-\alpha)\frac{y}{n_t}$$
$$\frac{\Psi(1-n_t)^{-\eta}}{\lambda_t} = (1-\tau_t^w)(1-\alpha)\frac{y}{n_t}$$

Remember that $l_t = 1 - n_t$. The approximation is:

$$\frac{\Psi(l^{ss})^{-\eta}(1+\hat{l}_t)^{-\eta}}{\lambda^{ss}(1+\hat{\lambda}_t} = (1-\tau^w)(1-\alpha)\frac{y^{ss}}{n^{ss}}(\frac{1+\hat{y}_t}{1+\hat{n}_t}) \tag{3.80}$$

Since the steady state is $\Psi(1-n^{ss})^{-\eta} = \lambda^{ss}(1-\tau_t^w)(1-\alpha)\frac{y^{ss}}{n^{ss}}$, we can divide the both sides for the steady state.

$$\frac{(1+\hat{l}_t)^{-\eta}}{1+\hat{\lambda}_t} = \frac{1+\hat{y}_t}{1+\hat{n}_t} \tag{3.81}$$
$$-\eta\hat{l}_t - \hat{\lambda}_t = \hat{y}_t - \hat{n}_t \tag{3.82}$$

Since $l_t = 1-n_t$, the approximation is: $l^{ss}(1+\hat{l}_t) = 1-n^{ss}(1+\hat{n}_t)$. So, $\hat{l}_t = -(\frac{n^{ss}}{l^{ss}})\hat{n}_t$. Making the substitution:

$$-\eta(\frac{-n^{ss}}{l^{ss}})\hat{n}_t - \hat{\lambda}_t = \hat{y}_t - \hat{n}_t$$
$$(1+\eta\frac{n^{ss}}{l^{ss}})\hat{n}_t = \hat{y}_t + \hat{\lambda}_t \tag{3.83}$$

From the Euler equation $\lambda_t = \beta E_{t+1}(1+r_t)\lambda_{t+1}$, we can loglinearize around the steady state:

$$\lambda^{ss}(1+\hat{\lambda}_t) = \beta\lambda^{ss}r^{ss}(1+E_{t+1}\hat{\lambda}_{t+1})(1+\hat{r}_t) \tag{3.84}$$

Since the steady state is $\lambda^{ss} = \beta(1+r^{ss})\lambda^{ss}$, and dividing the both sides, we can have:

$$\hat{\lambda}_t = \hat{r}_t + E_{t+1}\hat{\lambda}_{t+1} \tag{3.85}$$

The monetary policy is controling the money supply:

$$M_t = M_{t-1}(1+\theta),$$

in that θ is the rate of the money growth.

In real level, we have:

$$m_t = \tfrac{m_{t-1}}{1+\pi_t}(1+\theta).$$

The log-linearization takes the form:

$$m^{ss}(1+\hat{m}_t) = \frac{m^{ss}}{1+\pi^{ss}(1+\hat{\pi}_t)}(1+\hat{m}_{t-1}+\hat{\theta}_t) \tag{3.86}$$

Since the steady is $m^{ss} = \frac{m^{ss}}{1+\pi^{ss}}(1+\theta^{ss})$, in terms of the deviation, we have:

$$\hat{m}_t = \hat{m}_{t-1} + \hat{\theta}_t - \hat{\pi}_t \tag{3.87}$$

From the equation (3.10) and (3.12), we can get:

$$\lambda_t = \beta E_{t+1}\left(\frac{c_{t+1}^{-\Phi}}{(1+\tau_t^c)(1+\pi_{t+1})}\right) \tag{3.88}$$

The log-linearization is:

$$\lambda^{ss}(1+\hat{\lambda}_t) = \beta\tfrac{(c^{ss})^{-\Phi}(1-\Phi\hat{c}_{t+1})}{(1+\tau^c)(1+\pi^{ss})(1+\hat{\pi}_{t+1})})$$

Since the steady state is $\lambda^{ss} = \beta(\frac{(c^{ss})^{-\Phi}}{(1+\tau_t^c)(1+\pi^{ss})})$, the approximation is given by:

$$\hat{\lambda}_t = -\Phi E_{t+1}\hat{c}_{t+1} - E_{t+1}\hat{\pi}_{t+1} \tag{3.89}$$

From the production function $y_t = A_t k_{t-1}^{\alpha} n_t^{1-\alpha}$, in that $A_t = e_t^z$.

The loglinearization is:

$$y^{ss}(1+\hat{y}_t) = (1+z_t)(1+\hat{k}_{t-1})^{\alpha}(1+\hat{n}_t)^{1-\alpha} \tag{3.90}$$

Since the steady state for the production function is $y^{ss} = (k^{ss})^{\alpha}(n^{ss})^{1-\alpha}$. So, dividing the both sides for the steady state:

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + \hat{z}_t \tag{3.91}$$

From the the capital accumulation equation (3.17), we can get:

$$k^{ss}(1+\hat{k}_t) = (1-\delta)k^{ss}(1+\hat{k}_{t-1}) + I^{ss}(1+\hat{I}_t) \tag{3.92}$$

Since the steady state is $k^{ss} = (1-\delta)k^{ss} + I^{ss}$, after to divide the both sides for the steady state we have:

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{I}_t \tag{3.93}$$

Note that, from the steady state we have that $\frac{I^{ss}}{k^{ss}} = \delta$

From the government constraint 3.25, we can get the loglinearization form: first we divided the both sides by the output:

$$-\frac{b_t}{y_t} + \frac{(1+i_{t-1})}{(1+\pi_t)y_t}b_{t-1} = \frac{\tau_t^c}{y_t}c_t + \frac{\tau^w w n_t}{y_t} + \frac{\tau^k r_t k_{t-1}}{y_t} + \frac{\tau}{y_t} - \frac{g_t}{y_t} + \frac{m_{t-1}}{(1+\pi_t)y_t} - \frac{m_t}{y_t}$$

Making the log-linearization:

$$-\frac{b^{ss}}{y^{ss}}(1+\hat{b}_t-\hat{y}_t) + \frac{(1+i^{ss})(1+\hat{i}_{t-1})b^{ss}(1+\hat{b}_{t-1})}{(1+\pi^{ss})y^{ss}(1+\hat{\pi}_t+\hat{y}_t)} = \frac{\tau_t c^{ss}}{y^{ss}}(1+\hat{c}_t-\hat{y}_t) +$$
$$\frac{\tau^w w^{ss} n^{ss}}{y^{ss}}(1+\hat{n}_t-\hat{y}_t) + \frac{\tau^k r^{ss} k^{ss}}{y^{ss}}(1+\hat{r}_t+\hat{k}_t-\hat{y}_t) + \frac{\tau^{ss}}{y^{ss}}(1+\hat{\tau}_t-\hat{y}_t) - \frac{g^{ss}}{y^{ss}}(1+\hat{g}_t-\hat{y}_t) +$$
$$\frac{m^{ss}}{(1+\pi^{ss})y^{ss}(1+\hat{\pi}_t+\hat{y}_t)}(1+\hat{m}_{t-1}) - \frac{m^{ss}}{y^{ss}}(1+\hat{m}_t-\hat{y}_t)$$

Since the steady state for the government is:

$$-\frac{b^{ss}}{y^{ss}}+\frac{(1+i^{ss})}{(1+\pi^{ss})y^{ss}}b^{ss}=\frac{\tau_t^c c^{ss}}{y^{ss}}+\frac{\tau^w w^{ss}n^{ss}}{y^{ss}}+\frac{\tau^k r^{ss}k^{ss}}{y^{ss}}+\frac{\tau^{ss}}{y^{ss}}-\frac{g^{ss}}{y^{ss}}+\frac{m^{ss}}{(1+\pi^{ss})y^{ss}}-\frac{m^{ss}}{y^{ss}}$$

So, dividing the both sides for the steady state:

$$\begin{aligned}-\frac{b^{ss}}{y^{ss}}\hat{b}_t+\frac{b^{ss}}{y^{ss}}\hat{y}_t+\frac{1+i^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{i}_{t-1}+\hat{b}_{t-1}-\hat{\pi}^t-\hat{y}_t)=\frac{\tau_t^c c^{ss}}{y^{ss}}(\hat{c}_t-\hat{y}_t)+\\ \frac{\tau^w w^{ss}n^{ss}}{y^{ss}}(\hat{n}_t-\hat{y}_t)+\frac{\tau^k r^{ss}k^{ss}}{y^{ss}}(\hat{r}_t+\hat{k}_t-\hat{y}_t)+\frac{\tau^{ss}}{y^{ss}}(\hat{\tau}_t-\hat{y}_t)-\frac{g^{ss}}{y^{ss}}(\hat{g}_t-\hat{y}_t)+\\ \frac{m^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{m_{t-1}}-\hat{\pi}_t-\hat{y}_t)-\frac{m^{ss}}{y^{ss}}(\hat{m}_t-\hat{y}_t)\end{aligned}$$

Arranging and putting $(\hat{y}_t)$ in evidence:

$$\begin{aligned}-\frac{b^{ss}}{y^{ss}}\hat{b}_t+\hat{y}_t(\frac{b^{ss}}{y^{ss}}-\frac{1+i^{ss}}{(1+\pi^{ss})y^{ss}}+\frac{\tau_t^c c^{ss}}{y^{ss}}c^{ss}+\frac{\tau^w w^{ss}n^{ss}}{y^{ss}}+\frac{\tau^k r^{ss}k^{ss}}{y^{ss}}+\frac{\tau^{ss}}{y^{ss}}-\frac{g^{ss}}{y^{ss}}+\\ \frac{m^{ss}}{(1+\pi^{ss})y^{ss}}-\frac{m^{ss}}{y^{ss}})+\frac{1+i^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{i}_{t-1}+\hat{b}_{t-1}-\hat{\pi}_t)=\frac{\tau_t^c c^{ss}}{y^{ss}}\hat{c}_t+\frac{\tau^w w^{ss}n^{ss}}{y^{ss}}\hat{n}_t+\\ \frac{\tau^k r^{ss}k^{ss}}{y^{ss}}(\hat{r}_t+\hat{k}_t)+\frac{\tau^{ss}}{y^{ss}}(\hat{\tau}_t)-\frac{g^{ss}}{y^{ss}}\hat{g}_t+\frac{m^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{m}_{t-1}-\hat{\pi}_t)-\frac{m^{ss}}{y^{ss}}\hat{m}_t\end{aligned}$$

