WALBERTI SAITH

ESSAYS ON FISCAL POLICY AND INCOME INEQUALITY

Tese apresentada à Universidade Federal de Viçosa, como parte das exigências do Programa de Pós-Graduação em Economia Aplicada, para obtenção do título de *Doctor Scientiae*.

VIÇOSA
MINAS GERAIS - BRASIL
2017

Ficha catalográfica preparada pela Biblioteca Central da Universidade Federal de Viçosa - Câmpus Viçosa

T

S158e
2017

Saith, Walberti, 1988-
Essays on fiscal policy and income inequality / Walberti Saith. – Viçosa, MG, 2017.
xi, 115 f. : il. (algumas color.) ; 29 cm.

Texto em inglês.
Inclui apêndices.
Orientador: Leonardo Bornacki de Mattos.
Tese (doutorado) - Universidade Federal de Viçosa.
Referências bibliográficas: f. 107-115.

1. Política tributária. 2. Renda - Distribuição. 3. Desenvolvimento econômico. I. Universidade Federal de Viçosa. Departamento de Economia. Programa de Pós-Graduação em Economia Aplicada. II. Título.

CDD 22. ed. 336.3

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APROVADA: 21 de novembro de 2017.

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To my mother and father.

ACKNOWLEDGMENTS

First of all, I would like to thank God, without Him I would not be here. His presence was essential every step of the way. My family who spared no effort so that I could accomplish this step more in my academic career.

I would also like to thanks to professor Dr. Joanna Giorgios Alexopoulos, for presenting me the topics that I explore in this work, without which this work would not be possible, I appreciate your patience and suggestions along all this work.

I also express my gratitude to professor Dr. Leonardo Bornacki de Mattos, for his patience and assistance in the construction of this work.

A special thanks must also be made to professor Ian Michael Trotter, yours suggestions and help were essential in the construction of this work.

To my colleagues in the department of economy from the federal university of Rondônia - UNIR for releasing me which made the doctorate possible.

A very special and important acknowledgments has to made to my girlfriend Sabrina de Matos Carlos for handle my om times bad mood and my anxiety and for help made reading, suggestions and many other collaborations to this work.

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ABSTRACT

SAITH, Walberti, D.Sc., Universidade Federal de Viçosa, November, 2017. **Essays on Fiscal Policy and Income Inequality**. Adviser: Leonardo Bornacki de Mattos.

One of the main problems of economic growth in developing economies is income inequality. Thus many studies in macroeconomic theory have attempted to determine what are the main ways in which inequality can be reduced. Redistributive fiscal policy has been considered an important way to reduce inequality and increase economic growth at same time. Considering the relationship between these variables, this study seeks to clarify how fiscal policy affects income inequality and economic growth. To perform such analysis, we used three different approaches. First we estimate the impacts of fiscal policy on income inequality and economic growth among Brazilian states using a set of panel data models. The analysis covers the years from 1996 to 2011, comprising 16 years for 26 of the 27 Brazilian states. We estimated an individual equation to explain economic growth and two individual equations for income inequality, each with a different set of explanatory variables. Based on panel data models, we present evidence that the relationship between Tax Burden and economic growth and income inequality is not linear. We show that when the Tax Burden corresponds to 23% of GDP the economic growth is maximum and when the Tax Burden corresponds to 19% of GDP the inequality is minimal. Second, we construct a model and analyze the effects of a fiscal policy of income redistribution for Brazilian economy. Specifically, we try to show the effects of an income transfer for the poorest part of the population. We build a dynamic stochastic model calibrated for Brazil. The results show the optimal taxes on capital income and labor income in opposite way in both shocks (government spending and productivity). The composition of the government budget changes according to the favoritism towards the poor. The simulations show that the existence of income inequality changes the optimal level of taxes and the reactions to supply and demand shocks, although the fiscal policy has limits. Also, we present evidence that reducing poverty can increase output, eliminating the necessity of transfers and reducing considerably the fluctuations of taxes. Third, we propose a model that is a version of a competitive equilibrium of the basic neoclassical growth model, which incorporate income inequality endogenously and heterogeneous agents: poor and rich, allowing us to understand this problem in a dynamic way. We use the Ramsey problem to determine the optimal sequences for the three types of flat-rate tax: capital income, labor income and consumption, in a non-stochastic economy. The analytical solution suggests that in the steady state, optimal tax on capital should always be zero, regardless of the government's favoritism towards particular agents. Also, the government should finance the transfers to the poor agent using different combinations of taxes on consumption and labor income.