So, the final equation log-linearization of the government is:

$$\begin{aligned}-\frac{b^{ss}}{y^{ss}}\hat{b}_t+\frac{1+i^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{i}_{t-1}+\hat{b}_{t-1}-\hat{\pi}_t)=\frac{\tau_t^c c^{ss}}{y^{ss}}\hat{c}_t+\frac{\tau^w w^{ss}n^{ss}}{y^{ss}}\hat{n}_t+\\ \frac{\tau^k r^{ss}k^{ss}}{y^{ss}}(\hat{r}_t+\hat{k}_t)+\frac{\tau^{ss}}{y^{ss}}\hat{\tau}_t-\frac{g^{ss}}{y^{ss}}\hat{g}_t+\frac{m^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{m}_{t-1}-\hat{\pi}_t)-\frac{m^{ss}}{y^{ss}}\hat{m}_t\end{aligned} \tag{3.94}$$

The resulting fourteen equation solve for $z_t, u_t, g_t, \hat{y}_t, \hat{k}_t, \hat{n}_t, \hat{\lambda}_t, \hat{c}_t, \hat{I}_t, \hat{m}_t, \hat{r}_t, \hat{i}_t, \hat{b}_t$, and $\hat{\pi}_t$. To summarize, the linearized model consists of equations below, together with the process for the three exogenous shocks. The collection of equilibrium conditions are:

$$-\Phi\hat{c}_t = \hat{\lambda}_t + \hat{i}_t \tag{3.95}$$

$$\frac{y^{ss}}{k^{ss}}\hat{y}_t = \frac{c^{ss}}{k^{ss}}\hat{c}_t + \delta\hat{I}_t + \frac{g^{ss}}{k^{ss}}\hat{g}_t \tag{3.96}$$

$$\hat{c}_t = (1+\tau_t^c)\hat{m}_t \tag{3.97}$$

$$\hat{r}_t = \alpha\frac{y^{ss}}{k^{ss}}(E_t\hat{y}^{t+1} - \hat{k}_t) \tag{3.98}$$

$$(1+\eta\frac{n^{ss}}{l^{ss}})\hat{n}_t = \hat{y}_t + \hat{\lambda}_t \tag{3.99}$$

$$\hat{\lambda}_t = \hat{r}_t + E_{t+1}\hat{\lambda}_{t+1} \tag{3.100}$$

$$\hat{m}_t = \hat{m}_{t-1} + \hat{\theta}_t - \hat{\pi}_t \tag{3.101}$$

$$\hat{\lambda}_t = -\Phi E_{t+1}\hat{c}_{t+1} - E_{t+1}\hat{\pi}_{t+1} \tag{3.102}$$

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + \hat{z}_t \tag{3.103}$$

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{I}_t \tag{3.104}$$

$$-\frac{b^{ss}}{y^{ss}}\hat{b}_t + \frac{1+i^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{i}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t) = \frac{\tau_t^c c^{ss}}{y^{ss}}\hat{c}_t + \frac{\tau^w w^{ss} n^{ss}}{y^{ss}}\hat{n}_t +$$
$$\frac{\tau^k r^{ss} k^{ss}}{y^{ss}}(\hat{r}_t + \hat{k}_t) + \frac{\tau^{ss}}{y^{ss}}\hat{\tau}_t - \frac{g^{ss}}{y^{ss}}\hat{g}_t + \frac{m^{ss}}{(1+\pi^{ss})y^{ss}}(\hat{m}_{t-1} - \hat{\pi}_t) - \frac{m^{ss}}{y^{ss}}\hat{m}_t \tag{3.105}$$

3.5 The data

The data series was constructed in order to obtain the behavior of macroeconomic aggregates of the Brazilian economy. All variables used in this study were taken from "Instituto de Pesquisas Econômicas Aplicadas" (IPEADATA) or Central Bank of Brazil. Additionally, all the following variables were available for all years of the period after 2000 until 2013. The aggregate output data was built by the sum of the variables of consumption and aggregate investment and government spending. This implies that exports and imports do not enter the product composition, once the model it is a closed economy. The consumption variable is the final consumption of households, calculated for IBGE and available in the database of . The investment was the Gross fixed capital formation (IPEADATA). The government spending is the annual final consumption of the government with reference of 2010 calculated by IBGE/SCN and available in the IPEADATA.

For money supply, it was taken into account the restricted concept of M1, that refers to the portion of the means of payment, according to the restricted concept of money, which includes paper money held by the public and demand deposits effectively moved by checks, readily available for payment of goods and services and accepted as currency for the consumption. This variable is available in the Central Bank of Brazil.

The number of hours worked was built from the procedure adopted by Val & Ferreira (2001) and (FILHO et al., 2012). A particularly important issue in Brazil is the fact that national accounts do not have hours worked series. Thus, due to non-availability of a number of hours worked, it was used the number of hours worked calculated by "Confederação Nacional da Indústria" (CNI), and multiplied by an average of working days in the month, 19,25, as in (VAL; FERREIRA, 2001), (FILHO et al., 2012).

The nominal interest rate was considered the accumulated SELIC, available at the Central Bank. The inflation rate used was the IPCA, available in IPEADATA. And the real interest rate was achieved by the difference between nominal interest rates and inflation. The resume of the variables can be shown in the Table 1.

Finally, as Ornellas & Portugal (2011), some variables must have their values in steady state. The corresponding taxes were calculated by the ratio of these variables to proportion of the variable correspondent. The consumption tax, τ^c, is the average of ratio between taxes on the final consumption. The tax of the income, τ^w is the average of the ratio between taxes on income and property on the income of wages. The capital tax, τ^k, is the average of the ratio between tax on capital and GDP. The letters can be also found in the IPEADATA database.

All the names, sources and characteristics of data are provided in the Table 1. After collecting the original data, it was necessary to apply the following data processing procedure: 1) Convert monthly data to annual data, when necessary. 2) Express each variable as its natural logarithm.

Table 1 – Variables

Variable	Description	Sources
Y	GDP	IPEADATA
I	Investment (FBKF)	IPEADATA
C	households consumption	IPEADATA
G	government spending	IPEADATA
M	money suppy (M1)	CENTRAL BANK
n	averagy annual hours	IPEADATA
τ^c	average tax rate	IPEADATA
τ^w	average tax rate	IPEADATA
τ^k	average tax rate	IPEADATA
i	nominal interest rate (overnigth-selic)	CENTRAL BANK
r	real interest rate	CENTRAL BANK
π	Inflation (IPCA acumulated)	IPEADATA

Source: Research results.

In order to extract the cyclic component of the series, it was used the Hodrick and Prescott (HP) filter with smoothing parameter of 100, that is conventional for the annual data, where in the variables are in logarithm, as in Mussolini & Teles (2012).

3.6 Parameter settings of calibration

In calibration, the value of the structural parameters is fixed to those estimated in previous microeconometric studies and/or those computed using long-run averages of aggregate data. Then, the model is simulated using a synthetic series of shocks, and the unconditional moments of the simulated economic series are computed and compared with the ones of actual data. The model is usually evaluates in terms of the distance between these two set of moments (RUGE-MURCIA, 2003).

Before the predictions by (CIA) model, it is necessary to set the parameters of the model. In order to simulate the model, most of parameters for the standard (RBC) model can be followed Walsh (2010), and others works. After setting up the parameters, the computational method will be chosen. Then, the model will be simulated using as computational method the dynare program to solve the dynamics of the artificial economy. Our goal is to show the statistics that include all of variables of standard deviation, correlation and standard deviation relative with shocks of technology, fiscal and monetary policy.

In order to assess the effects of money in this CIA model, and evaluate the model, values must be assigned to the specific parameters. The model must be calibrated. The steady state depends on the values of $\alpha, \beta, \eta, \Psi$, and Φ.

The calibration of the model will be constructed from series of Brazil. Furthermore, certain parameters can be replicated in other studies, mainly those that do not have data observed. All series will be found at IPEADATA and Central Bank of Brazil, comprising the period of 2000 and 2013. The Table 2 summarizes the chosen baseline calibration. In order to study the competitive equilibrium, the following values were selected on the basis of a priori information.

In the calibration, the parameters α, δ, β and θ, they were obtained directly from the data of the Brazilian economy. It was necessary econometric estimates for other parameters, except the parameters η and Φ, that from Walsh (2010). The setting of Ψ makes the steady state value of n^{ss} equal to 1/3. Following Cooley & Prescott (1995), mentioning macroeconomic evidence realizing that people allocate about 1/3 of the total time the production process. Gomes, Bugarin & Ellery (2005), found evidence that Brazil's workers allocate 1/3 of the time in the labor market based on data from the "Pesquisa Nacional de Amostra Domiciliar". The inverse of the intertemporal elasticity of substitution, Φ, is set equal 2 in the artificial simulation. η is set equal to 1. With this, yields a labor supply elasticity of $[\eta n^{ss}/(1 - n^{ss})]^{-1}$, as is Walsh (2010).

The coefficient of capital's share of income α was obtained by reason of gross operating excess and GDP. The depreciation rate δ was obtained using the capital accumulation equation in steady state, making the share $\delta = (I/Y)/(K/Y)$. The reason I/Y

was 19% and K/Y was 2,95. So, the tax of depreciation assumed the value equal 0.0644.

The factor of discount was calculated from the Euler equation in the steady state $1 + r^{ss} = 1/\beta$. the value of r^{ss} was equal 0.051. Assuming $\eta = 1$ implies that the utility is log-linear in leisure. The average growth rate of the money stock, M1, was 11% in the period.

Table 2 – Parameters setting

Parameter	Value	Description
β	0,9515	discount parameters
η	1	leisure parameter
α	0.34	capital share
δ	0.0644	depreciation rate for capital
Ψ	1.56	Parameter in utility function
Φ	2	coefficient of relative risk aversion
θ	0.11	rate of money growth
ρ_z	0.86	autocorrelation of techology shock
ρ_u	0.88	autocorrelation of money shock
ρ_g	0.70	autocorrelation of government shock
σ_z	0.0075	variance of techology shocks
σ_u	0.049	variance of money shocks
σ_g	0.018	variance of shocks government
τ^c	0.23	tax of consumption
τ^w	0.27	tax of income
τ^k	0.023	tax of capital

Source: Research results.

The value of σ_e^2 is set to match the standard deviation of annual HP-filtered log Brazil GDP for the 2000-2013 period. Consistent with the real-business-cycle literature, a stochastic disturbance to total factor productivity is incorporated that follows an AR(1) process:

$$logz_t = \rho_z logz_{t-1} + e_t \tag{3.106}$$

With $e_t \sim^{iid} N(0, \sigma_e^2)$. It's assumed that e_t is a serially uncorrelated mean zero process and $|\rho_z| < 1$. The timing convention is: the capital carried over from period t-1, k_{t-1}, is available for use in producing output during period t.

The parameters of persistence of productivity, ρ_z and productivity shock,$\sigma_z$were obtained from the estimation of the Solow residual, z. The parameters were obtained through the Ordinary Least Squares (OLS) method.

The estimating process of productivity shock yields ρ_z=0.86 with a value of 0.00751 for σ_z, the standard deviation of innovations to the productivity shocks. In Walsh (2010)

Table 3 – Process of productivity shock estimation

	coef.	std. err.	t
$log z_{t-1}$	0.86	0.0074	116.22
const	0.25	0.0098	25.46
$R^2 = 0.99$			
σ_z=0.00751			

Source: Research results.

and Cooley & Hansen (1995) the value of ρ_z assumed the value 0.95. Works that were calibrated for the Brazil we have: Filho et al. (2012) find 0.80; Pereira & Junior (2011);

It is also necessary and very important to specify the process followed by nominal stick of money. For the period from 2000 to 2013, the average annual growth rate of M1 in Brazil was 11 percent. The table 4 shoes the estimated AR(1) process for M1 growth that yield ρ_u=0.88, with a value of 0.04919 for σ_u, the standard deviation of innovations to the nominal money growth rate. While the value of the ϕ coefficient was statistically equal to zero.

Table 4 – Process of money shock estimation

	coef.	std. err.	t
$\Delta log M_{t-1}$	0.88	0.09	9.54
$\Delta log z_{t-1}$	0.27	0.19	1.43
const	1.28	1.01	1.26
$R^2 = 0.99$			
$\sigma_u = 0.04919$			

Source: Research results.

The Table 5, show the process of the expending government shock whit monetary and productivity shocks. The value of ρ_g assume the value 0.70, and the values of ϕ_g and ϕ_u assumed the value zero due the significance of the t test. The value of σ_g assume the value 0.01762 in this process.

The model used can not be solved analytically, so it was chose to follow the computation solution method described by Uhlig (1999) to solve the dynamic system, in which a Taylor expansion is made of the equilibrium conditions around the deterministic steady state. From the linearized model it is possible to obtain the policy functions and calculate the second theoretical moments of the variables and funcitons, impulse response and variance decomposition.

After setting up the parameters, we use the computational method by in the CIA model from equations of linear approximation around the steady state. From the CIA

Table 5 – Process of government expending shock estimation

	coef.	std. err.	t
$\Delta logG_{t-1}$	0.70	0.35	2.00
$\Delta logz_{t-1}$	0.13	0.17	0.78
$\Delta logM_{t-1}$	0.15	0.20	0.77
const	1.89	1.94	0.98
$R^2 = 0.99$			
$\sigma_g = 0.01762$			

Source: Research results.

constraint, we can get the relationships between c_t and m_t indirectly in the system. $\hat{\lambda}_t$ denotes the marginal utility of wealth.

4 Computational Results and Discussion

As showed in the previous chapter, parameters model were calibrated for the steady state. In this chapter, we will feed the model with productivity, monetary and fiscal policy shocks in order to get the second moments (standard deviation and correlation coefficient) and the response functions for the main variables. Although, monetary and fiscal policy shocks and the interaction between both policies are the main goal for this work, we will use productivity shocks to test the model, i.e., we will compare how well the real business cycle model with CIA constraint (artificial economy) matches Brazilian data second moments.

This chapter is divided in five main sections. First section will present the stylized facts of Brazilian economy. Second section will present the artificial economy and test the model; third section will show results for just monetary shocks; four section will show results for just fiscal policy shocks; and, finally, last section will present results for monetary and fiscal policy shocks simultaneously.

4.1 Stylized facts of the Brazilian economy

Before discussing the simulations, it is interesting to make a brief discussion about Brazilian data volatilities and the relationship of macroeconomic variables movements and the main actions of monetary and fiscal policy during the studied period.

According to Moreira & Junior (2013), Brazil has systematically shown internal and/or external macroeconomic disequilibrium since the 70s and this condition generated substantial inflation. To attenuate this effect, policy makers relied on stabilization policies. These policies can frequently result in internal or external debt. On the possible explanations to this debt are the inconsistencies between fiscal and monetary policies.

More recently, after the implementation of the "Plano Real", both monetary policy and the increase in Brazilian public debts have been debated in the academic area and by policy makers. Monetary policy was successful in curbing inflation, despite some shortcomings, whereas the Brazilian public debts sharply increased, mainly in 1998/1999 and 2002/2003, when their sustainability was called into question (NUNES; PORTUGAL, 2009), and in the last years, after 2012.

According to Giambiagi (2004), Brazil presented two major inflection points in relation to fiscal policy: in 1999, a primary fiscal adjustment was made that characterized one of the moments that allowed the division of economic policy into "before" and "after" a certain event, and in the year 2004, when it was expected that a ten-year cycle of increasing public debt/GDP ratio had ended, with the first decline in this ratio since 1994. Furthermore, this period presents a change in the treatment of public accounts in Brazil, where it was introduced a floor for the primary surplus. In 1999, it occurred an important turning point not only for monetary and exchange rate policy, but also for fiscal policy, with the adoption of a stricter budget constraint in agreement with the International Monetary Fund (IMF). Consequently, in 2001, the primary surplus reached 3,5 % of GDP.

The inflation targeting regime has been adopted since 1999 in Brazil, with the objective to control inflation, using interest rate (Selic) as a control instrument of the economy's price level. At the same time, the government committed to achieve a primary surplus target, in order to guarantee the sustainability of the public debt. These measures aimed to signal the government's commitment in order to decrease interest rates and inflation.

According to Ornellas & Portugal (2011), in the case of the Brazilian's economy, the possible existence of a conflict of interest between Central Bank and the National Treasury reinforces the importance of analyzing the interaction between monetary and fiscal policies. On the one hand, the central bank is responsible for controlling the price level of the economy and uses the short-term interest rate as the main monetary policy instrument that responds to changes in the money supply. On the other hand, the responsibility of the National Treasury lies in the administration of public debt, both internal and external. In this way, the National Treasury should have the best financing for the execution of government's actions: lower cost debt and longer terms. Lopreato (2006) argues that if an increase in interest rates damages debt sustainability, the alternative is to broad the fiscal effort required for intertemporal adjustment of public accounts, as a central element of the fiscal policy.

However, the importance of public debt is also crucial in discussing the effectiveness of monetary policy in Brazil. When the interest rate rises, the bond holders' income increases, raising the expectation of disposable income, which can generate a positive effect on consumption and stimulate aggregate demand through the income effect.

According to Filho (2015), Brazil suffers a period of slow GDP growth and high inflation. This phase began in 2012 - when Brazilian terms of trade started to fall and exchange rate began to rise - due to changes in international economic conditions. The slowdown in economic growth has made it difficult to meet government's fiscal targets in recent years. On the one hand, the revenue has grown slowly due to lower GDP growth and tax incentives needed to mitigate the loss of industry competitiveness and encourage investment. On the other hand, the spending continue to grow above GDP growth due to the expansion of the social programs, for instance, direct income transfers and public spending on housing, health and education.

From december 2001 to July 2015, the average debt/GDP ratio in Brazil was 42.7%

with a peak in the debt/GDP ratio in september 2002, reaching 62.35% of GDP. In this period, due to uncertainties generated by presidential election, there were disturbances that led to a depreciation of the exchange rate, which resulted in the increase of the debt to GDP ratio. Since then, a period can be highlighted, when debt/GDP fell, hitting 37 % in november 2008. This process still suffered a slight interruption at the end of 2009, when it grew again in the context of the world's crisis. However, the process of falling was resumed until the end of 2013, when from this year again increased[1]

In order to characterize the cyclical behavior of the macroeconomic variables used, some procedures were adopted with the objective of obtain the cyclical movements through their respective volatilities and additionally, their correlation with output. All series have been filtered through the HP filter, which, as previously mentioned, it will allow us to work only with the cyclic component of the original series.

The dynamic characteristics of the Brazilian economy were analyzed in the period between 2000 and 2013. The stylized facts obtained from the Brazilian macroeconomic variables are shown in the Table 6. The first column corresponds to the standard deviations of each macroeconomic variable (σ_x); the second corresponds to the ratio between the standard deviations of each variable and the output (σ_x/σ_y); and the third and last corresponds to the correlations of the product with each of the variables along the cycle ($\rho(xy)$).