RESUMO

SAITH, Walberti, D.Sc., Universidade Federal de Viçosa, novembro de 2017. **Ensaios sobre Política Fiscal e Desigualdade de Renda**. Orientador: Leonardo Bornacki de Mattos.

Um dos principais problemas do crescimento econômico nas economias em desenvolvimento é a desigualdade de renda. Assim, muitos estudos sobre a teoria macroeconômica tentaram determinar quais são os principais modos pelos quais a desigualdade pode ser reduzida. A política fiscal redistributiva tem sido considerada uma maneira importante de reduzir a desigualdade e aumentar o crescimento econômico ao mesmo tempo. Tendo em vista a relação entre essas variáveis, este estudo procura esclarecer como a política fiscal afeta a desigualdade de renda e o crescimento econômico. Para realizar tal análise, nós utilizamos três diferentes abordagens. Na primeira estimamos os impactos da política fiscal sobre a desigualdade de renda e o crescimento econômico entre os estados brasileiros utilizando um conjunto de modelos de dados em painel. A análise abrange os anos de 1996 a 2011, último ano disponível, compreendendo 16 anos para 26 dos 27 estados brasileiros. Estimamos uma equação individual para explicar o crescimento econômico e duas equações individuais para a desigualdade de renda, cada uma com um conjunto diferente de variáveis explicativas. Com base em modelos de dados em painel, apresentamos evidências de que a relação entre carga tributária, crescimento econômico e desigualdade de renda não é linear. Mostramos que quando a carga tributária corresponde a 23% do PIB, o crescimento econômico é máximo e quando a carga tributária corresponde a 19% do PIB, a desigualdade é mínima. Na segunda, construímos um modelo e analisamos os efeitos de uma política fiscal de redistribuição de renda para a economia brasileira. Especificamente, tentamos mostrar os efeitos de uma transferência de renda para a parte mais pobre da população. Criamos um modelo estocástico dinâmico com parâmetros calibrados para o Brasil. Os resultados mostram que os impostos ótimos sobre a renda do capital e a renda do trabalho se comportaram de maneiras opostas em ambos os choques (gastos governamentais e produtividade). A composição do orçamento do governo muda de acordo com seu favoritismo para os pobres. As simulações mostram que a existência de desigualdade de renda altera o nível ótimo de impostos e as reações aos choques de oferta e demanda, embora a política fiscal tenha limites. Os resultados também mostram evidências de que reduzir a pobreza pode aumentar o produto, eliminar a necessidade de transferências e reduzir, consideravelmente, as flutuações nos impostos. Na terceira abordagem, propomos um modelo que seja uma versão de um equilíbrio competitivo do modelo de crescimento neoclássico básico, que incorpora desigualdade de renda endogenamente e agentes heterogêneos: pobres e ricos, nos permitindo compreender esse problema de forma dinâmica. Utilizamos o problema de Ramsey para determinar as sequências ótimas para os três tipos de impostos distorcivos, sobre a renda do capital, sobre a renda do trabalho e sobre o consumo em uma economia não estocástica. A solução analítica encontrada sugere que, no estado estacionário, o imposto ideal sobre o capital deve sempre ser zero, independentemente do favoritismo do governo em relação a um agente em particular. Além disso, o governo deve-

ria financiar as transferências para o agente pobre usando diferentes combinações de impostos sobre consumo e renda do trabalho.