Table 6 – Stylized facts of the Brazilian economy between 2000 and 2013

Real economy			
Variable	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$
y	1.42	1.00	1.00
c	1.22	0.86	0.99
I	4.37	3.07	0.99
n	2.33	1.64	0.93
π	28.31	19.87	-0.39
i	13.29	9.33	-0.90
m	4.46	3.13	0.99
r	26.39	18.53	-0.83
b/y	6.10	4.28	-0.96

Source: Research results.

When analyzing the cyclical behavior of output and consumption it is possible to see that their volatilities are very close. The volatility of consumption is 14% lower than the volatility of output. The characteristic of these variables is the strong positive correlation between them, which indicates that the aggregate consumption shows a very close behavior to the product, increasing in expansions and decreasing during the economic contractions.

1 According to data available in Ipeadata.

An explanation for this high correlation between consumption and output, as discussed in Val & Ferreira (2001), would be linked, in addition to other factors, a restriction on liquidity, which would prevent Brazilian's households from smoothing consumption as desired.

The aggregate investment showed high volatility when compared to output and consumption. This variable showed a volatility of just over three times the volatility of the output, and strong correlation with the output.

The series of hours worked showed higher volatility than the output according to the Table 6. Moreover, the relation between the product and the hours worked was positive, indicating that the variable is procyclical. The volatility of this series was slightly higher than other works for Brazil. This can be related to the fact that there are not a series of hours worked for the Brazilian economy, being necessary its creation.

The nominal variables show different characteristics of the real variables. The volatilities are larger in comparison with the volatilities of the real variables. Among the volatilities of the nominal variables, the monetary balances are the ones with the lowest volatility, although it represents a little more than three times the cyclical variations of the output.

Monetary balances in the M1 concept presented volatility higher than product volatility, in addition to being strongly correlated with output. There are similarities with other works carried out for Brazill, as Ellery-Jr & Gomes (2005), Filho et al. (2012). The average growth rate of this series was of the order of 11% per year during the period studied.

In addition, inflation shows a significantly high volatility. Thus, the inflation rate has a countercyclical relationship with the product, indicating that in times of economic expansion these variable may decrease, and on the other hand, when there are economic contractions, the price level increases.

Real interest has also showed significantly high volatility. Its correlation with output is countercyclical, as the nominal interest rate. According to Kanczuk (2002), the interest rate affects labor supply through income and substitution effects. In a country where the government has a net debt, such as Brazil, an increase in interest rates has ambiguous effects. As leisure is a superior commodity and families get richer with a higher interest rate, the income effect reduces labor supply. The substitution effect points in the opposite direction: it is more attractive to work when the returns on accumulated assets are greater.

According to Wichmann & Portugal (2013), it is expected fiscal policy to be countercyclical, at least in recession episodes. In other words, government allows debt-to-GDP ratio to grow over time when there is a recession.

4.2 Artificial economy

In order to build the artificial economy, we set all distortionary taxes to zero. There is no shock to fiscal and monetary policies. Therefore, fiscal policy is fully described by the long-run ratio of government expenditures-GDP. It is assumed that government revenue comes from a lump-sum tax on households budget constraint. Our artificial economy differs from previous Brazilian empirical studies because we added the CIA constraint in the real business cycle model; therefore, we need to assume a rule for monetary policy in the artificial economy. Let the monetary policy be fully described by the long run rate of money growth.

4.2.1 Model goddness of fit

The basic statistics of the Brazilian cyclical behavior are described in Table 7. All series are in logarithms and then it was used the HP-filter to remove the trend. As Ellery.Jr, Gomes & Sachsida (2002) pointed out, a feature of particular interest in Brazilian cycle is the high volatility of the series, mainly, consumption and hours worked. The standard deviation of consumption is 1.22% and correlation with the GDP is 0.99. The standard deviation of hours worked is 2.33% and correlation with the GDP is 0.93.

Table 7 – Brazilian cyclical behavior

Variable	σ_x	σ_x/σ_y	ρ_{xy}
Y	1.42	1.00	1.00
C	1.22	0.86	0.99
I	4.37	3.07	0.99
n	2.33	1.64	0.93

Source: Research results.

For computational simulations, we log-linearized the model equations around the steady state. Then, using the parameters calibrated for the steady state, we feed the artificial economy with productivity shocks in order to test how well it matches Brazilian economic fluctuations. Table 8 shows the statistical moments for key variables for Brazilian data and artificial economy.

Comparing Brazilian data statistics to the artificial economy statistics, we claim that the model provides a very good fit for the variability in output and investment. For instance, the standard deviations of output and investment, in data, are 1.42% and 4.37%, respectively, while, in the simulated model, they are equal to 1.48% and 4.82%, respectively. Also, the correlation between investment and GDP is close enough, it is equal to 0.99 in data and 0.94 in the simulated economy. It is worthwhile to notice that the artificial economy slightly overestimated the effect of productivity shock on output and investment volatility. This comes from the CIA constraint assumption since an increase

Table 8 – Real and artificial economy

	Real economy			Artificial economy		
Variable	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$
Y	1.42	1.00	1.00	1.48	1.00	1.00
C	1.22	0.86	0.99	0.86	0.58	0.89
I	4.37	3.07	0.99	4.82	3.25	0.94
n	2.33	1.64	0.93	0.40	0.27	-0.10

Source: Research results.

in money supply, in this model, works as taxation on consumption goods relatively to investment goods because just consumption expenditures are subject to cash in advance. Therefore, this model provides an additional motive for households to trade consumption for investment. For the same reason, our artificial economy fits consumption volatility better than the basic real business cycle model.

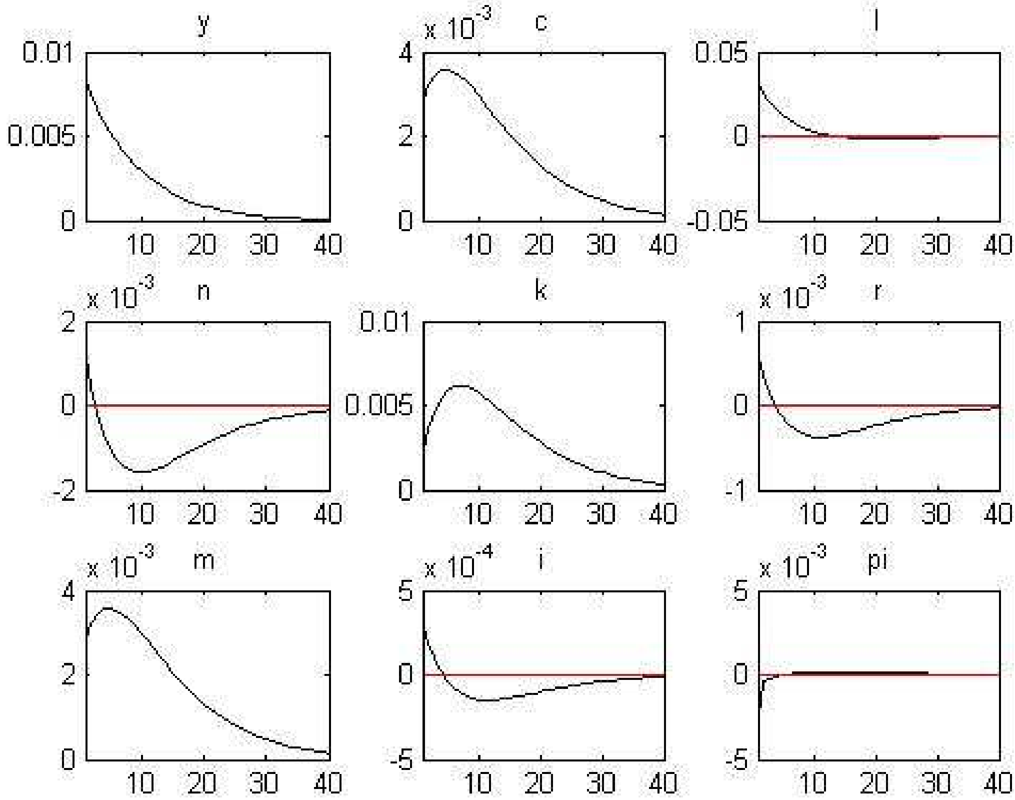
As showed in Table 8, the artificial economy accounts for approximately 70% of consumption variability. This is a great improvement on the basic RBC models where consumption is much smoother than data. For instance, Ellery.Jr, Gomes & Sachsida (2002) found that the standard RBC model can account for just 55% of Brazilian consumption variability.

The model does not provide a good fit for hours worked series. The standard deviation of hours worked in the artificial economy accounts to approximately just 17% of hours worked standard deviation in data. This result is in line with standard real business cycle models. Hence, just productivity shocks are not capable of describing volatility in the labor market, even when we consider that households are subject to CIA constraint. Therefore, in the CIA-RBC model with productivity shock, households are more willing to trade consumption for investment than consumption for leisure. This result was expected since productivity shocks have stronger effects in the real variables than in nominal variables, as showed in the next subsection. Finally, in the artificial economy, the work hours series basically is near zero, i.e., it is acyclical, while, in the data, it is strongly procyclical.

4.2.2 Productivity shock effects

Figure 2 shows the dynamics to the artificial economy following a technology shock. It reports the percentage deviation of the selected variables from their steady state value. Investment has a higher volatility than GDP. Also it is strongly pro-cyclical. For the artificial economy, the technological shock has an initial positive effect on output, consumption and investment as well as leading to an increase in wage and labor. Output increases for given factors of production leading to a household wealth effect.

Figure 2 – Impulse Response Function: Technology shock



As would be expected, a positive shock to productivity leads to an increase of output. As output rises, the shock causes consumption and investment both to increase. With an increase in capital stock, rental rate falls below the steady state rate.