1. Introduction

There is a substantial body of literature examining the role of fiscal policy in economic theory. Thus, consensus has been that fiscal policy, whether it is tax or expenditure can be important to determinants of the level of economic growth, but can also affect income inequality, being able to reduce poverty. The study, of the effects of fiscal policy gains even more importance in developing countries such as Brazil, mainly because this has been pointed out as one of the main instruments, capable of eliminating obstacles to economic development.

In fact, the Brazilian economy has shown evidence of the intensive use of fiscal policy in the decade of 2000-2010 for income redistribution. However a more detailed research is necessary to understand how effective the economic policy of redistribution is. A better understanding of this relationship can help in identifying (adjusting) the best level of fiscal policy.

The questions that we are raising in these studies are: what is the relationship between fiscal policy, income inequality and economic growth in Brazil? How does the optimal fiscal policy respond to the business cycles in Brazil? and How to determine optimal fiscal policy through the Ramsey problem in a model that considers inequality? The hypotheses are the following: first, fiscal policy is able to reduce income inequality, and second, a lower level of income inequality is capable of raising the income level of the economy. Furthermore, it is possible that fiscal policy can affect these two variables simultaneously.

Thus the objective of this study is to analyze the impact of fiscal policy on inequality and economic growth in Brazil. Specifics objectives are: First, build a model to capture the effect of a re-distributive fiscal policy on income inequality. Second, calibration of a parameterized DSGE model for the Brazilian economy. Third, verify if different combinations of taxes can affect the growth and inequality.

In the first study we analyzed the relationship between fiscal policy and income inequality. This study contributes to the literature by presenting a more disaggregated approach to the Brazilian economy. The results presented evidence that corroborates the initial hypotheses, which postulates that fiscal policy is capable of reducing income inequality and also increasing economic growth. Considering the state level of our data and a panel data estimation, we also show evidence that there is a limit to which fiscal policy can reduce inequality and increase economic growth. After passing this limit, fiscal policy increases inequality and decreases economic growth.

The second study tried to answer the question of how the optimal fiscal policy should behave over the business cycle in Brazil? Therefore, this work intends to contribute to the literature by presenting a Dynamic Stochastic General Equilibrium - DSGE model that captures the effect of optimal fiscal policy on inequality over the businesses cycle in a model that incorporates inequality. In particular, calibrated with Brazilian parameters. DSGE models are widely used for this type of analysis because they capture the key interactions between agents and the economic system.

We show that given the shock of productivity and government spending, the reduction of poverty leads to a higher level of output and at same time diminishes the economic fluctuations during the Real Business Cycles - RBC in Brazil. Furthermore, the level of inequality is directly linked to how fiscal policy is conducted over the RBC.

The third, we try to generalize the previous findings, building a theoretical model closer to reality that addresses the relationship between fiscal policy and income inequality. We relax the assumption of the heterogeneous agent model by enabling both agents to have physical capital. The analytical results of the mathematical model show that the difference between the marginal propensity to save between the rich and the poor is what makes the income transfer policy diminish the inequality.

In general, these studies show that fiscal policy can, up to a certain level, reduce inequality and increase economic growth. Moreover the level of inequality is crucial in determining fiscal policy. Reduced income inequality is also capable of increasing economic growth.

2. Tax Burden, Income Inequality, and Economic Growth: an Analysis for Brazil.

Abstract

One of the main problems of economic growth in developing economies is income inequality. In Brazil there is a high level of inequality between regions featuring an unequal distribution of income, which makes necessary a combined analysis of fiscal policy, economic growth and income inequality at the regional level. The objective of this paper is to estimate the impacts of fiscal policy on income inequality and economic growth among Brazilian states using a set of panel data models. The analysis covers the years from 1996 to 2011, last year available comprising 16 years for 26 of the 27 Brazilian states. We estimate one individual equation for economic growth and two equations for income inequality to determine the effect of fiscal policy on these variables. Based on panel data models, we present evidence that the relationship between Tax Burden and economic growth and income inequality is not linear. We show that when the Tax Burden corresponds to 23% of GDP, the economic growth is maximum and when the Tax Burden corresponds to 19% of GDP, the inequality is minimal. By exceeding these limits the relations between fiscal policy economic growth and income inequality reverses. We also find evidence that income inequality and Population Growth have a negative impact on economic growth. The number of people without education reduces economic growth and increases income inequality. We also find evidence that economic growth can reduce income inequality.

keywords: Inequality, Fiscal Policy, Economic Growth, Income Distribution.