The inflation rate declines in the first periods, but returns quickly for the steady state. Money balances grow in magnitude greater than the reduction in inflation and adjusts more slowly. Real and nominal interest rates slightly rise initially, and then fall below the steady state in the following periods. The effect is higher for real interest rate than for nominal interest rate since inflation persistence is very low. This experiment shows that the productivity shock has higher influence in the real variables than in nominal variables.

A productivity shock increases the real wages and thus the real income of households that produces an increase of real consumption. In the other hand, expected return on capital increases and, along with it, investment. Stimulated by a decrease in real marginal cost, output rises because of the increased supply of labor and capital.

4.3 Monetary shocks and economic cycles

To examine how monetary changes affect the cyclical properties of the Brazilian macroeconomic variables, initially it was added shocks in monetary policy, as presented previously. The estimated value of the persistence parameter was equal to 0.88. The relationship between unanticipated changes in productivity and money was statistically null [2]. The monetary shock standard deviation was set equal to 0.049. Other parameters used in the simulation were similar to those used in the artificial economy.

The results of the monetary economy, with the nominal money supply following a stochastic autoregressive process and a monetary shock, are shown in the Table 9. It shows statistics that include all (real and nominal) variables standard deviation, correlation with output, and standard deviation relative to output. First, the comparisons were performed with respect to the results obtained from the artificial economy simulation. Then, this model was compared with the real economy moments. It is assumed that the quantity of money is set exogenously.

Table 9 – Effects of monetary shocks on the model

	Artificial economy			Monetary economy		
Variable	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$
Y	1.48	1.00	1.00	2.60	1.00	1.00
C	0.86	0.58	0.89	3.17	1.22	0.90
I	4.82	3.25	0.94	5.99	2.30	0.24
n	0.40	0.27	-0.10	3.28	1.26	0.64
π	0.29	0.19	-0.56	10.82	4.16	-0.71
i	0.06	0.04	0.39	8.18	3.15	-0.71
m	0.86	0.58	0.89	3.17	1.22	0.90
r	0.12	0.08	0.27	0.34	0.13	0.56

Source: Research results.

The most important result of Table 9 is the fact that in order to have higher volatility of nominal variables, it is necessary to consider unanticipated monetary policy shocks. Moreover, in this model, correlation between inflation and nominal interest rate with output, is strongly negative, i.e., inflation and nominal interest rate are clearly countercyclical.

Unsurprisingly, the inclusion of monetary shocks has significantly altered the volatility of real macroeconomic aggregates, which showed higher fluctuations to those observed in artificial economy. This comes directly from the fact that, in this section, we added a new source of uncertainty, monetary shocks.

When one takes into account the real variables only, investment was the variable

[2] See Table 5

that showed the greatest variation in the short run, as well as in the real economy and also in the artificial economy. The simulated time series of investment provides a standard deviation of 5.99 and a cross-correlation of 0.24 with output. This value is higher than artificial and real economy, 4.82 and 4.37, respectively. On the other hand, the standard deviation of investment amounts to 2.30 times the standard deviation of output. This value is smaller than artificial and real economy. This happened due to the increased volatility of the product with the inclusion of monetary shocks.

More interesting is the hours worked series, with volatility equals to 3.28%. This variable showed closed to the real economy and much higher than the artificial economy with just productivity shocks. From the artificial economy to the economy with monetary shocks, the volatility grew almost eight times. This represents the influence of monetary policies on the household's decision. This happens because nominal interest rate and inflation affects the relative price between consumption and leisure for households in the CIA model. Since with monetary shocks, nominal variables showed greater volatility compared to artificial economy, especially inflation, nominal interest rates and cash balances, then, monetary shocks induces household to change consumption for leisure and increases labor market volatility.

The consumption showed higher volatility, showing that unpredictable monetary policy has greater effect by changing the inflation expectations and interest rate. This was evidenced in Cooley & Hansen (1995) monetary shocks tend to increase the volatility of consumption and investment. With both shocks, monetary and technological shocks, consumption volatility increases due to two effects: (i) from the technological shock, because it changes the relative price between consumption and investment and (ii) from the monetary shock, throughout changes in nominal interest rate that is the cost of demanding real money balances and, therefore, changes the relative price between consumption and leisure.

Contrary to the artificial economy, nominal variables play an important role in the model with monetary shocks. The inflation rate showed a significant rise in short run fluctuations, from a standard deviation of 0.29 in the artificial economy to 10.82 in the economy with monetary shock. However, inflation showed a standard deviation in the real economy even higher than the monetary economy, as showed by table 10. Monetary shocks can explain just 38% of volatility in the Brazilian inflation rate in the period. Moreover, the correlation between inflation and output was stronger in the model with the monetary shock (-0.71%). This result implies that unanticipated monetary policy had little effect in Brazilian inflation rate.

With the inclusion of monetary shocks, the standard deviation of the variable cash balances increased from 0.86 to 3.17. The economy with monetary shocks approached the real economy, in which money balances showed a standard deviation of 4.46. This significant change is linked to the fact that the artificial economy does not take into account the unanticipated monetary changes, which increases volatility in the short run.

Table 10 – Real economy and monetary shocks simulation

	Real economy			Monetary economy		
Variable	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$
y	1.42	1.00	1.00	2.60	1.00	1.00
c	1.22	0.86	0.99	3.17	1.22	0.90
I	4.37	3.07	0.99	5.99	2.30	0.24
n	2.33	1.64	0.93	3.28	1.26	0.64
π	28.31	19.87	-0.39	10.82	4.16	-0.71
i	13.29	9.33	-0.90	8.18	3.15	-0.71
m	4.46	3.13	0.99	3.17	1.22	0.90
r	26.39	18.53	-0.83	0.34	0.13	0.56

Source: Research results.

It's possible to check the increase in the standard deviation of the cycle of the nominal interest rate when compared to the artificial economy. The inclusion of monetary shocks increased the volatility of interest rate (8.18%), although not fully capturing the volatility as captured the real economy (13.29%), in other words, unanticipated monetary shocks can explain nearly 62% of nominal interest rate volatility. On the other hand, this model, can explain more than 70% of real money balances volatility. Moreover, the correlation between real money balances and GDP is close, in the model, it is equal to 0.90 while in data is 0.99.

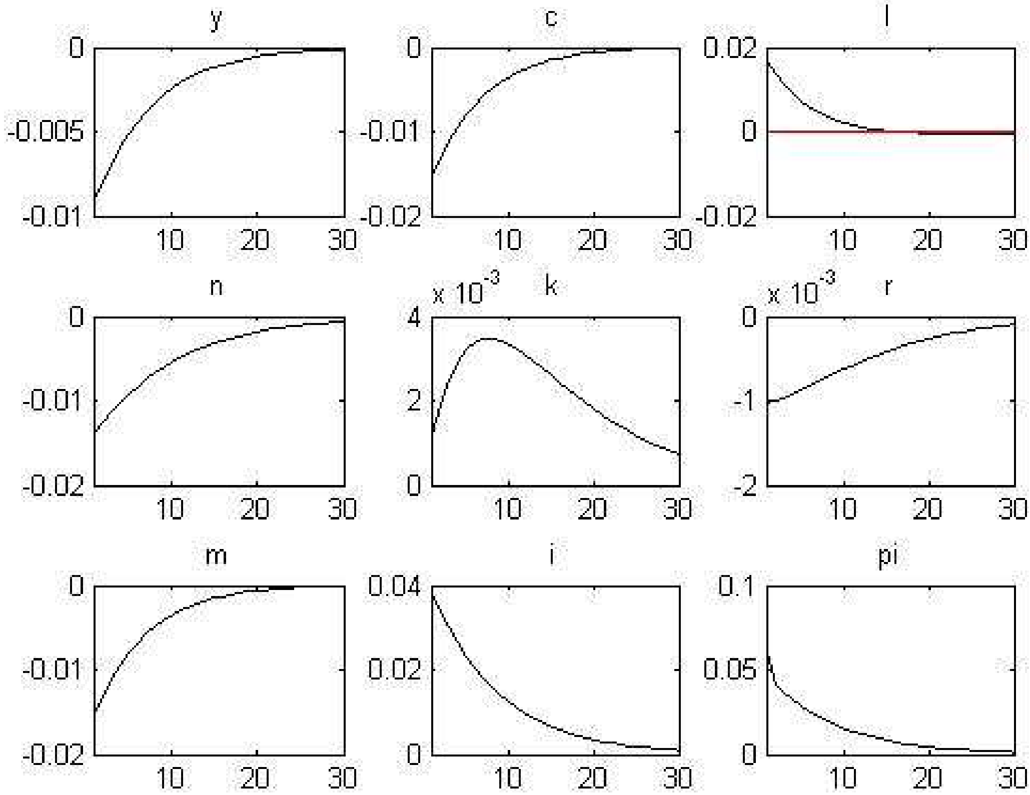
Figure 3 shows the magnitude of the effect of one standard deviation monetary shock on the variables. The effects depend on the degree of persistence in the money growth process. Higher values of ρ_u[3] generate larger effects. When $\rho_u > 0$, money growth rate has a positive autocorrelation and the monetary shocks does make a negative impact on economic activity.

As shown in Figure 3, a positive monetary shock increases the nominal interest rate. Monetary policy is usually thought to reduce nominal interest rates, at least initially. The negative effect of money on nominal interest rates is usually called the liquidity effect, and it arises if an increase in the nominal quantity of money also increases the real quantity of money because nominal interest rates would need to fall to ensure that real money demand also increased (WALSH, 2010).

In general, a positive monetary shock may cause liquidity effect or anticipated inflation effect. The liquidity effects is characterized by a drop in nominal interest rate and a rise in output. The anticipated inflation effect occurs when there is an increase in the nominal interest rate and a decline in output.

[3] ρ_u is the coeficient of persistence of the monetary shocks.

Figure 3 – Impulse Response Function: Monetary shock



The prices have been assumed to be perfectly flexible. So, the main effect of money growth rate shocks when $\rho_u > 0$ is to increase expected inflation and raise the interest rate. The monetary shock generates a jump in the price level immediately.

Inflation rises instantaneously. Thus, the purchasing power of the transferred monetary balances decreases, and so does consumption via CIA constraint. It happens because inflation is a tax on the consumption. So, agents will transfer their consumption to future period, in which inflation tax will be lower. The decrease in hours worked implies that output will instantaneously be below its steady state level, since capital is predetermined.

After analyzing the response functions to impulses, it is interesting to show the participation of shocks in the variables through variance decomposition. The variance decomposition analysis suggests that monetary and technological shocks are a very important source of fluctuations in output. Table 11 indicates that the fluctuations of investment and capital are explained mostly by productivity shocks. On the other hand, consumption, hours worked, real money balances and nominal variables, interest rate and inflation are mostly explained by monetary shocks.

Table 11 – Variance decomposition

Variables	eu	ez
Y	49.80	50.20
c	84.25	15.75
I	22.94	77.06
n	96.57	3.43
k	25.36	74.64
r	80.24	19.76
m	84.25	15.75
i	99.99	0.01
π	99.93	0.07

Source: Research results.

4.4 Fiscal shocks and economic cycles

To examine how fiscal changes affect the cyclical properties of Brazilian macroeconomic variables, initially was added shocks in government spending conform previously presented equation. In addition to the persistence process, the cycles began to be influenced by stochastic fiscal shocks.

The estimated value of the persistence parameter was equal to 0.70. The relationship between unanticipated changes in productivity and fiscal policy was statistically null [4]. For the fiscal shock was obtained value of 0.018. The others parameters used in the simulation were similar to those used in standard simulation.

Table 12 reports the model generated second moments with government expenditures shocks. First, comparing with the data in artificial model, it's possible to see that the volatility of all variables is larger in the fiscal economy, with the exception of output. The introduction of government expenditure shocks smooth out the volatility of the output, this comes from the crowding out effect, as it will be showed by the impulse functions latter in this section. The simulation showed that the volatility decreased by 11%. In the artificial economy, the volatility was 1.48%, and with fiscal shocks effects the volatility declines to 1.32%. On the other hand, the variation in volatility of consumption almost doubles. In the artificial model, the volatility of consumption was 0.86, going to 1.43 in an economy with government, approaching the actual data. On the other hand, the volatility of the government debt is about the double of households' consumption.

The simulation time series of investment provides a standard deviation of 5.10% and correlation of 0.80 with output. From Brazilian data, see table 13, the standard deviation of investment amounts to 3.07 times the standard deviation of output. The simulated model series were found a value of 3.86 to the same ratio. From the artificial

[4] See Table 5

Table 12 – Effects of fiscal shock on the model

	Artificial economy			Fiscal economy		
Variable	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$
Y	1.48	1.00	1.00	1.32	1.00	1.00
C	0.86	0.58	0.89	1.43	1.08	0.84
I	4.82	3.25	0.94	5.10	3.86	0.80
n	0.40	0.27	-0.10	0.87	0.66	-0.57
π	0.29	0.19	-0.56	0.48	0.36	-0.36
i	0.06	0.04	0.39	0.06	0.05	-0.12
m	0.86	0.58	0.89	1.68	1.27	0.84
r	0.12	0.08	0.27	0.16	0.12	-0.20
b/y	-	-	-	2.37	1.80	-0.22

Source: Research results.

model, the ratio is closer. For investment, its volatility presented a slight increase, still remaining the most volatile variable, more than three times the volatility of product.

Another series that had its volatility increased was hours worked. From the artificial economy to the economy with government, the volatility nearly doubled, from 0.40 to 0.87. Although, it is still short if compared with actual Brazilian data.

In this simulation, the inclusion of the fiscal policy contributes to an increase of 50% of the volatility of inflation. In the artificial model, the variation in standard deviation was 0.29 going to 0.48 in a simulation with government shocks. Although, it is still lower of actual data volatility and both simulations showed contercyclical relationship with the product, as seen in the actual data.

It was found that the real interest rate presented volatility 25% higher than in artificial economy, with significantly lower standard deviation to that found in the real economy. In the artificial economy, the correlation of real interest rate and the output was procyclical. However, with the introduction of fiscal shocks, the relationship has become countercyclical. On the other hand, the volatility of nominal interest rate presents similarity in both models. Although changing the cyclical relationship.

Shocks in fiscal spending caused changes in the volatility of cash balances, which grew about twice the volatility of the artificial economy, although representing less than half of what the actual facts revealed. Furthermore, the cash balances are highly correlated with the product, confirming the actual data procyclical relationship.

If statistics for model with government are compared to statistics for Brazilian data, presented by Table 13, numbers suggest that the model can account, in some sense, for the observed variability in output, investment and consumption. The standard deviation of output is 1.42%, and in the data and 1.32%. The inclusion of shocks in government spending favored a reduction in the volatility of the output. On the other hand, the model does not have a good match for inflation rate, nominal and real interest rate and, real balances. Moreover, it underestimates the debt standard deviation.

Table 13 – Real and fiscal economy effects

	Real economy			Fiscal economy		
Variable	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$	$\sigma_x(\%)$	σ_x/σ_y	$\rho(xy)$
Y	1.42	1.00	1.00	1.32	1.00	1.00
C	1.22	0.86	0.99	1.43	1.08	0.84
I	4.37	3.07	0.99	5.10	3.86	0.80
n	2.33	1.64	0.93	0.87	0.66	-0.57
π	28.31	19.87	-0.39	0.48	0.36	-0.36
i	13.29	9.33	-0.90	0.06	0.05	-0.12
m	4.46	3.13	0.99	1.68	1.27	0.84
r	26.39	18.53	-0.83	0.16	0.12	-0.20
b/y	6.10	4.28	-0.96	2.37	1.80	-0.22

Source: Research results.

The standard deviation of government debt in simulation it was only 39% of the volatility of the standard deviation in the real series of government debt. The negative correlation was verified, although much smaller (-22%). This find reveals the countercyclical relationship between government debt and the output. Among the fiscal variables, the simulated model overestimated the volatility of government spending. On the other hand, underestimated the volatility of debt.

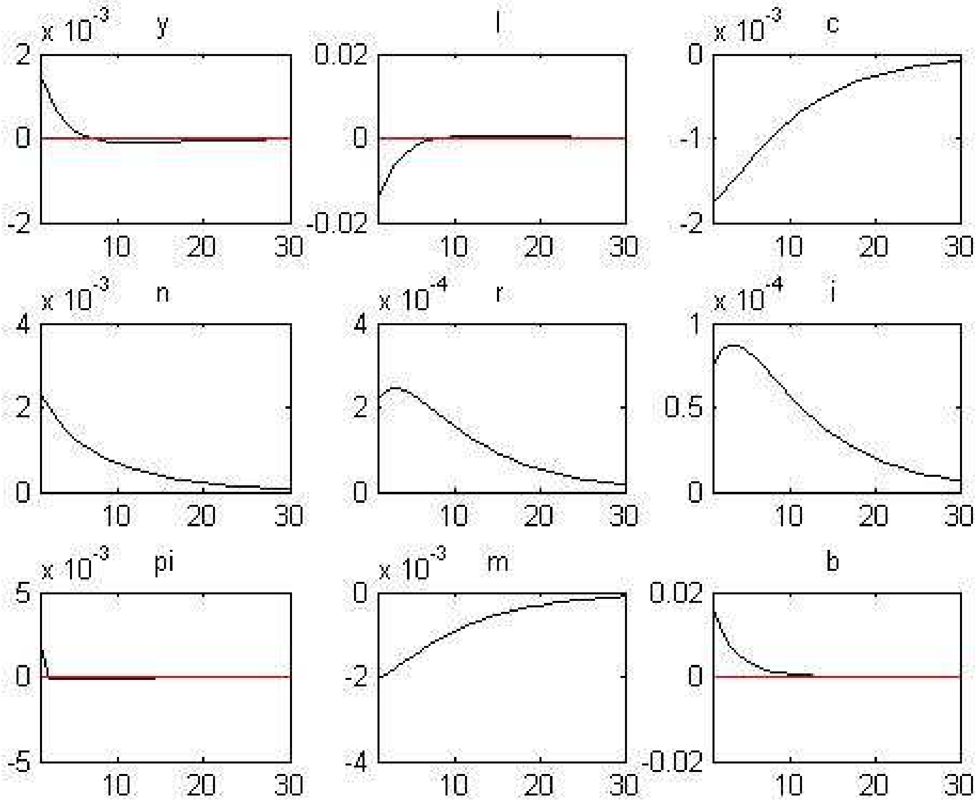
The model does not provide a good match for the labor market. Table 12 displayed that the model underestimates the volatility of hours worked. The simulated standard deviation is a third of the real economy.

Looking at the consumption properties, it is possible to see that with inclusion of fiscal shocks, the volatility of consumption is 1.43%, very close to the volatitity of the real economy data. This increasing in the standard deviation can be linked the fact that households' consumption is affected when government spending increases, conform the Ricardian equivalence.

Following a shock to government spending, it is expected a drop in consumption and an increase in rental rate and hours worked. The shock to government expenditure has a positive effect on output, hours worked, debt of the government, as well as, an increase in inflation and the interest rates. On the other hand, it has negative effects on the investment and consumption.

Figure 4 shows the responses of output, consumption, inflation, nominal and real interest rate, hours worked, inflation, cash balanced, debt and investment following a fiscal shock. Figure 4 reveals that output, hours worked, real interest rate, inflation and the debt rise after a positive shock of government spending. The inflation returns more quickly for

Figure 4 – Impulse Response Function: Government's expenditure shock



the steady state.