JEL Codes: E62, D31, O47, C23.

2.1 Introduction

One of the main problems of economic growth in developing economies is income inequality. In fact, many studies have shown the negative relationship between them. Thus many studies in macroeconomic theory have attempted to determine what are the main ways in which inequality can be reduced. Redistributive fiscal policy has been considered an important way to reduce inequality and increase economic growth at same time.

Many of the empirical studies of fiscal policy, income distribution and economic growth focus on the differences between developed and developing countries. These relationships are also the subject of various theoretical models in the recent literature on endogenous growth. Despite the abundance of studies there is no consensus about how this relationship occurs. As highlighted by Shin (2012), this difference can be explained by the different levels of development of each country. Thus in developed countries this relationship is positive, while in developing countries such as Brazil, this relationship is negative. In fact this result is corroborated by several empirical studies, most notably in a study by Barro (2000).

In recent years, Brazilian fiscal policy has gained prominence for its relevance to economic growth. Many economists have attributed the low performance of the Brazilian GDP (IPEADATA, 2006) in the years of 2014 (0.1%) and 2015 (-3.85%) to failures in the conduct of fiscal policy[1]. While some authors argue that fiscal policy is essential for income distribution, there are others who argue that the increasing levels of public debt and fiscal exemption led the Brazilian economy to a recession and that an austere fiscal policy it is necessary for the retake of economic growth. Regardless of the argument, we note the importance of fiscal policy, either in terms of budget expenditure or tax revenues, and its influence on economic growth and income distribution in Brazil.

An analysis at the state level for the Brazilian economy becomes even more relevant when considering the fiscal crises in some states, such as Rio de Janeiro and Rio Grande do Sul. In addition, fiscal policy can provide an important explanation as total why there is an unequal distribution of income among Brazilian states. These facts show the importance of the analysis at the state level.

For decades, income inequality in Brazil has been ranked among the worst in the world. Given the scale of the problem, many debates have been had with the aim of evaluating the determinants of inequality, ways of reducing it and its implications for society. So reducing inequality can be important because high inequality constitutes a barrier to poverty reduction. Furthermore, countries with higher and persistent initial inequality tend to grow less in the medium and long term (AGHION; CAROLI; GARCIA-PENALOSA, 1999; MUINELO-GALLO; ROCA-SAGALÉS, 2014).

Although many of the empirical studies presented in the literature analyze inequality, economic growth and fiscal policy at country level, high levels of inequality can also appear within the same country. In Brazil there is a high level of inequality between regions featuring an unequal distribution of income, which makes necessary a combined analysis of fiscal policy, economic growth and inequality at the regional level. An important apparent paradox is also

[1] A factor that was used as the basis for the impeachment of President Dilma Rousseff in 2016.

evident in the Brazilian economy characterized by high inequality and a relatively big public sector.

Most of national and international studies are based on separately estimated regressions (mostly panel data), focusing on the effects of fiscal policies on economic growth (MUINELO-GALLO; ROCA-SAGALÉS, 2014). In addition, these studies are based on the analysis at national level, showing how different variables affect the economic growth of developing and developed countries.

When analyzing inequality, fiscal policy and growth at the national level, Panizza (2002) points out that one of the major problems faced empirically with country level studies is the quality and comparability of inequality data cross-country, also corroborated by Atems (2013). Thus, studying the dynamics of inequality, economic growth and fiscal policy within the same country can provide a better understanding and estimates of the relationships.

Studies on income inequality in Brazil are concentrated mainly in the analysis of income distribution conditions, that is, showing whether the income gap between rich and poor is increasing or decreasing. Thus there is a lack of studies showing the joint effects of inequality and fiscal policy on economic growth in Brazil, especially at the state level.