On the other hand, the graphs of the impulses of government spending show that a positive and unanticipated innovation in government spending will decline the investment and consumption. Investment declines in the first period and then starts increasing. It is necessary ten periods for the investment to return to its steady state level. Consumption returns after about thirty periods to its long run level.

The economy displays the usual features following a shock to government spending such as a crowding out effect on consumption and investment. The decreased consumption leads to households choosing work over leisure as the marginal utility of consumption increases. The additional supply of labor causes a fall in wage. Moreover, in order to finance its expenditures, government increases its debt. Households, in the short run equilibrium, increase their savings in bonds and decrease investment. The drop in investment leads to a fall in capital stock and a rise in rental rate, and the interest rate.

Hence, there is a strong crowding out effect on private consumption and investment following an increase in government expenditure. When government spending goes up, the household feels poorer because it has to pay more in taxes, in the future. Households will consume less and work more. Working more raises output. Thus, the mechanism by which government spending impacts the output in the economy is not to stimulate demand, but throughout the wealth effect, in which people feel poorer and supply more work hours.

Increases in government consumption produce a negative wealth effect, so that families respond decreasing leisure, or increasing the supply of work and therefore, output. Thus, the marginal product of capital increases, which encourages investment. The same negative wealth effect generates a lower consumption. As the shock is temporary, but shows considerable persistence, variables slow down to return to its steady-state value. (MUSSOLINI; TELES, 2012)

4.5 Fiscal and Monetary effects on the economic cycle

The aim of this section is to show the results of the simulations and discussion about different results regarding the interaction of monetary and fiscal policies in a RBC model with CIA constraint. To examine how monetary and fiscal policies can influence the cyclical changes in macroeconomic variables were considered different situations. Unlike the previous section, the shocks were performed simultaneously, technological, fiscal and monetary, taking into account the value of their persistence and standard deviations estimated and presented in the Table 2.

The persistence of the technological shock parameter was equal to 0.86. The estimated value for the persistence parameter for the monetary shock was equal to 0.88, and according to estimates, the unanticipated changes in productivity was not statistically different from zero. The parameter of shock persistence in government spending was equal to 0.70, and unanticipated changes in productivity and monetary policy were statistically zero. The monetary and fiscal shocks assumed the values 0.049 and 0.016, respectively. The others parameters remained.

This experiment illustrates on of the basic channels through which fiscal and monetary policy affects economic activity in general equilibrium models. If government's spending exceeds its revenue, the resulting deficits have to be financed either through borrowing or issuing money. However, borrowing is limited by the public's capacity or willingness to hold additional government debt, and monetary expansion leads to inflation (NEYAPTI, 2003), i.e., short run monetization of public debt.

This section describes the monetary and fiscal policy problem. The requirement to finance government expenditure and interest payments on government debt with distortionary taxes can depress labor supply and output, besides, depress capital and consumption.

Table 14 shows the results of short term simulations with the simultaneous inclusion of the three shocks in comparison with the real economy and also sets the artificial economy with just technological shocks. The inclusion of monetary and fiscal factors, through persistence and shock, has changed significantly the volatility of the variables, except for the product, which is lower than artificial economy. For output, consumption and investment, this experiment shows that technological shocks can explain most of economic fluctuations of these variables.

The model is consistent with the large (percentage) variability in inflation data, though its magnitude is smaller. The model can capture approximately 32% of Brazilian inflation variability. Therefore, most of inflation variability of this period cannot be explained by unanticipated fiscal or monetary policy. Moreover, comparing this experiment with Table 16, unanticipated fiscal policy has little effect on short run inflation variability. The model inflation variability is generated, mostly, by shocks in monetary policy.

Table 14 – Simultaneous Technological, Monetary and Fiscal shocks

Variable	Real economy			Artificial economy			Simultaneous shocks		
	σ_x %	σ_x/σ_y	ρ_{xy}	σ_x %	σ_x/σ_y	ρ_{xy}	σ_x %	σ_x/σ_y	ρ_{xy}
Y	1.42	1.00	1.00	1.48	1.00	1.00	1.49	1.00	1.00
C	1.22	0.86	0.99	0.86	0.58	0.89	2.71	1.82	0.82
I	4.37	3.07	0.99	4.82	3.25	0.94	5.81	3.90	0.42
n	2.33	1.64	0.93	0.40	0.27	-0.10	1.87	1.26	0.36
pi	28.31	19.87	-0.39	0.29	0.19	-0.56	8.84	5.93	-0.58
i	13.29	9.33	-0.90	0.06	0.04	0.39	6.36	4.27	-0.59
m	4.46	3.13	0.99	0.86	0.58	0.89	3.18	2.13	0.82
b/Y	6.10	4.28	-0.96	-	-	-	5.43	3.64	0.03
r	26.39	18.53	-0.83	0.12	0.08	0.27	0.25	0.17	0.43

Source: Research results.

Interesting, the simulation reveals that the cyclical changes in hours worked in this model has a better match with actual Brazilian data, representing 80% of the volatility of real series of hours worked. Furthermore, it presents a positive correlation with GDP, although smaller than data. This result shows the importance of considering CIA constraint in order to explain how households change consumption and leisure throughout nominal interest rate changes.

Government deficit volatility in this model represents almost 90% of the volatility in real series. For real money balances, the model with simultaneous shocks captures almost 71% of the actual volatility. This is an interesting result. Comparing government debt in this experiment with the experiment when there is just fiscal policy shocks, government debt more than doubles. In other words, monetary policy constraints fiscal policy by affecting nominal interest rates, i.e., monetary policy is the active policy while fiscal policy is the passive policy in the short-run fluctuations in Brazil.

Although the volatility of real interest rate has doubled from the artificial economy to this simulation, this value is still far from the volatility of the actual series, the model tends to smooth real interest rate, unlike the real economy, where real interest rate volatility in terms of output is higher than eighteen times.

Although many austerity plans have been created over the decades of 80 and 90, the landmark of the incorporation of the new Brazilian economic policy management took place from 1999. The introduction of inflation targets regime and the regime of primary surplus targets restrict the discretion of the authorities (JORGE; MARTINS, 2013).

According to Mendonca & Silva (2008), the use of interest rates as the main instrument for achieving the inflation target in Brazil and the fact that much of the public debt is indexed to this rate means that there is a reciprocity between driving monetary policy and public debt management. It was observed that the use of a monetary policy focused on a disinflationary process caused an effect that cannot be overlooked on the fiscal balance. Therefore, for an economic policy based on fiscal balance and achieving the inflation target to be successful, it is necessary to combine an adequate management of public debt with the definition of an interest rate that causes the converging expectations for the announced target.

Increase in interest rates (Selic) to contain inflationary pressure leads to an increase in Brazilian public debt. This increase reflects the fact that much of the public debt is indexed to the interest rate that the central bank determines. From 2001 to July 2015, the average share of government securities indexed to interest rate was 59 % of the total public debt. That is, more than half of Brazil's public debt is indexed to interest rate that determines monetary policy.

The fiscal policy pegged to rigid rules and standards compliance of the planned targets. It became mandatory to define the Budget Guidelines Law, the primary surplus target next year and the indication for the next two years. The commitment explanation with the trajectory of public debt enthroned in budget execution practice contigency of public expenditure in the amount required for the primary surplus target was achieved. The primary surplus earned compulsory expenditure of character and discretionary spending took wastepaper, always liable to be adjusted in the name of compliance with the fiscal target. Moreover, the approval of the Fiscal Responsability Law reinforced the control of states and municipalities accounts, defined legal limits of consolidated debt and specific costs, standards of concentration of credit operations and disposal criteria of any excess debt (LOPREATO, 2007).

In order to test the model and to find possible answers for the failures in the results in comparison with the real data of the Brazilian economy, other simulations had made with changes in the main parameters. Therefore, by simulating again the volatilities for the Brazilian macroeconomic series, leaving from the model with simultaneous monetary and fiscal policy shocks, and keeping all other parameters constant, obtained the results described in Table 15. This exercise through the performance of the sensitivity analysis gave us a measure of how the results of volatilities would change if the model parameters were calibrated with different values. In the table 15, we have simulations to change to four main parameters, as in Pereira & Ellery-Jr (2011).

The volatility of hours worked is not well captured in the model against the real data. As in Kanczuk (2002), also presents a Volatility lower than it was observed. Even with the inclusion of different shocks, the simulations were not able to replicate the volatility of the series of hours worked in the Brazilian economy, and according to Pereira & Ellery-Jr (2011), there are two non-exclusive arguments for this result. The first would be the construction of the series that does not match the model's need; the second, the intertemporal substitution elasticity in this work calibrated with Walsh (2010), with date of the United States, may not represent the national reality. Thus, the first argument is widely used in the national literature, since the data used are indexes, where some assumptions are needed. In addition, the data are formal employment, among other difficulties.

As regards labor supply, the results indicate that formal work in Brazil implies the need to use an intertemporal substitution elasticity of labor higher than that commonly employed in the international literature. Likewise, the table 15 shows that the volatilities of the other variables seem to follow a behavior towards the volatilities of the real series, when the intertemporal substitution elasticity increases.

The parameter of the intertemporal substitution elasticity indicates the higher valor of this parameter, the volatility of hours worked become more approximate to the volatility of the real series. The simulations show that with an increase of 10% on the value of the parameter, the volatility increases to 1.92, which means 80% of the volatility of the real series.

The value used in the simulations calculated previously through the steady state equations for the intertemporal discount rate, β, which measures the impatience of the representative household agent, was 0.9515. This rate relates the decisions of present and future consumption, so that reductions in β increase the weight of future consumption in the utility of the agents, indicating that this would represent stimulus to the present savings. Therefore, variations in this parameter may influence the decisions between present and future consumption. The simulations show that an increase of 5% in the value of this parameter, the volatility of consumption changes to 4.39 compared to 2.71 in the artificial economy. In addition, this volatility is three times the volatility of the real series. Also, the money volatility is also very sensitive to changes in this parameter.