This work stands out from the others in the literature for focusing on the relationship between fiscal policy, economic growth and income inequality at state level (within a country), in this case Brazil. The studies in the literature are mostly focused at the national level, but they neglect important aspects and characteristics of each political system. Brazilian states, for example, have autonomy in terms of taxation.

Thus, an aggregate (country level) analysis could neglect the distinct impacts that differing state levels of tax burdens have on economic growth and income inequality, that is, the effects of fiscal policy may differ significantly between states, resulting in different effects on economic growth and income inequality.

This study contributes to the literature by verifying whether the relationships commonly studied at the country level also occur at a more disaggregated level, in particular at state level in Brazil. In the national literature, our work stands out by presenting an approach for how the size of the state (fiscal policy) has affected economic growth and income inequality in the states, thus considering local factors that are neglected by a national analysis. We also take a methodological approach different from what is normally employed in this analysis.

The objective of this paper is to estimate the impacts of fiscal policy on inequality and economic growth among Brazilian states using panel data. This paper intends to contribute to the literature in two ways: First, with an unprecedented application to the Brazilian economy in a state level, focusing an regions of a single country also avoids data comparability issues, presented in panel data of developed and developing countries. Second, with a modeling, not yet applied, that allows us to analyze if the fiscal policy impacts economic growth and inequality, and how income inequality impacts the economic growth of the Brazilian states.

2.2 Inequality, Fiscal Policy and Economic Growth

One of the first studies to relate economic growth and inequality was the seminal study of Kuznets (1955) which showed that in a developing economy, market forces first increase and then decrease economic inequality. This implies that economic growth would have a positive initial relationship with inequality, that is, when the country's income grows inequality also increases, but when exceeding a turning point this relationship becomes negative with growth decreasing inequality[2].

In a non-direct way the first studies on economic growth, also showed a concern for understanding the relationship between economic growth and inequality. By focusing on the question of why some countries grow more than others, these theoretical models were essentially concerned with the dynamics between growth and inequality across countries. In this context, we can highlight the growth model of *Solow*, in which there is the assumption that all the countries of the world will converge to the same levels of *per capita* income. There are also others studies that contributed significantly to the theme with similar models to *Solow* such as the *beta* convergence studies of Baumol (1986) and economic growth of Barro and Sala-i-Martin (1992).

An important factor to note in subsequent studies is that the relationship between income inequality and economic growth seems to depend on the degree of development of the region under study. In his study, Barro (2000) shows that inequality in poor and underdeveloped countries decreases economic growth, but inequality in rich and developed countries increases economic growth.

From the theoretical point of view, the positive relation can be explained as pointed out by Shin (2012). The savings rate of rich people is higher than that of the poor in developed countries. A fiscal policy that redistributes income from rich to poor people reduces the savings rate of the economy as whole and thus economic growth can decrease . On the other hand, the negative relation presented in developing countries, can be explained by the credit constraint which poor face. They do not have the opportunity of investing, and extremely poor cannot even participate in productive activity. As a result, income inequality makes economic growth lower and income equality makes it higher.

Since the 1980s, several authors began to study the impacts of fiscal policy on income inequality and especially on economic growth. For the most part, these studies seek to show how the economy reacts to government spending or how taxation can affect the rate of economic growth.

[2]This theoretical framework became very popular in Brazil during the 1970s.

Neoclassical basis models show the growth as determined by external factors, most notably by population growth and technological progress. More recently the models of endogenous growth have shown that investment in human and physical capital affect the steady-state, which leads to a more important role for the government expenditure and tax in economic growth (BARRO; Sala-i-Martin, 1990; GARCIA-PENALOSA; TURNOVSKY, 2007).

Several authors, such as Aghion and Bolton (1992) and Galor and Zeira (1993) point out the role of fiscal policy as a transmission channel between inequality and growth. The evidence found by these studies show that initial inequality could slow growth, thus these two variables have a negative relationship. In unequal economies, common in developing countries such as Brazil, there are greater demands for redistribution policies, which leads these governments to have high levels of taxation, which directly affects investment decisions (Crowding Out) and also economic growth (ALESINA; PEROTTI, 1996).