As for the changes in the α parameter, the real series were more sensitive, showing more changes in volatility. On the other hand, the nominal variables suffered marginal variations in their respective volatilities.

Altogether, the effects were less significant for variations of the η parameter. In addition, the nominal variables were not very sensitive to changes in the parameters of the model, mainly inflation, nominal interest rate and public debt.

Finally, the real interest rate presented a constant behavior, it did not respond to the process of changing the parameters, except only when in the occurrence of changes in the β parameter, which it was expected. Moreover, the model did not adequately capture the standard deviations of the real and nominal interest rate business cycles observed in the Brazilian economy and, therefore, the results related to these variables should be analyzed with prudence.

Table 15 – Sensitivity analysis

	Variation in Ψ				Variation in η				Variation in β			Variation in α			
	-5%	-10%	5%	10%	-5%	-10%	5%	10%	-5%	-10%	5%	-5%	-10%	5%	10%
Variable	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %	σ_x %
Y	1.48	1.46	1.49	1.50	1.49	1.50	1.48	1.47	1.83	2.06	1.38	1.52	1.56	1.45	1.41
C	2.7	2.69	2.72	2.73	2.72	2.73	2.71	2,70	2.52	2.48	4.39	2.68	2.65	2.75	2.79
I	5.84	5.87	5.78	5.75	5.78	5.76	5.83	5,84	5.40	5.16	5.59	5.91	6.01	5.71	5.61
n	1.84	1.81	1.89	1.92	1.89	1.91	1.85	1.83	2.51	2.79	0.48	1.89	1.92	1.84	1.82
pi	8.83	8.83	8.84	8.85	8.84	8.84	8.83	8.83	8.77	8.74	9.04	8.83	8.82	8.84	8.85
i	6.36	6.36	6.36	6.36	6.36	6.36	6.36	6.36	5.99	5.65	6.74	6.35	6.35	6.36	6.36
m	3.16	3.15	3.18	3.19	3.18	3.19	3.17	3.16	2.95	2.9	5.13	3.13	3.1	3.22	3.27
b/Y	5.42	5.42	5.44	5.45	5.43	5.43	5.43	5.43	5.52	5.97	5.73	5.46	5.5	5.42	5.43
r	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.35	0.43	0.18	0.26	0.26	0.25	0.24

Source: Research results.

Table 16 – Real Economy and Simulation's Economy

Variable	Real economy			artificial economy			Monetary Policy			Fiscal Policy			Simultaneous shocks		
	σ_x %	σ_x/σ_y	ρ_{xy}	σ_x %	σ_x/σ_y	ρ_{xy}	σ_x %	σ_x/σ_y	ρ_{xy}	σ_x %	σ_x/σ_y	ρ_{xy}	σ_x %	σ_x/σ_y	ρ_{xy}
Y	1.42	1.00	1.00	1.48	1.00	1.00	2.60	1.00	1.00	1.32	1.00	1.00	1.49	1.00	1.00
C	1.22	0.86	0.99	0.86	0.58	0.89	3.17	1.22	0.90	1.43	1.08	0.84	2.71	1.82	0.82
I	4.37	3.07	0.99	4.82	3.25	0.94	5.99	2.30	0.24	5.10	3.86	0.80	5.81	3.90	0.42
n	2.33	1.64	0.93	0.40	0.27	-0.10	3.28	1.26	0.64	0.87	0.66	-0.57	1.87	1.26	0.36
π	28.31	19.87	-0.39	0.29	0.19	-0.56	10.82	4.16	-0.71	0.48	0.36	-0.36	8.84	5.93	-0.58
i	13.29	9.33	-0.90	0.06	0.04	0.39	8.18	3.15	-0.71	0.06	0.05	-0.12	6.36	4.27	-0.59
m	4.46	3.13	0.99	0.86	0.58	0.89	3.17	1.22	0.90	1.68	1.27	0.84	3.18	2.13	0.82
b/Y	6.10	4.28	-0.96	-	-	-	-	-	-	2.37	1.80	-0.22	5.43	3.64	0.03
r	26.39	18.53	-0.83	0.12	0.08	0.27	0.34	0.13	0.56	0.16	0.12	-0.20	0.25	0.17	0.43

Source: Research results.

5 Concluding Remarks

In this study, we developed the interaction's rule of fiscal and monetary policy in a general equilibrium RBC model with a CIA constraint. Differing from conventional model, besides the isolated analysis of monetary and fiscal policies, we consider both monetary and fiscal policy at the same time. For computation simulation, it was necessary to log-linearize the model around the steady state and then, using parameters calibrated for the steady state, we tested how well it matches Brazilian economic fluctuations for the years between 2000 and 2013.

In this work , it was performed simulations about fiscal and monetary policies in a business cycle model to replicate the dynamics of the aggregate Brazilian economy. Then, we found a negative relationship between output and inflation rate, output and price level, output and nominal interest rate, when there is flexible prices in the economy.

The analysis focuses on cyclical changes in macroeconomic variables in a model of real business cycles with the inclusion of monetary and fiscal shocks. Although monetary and fiscal policy shocks and the interaction between both policies are the main goal, it was used productivity shocks to test the model in order to compare how well the real business cycle model with CIA constraint matches Brazilian data second moments.

The real business cycle model features monetary neutrality and arguments that there should not be active stabilization policy by governments. Unlike the New Keynesian model, where the prices have rigidity, this mean that the inflation variation in a short run is null, and the real and interest rate in short run are equivalent. In our model with flexible prices, the inflation presents a variability in short run when implementable to the monetary shocks, although smaller than the real economy. For RBC models, fiscal policy affects the households supply of labor and, therefore, affects output, while, for Keynesian models, fiscal policy affects households disposable income and therefore, affects the private consumption and aggregate demand.

The model responded relatively well without rigidity prices. This is connected directly to insert CIA restriction on consumption. The objective was to simulate the behavior of macroeconomic variables due to the shocks of monetary and fiscal policies. As our goal is different from the others, we incorporate monetary policy as well as fiscal and technology shocks to investigate how macroeconomic variables respond when these are changed.

The artificial economy simulated with productivity shocks and constant growth of money provides a good fit to the variability in output and investment. The artificial economy accounts for approximately 70% of the consumption variability what is a great improvement on the basic RBC model which cannot provide a good fit for consumption volatility. This result shows that CIA models can explain most households consumption profile even in the absence of shocks in monetary policy.

After the inclusion of monetary shocks, the simulation reveals a higher volatility of nominal variables in comparison with the artificial model. In this model, correlation between inflation and nominal interest rate with output was strongly negative. This monetary model showed significant volatility of hours worked, it was closed to the real economy and much higher than the artificial economy. This represent the influence of monetary policies on the household's decision, due to how nominal interest rate and inflation affects the relative price between consumption and leisure of households. As in the model the prices have been assumed to be perfectly flexible, the inflation rises instantaneously, unlike the New Keynesian model with sticky prices. Thus, the purchasing power of the transferred monetary balances decreases, and so does consumption via CIA constraint. Therefore, this model does not present the neoclassical dichotomy; nominal variables have an impact on real variables such as consumption. The model was able to demonstrate the high procyclicality between cash balances and output, as evidenced in real economy.

The effects of the inclusion of fiscal shocks through the government spending, contributed to smooth volatility in output, investment and consumption. On the other hand, does not have a good match for the nominal variables. Moreover, it underestimates the debt standard deviation. The result of simulation showed a strong crowding out effect on private consumption and investment following an increase in government expenditure.

The inclusion of simultaneous fiscal and monetary shocks presented important finds for both, real and nominal variables. First, this model matched more than 80% of hours worked volatility. It shows that both policies together with a CIA constraint have great impact on the labor market. On the other hand, this model decreased the volatility of inflation rate if compared with the model with just monetary policy shock. This suggests that monetary policy plays the most important rule for understanding Brazilian inflation rate. Moreover, the most important result shows that monetary policy, for the Brazilian time series analyzed, is the active policy while the fiscal policy is the passive one. Therefore, for Brazilian cycles, monetary policy constraints fiscal policy throughout nominal interest rates what varies government debt. Simulation with both, monetary and fiscal policies shocks, matches more than 85% Brazilian government debt while fiscal policy shocks alone can explain just 40% of Brazilian government debt.

This study analyzed the volatility of macroeconomic variables due to different shocks of fiscal and monetary policies, isolated and simultaneously. It was concluded that the results were in agreement with the assumption made in this work, i.e., monetary policy is active and constraints fiscal policy in the short run. This work has demonstrated the importance of monetary policy in controlling government spending, and in the end, controlling level of debt. Another important issue that the work presents, is the smoothing of the volatility of the variables when the fiscal policy are incorporated in the simulations, and when this simulations are compared with the simulation of isolated monetary policy. The results suggest that coordination between monetary and fiscal policies can be beneficial in reducing volatility of macroeconomic variables. These results have important effects on normative economics. Expansionary monetary policy increases inflation and nominal interest rate what worsen government expenditure on interest payments, increasing even more public debt. This result explains why many governments have adopted contractionary fiscal policies during recessions.

Finally, although it is not the goal of this work, the methodology built in this research can be used for different questions on both, fiscal and monetary policies. First, in this study the actions of economic policies were not addressed by rules, i.e, they were treated in a discretionary manner by unanticipated shocks. In the case of monetary policy, it could be added some type of rule, for instance, the Taylor rule. Second, prices did not show any kind of rigidity, i.e, the prices were fully flexible. For future studies, it would be interesting to incorporate rules for monetary and fiscal policies, and the treatment of prices with some kind of rigidity. Other possible field of study is to consider an open economy in order to investigate the rule of net exports and exchange rate on the interaction of fiscal and monetary policies. Surely this could contribute to the subject matter in order to bring closer the Brazilian reality.

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