Al-Shatti (2014) highlights that there is a positive relationship between the *per capita* income and fiscal policy such that the income elasticity of public expenditures is always more than one. However, other studies have shown that this relationship is not linear because some times the government expenditure in relation to GDP will decrease, thus the elasticity of GDP to government expenditure is less than one, so inelastic.

Thus inequality can affect economic growth through fiscal policy, which is precisely one of the mechanisms that we try to examine in this work. It is not clear, however, the direction in which this mechanism behaves. The most probable is that such variables are caused at the same time as they have other factors that determine them together.

2.2.1 Empirical Evidences for Brazil

In Brazil the studies on income inequality have been based mainly on the theory of income convergence. Several studies aimed to analyze the convergence of income between municipalities, states and regions. Among these we can highlight Ferreira and Diniz (1995), Ferreira (1998) and Azzoni (1997). Another line of study widely found in Brazil is related to the theory of Kuznets (1955), such as Ferreira and Lledó (1997), Bagolin, Gabe and Ribeiro (2004), Taques and Mazzutti (2010), Figueiredo, Junior and Jacinto (2011).

Ferreira and Lledó (1997), for example, estimate a panel data model to verify the long-run relationship between income distribution, fiscal policy and economic growth. The results showed that fiscal policy and economic growth have a non-linear long-run relationship. In addition, the differentiated growth of Brazilian states depends not only on the fiscal effort incurred by the respective states, but also of the portion of revenue collected that is intended for public expenditure and investments. Finally, states with lower per capita income have a higher income concentration confirming the hypothesis of Kuznets for the model made by these authors.

Several studies verify the hypothesis of income convergence for the Brazilian regions as emphasized by Ribeiro and Almeida (2012). However, persistence of income inequality is still observed between regions. These studies fail to explain what are the factors that would lead the regions with the lowest levels of per capita income to grow at rates higher than the richest regions and therefore reduce inequality.

Nevertheless the majority of studies in the Brazilian literature focus more on the relationship between economic growth and inequality, trying to find evidence for/on the Kuznets curve for the Brazilian economy, relegating the role of fiscal policy to second place. In addition, it uses a period (1970 to 1980) in which the fiscal policy of the states was not as independent as it is today. Our study seeks to show that fiscal policy can impact income inequality and that inequality can affect the economic growth of the states in the post-fiscal consolidation period.

One of the most commonly used ways to measure income inequality is through the Gini coefficient[3]. The Gini coefficient is formed in an aggregate measure and can range from 0 (perfect equality) and 1 (perfect inequality). According Todaro and Smith (2012), the Gini coefficient for countries with highly unequal income distribution, is usually between 0.50 and 0.70, while for countries with relatively equal distributions it is in the range of 0.20 to 0.35. Another measure of inequality used is the ratio between the 20% rich of the population and the 20% poor, such a measure can help us to understand the dynamics of income distribution between the extremes of the population.

Figure 2.1[4] shows the evolution of the Brazilian Gini coefficient from 1996 to 2011. We also present the evolution of real GDP growth, tax burden as a proportion of GDP and the ratio between 20% rich of the population and the 20% poor, for the same period. In general, it is observed that income inequality has been decreasing as can be observed by the trend line (blue). Despite this, it is observed that the Gini coefficients are still a considerable problem, showing that inequality is a major social and economic problem in Brazil. Although in recent years it has been observed that there is no significant variation of these coefficients which suggests that they are close to their steady state.

As we can see in 1996, Brazil has a higher level of inequality and a lower tax burden. In the early 1990s, Brazil still was suffering the economic consequences of the so-called lost decade (1980). The most striking factors being the high inflation rates, this made the inequality reach higher levels, even with a small tax burden. This happens because the inflation made the poorer labor suffer a great loss of purchasing power, more than rich part of population, which increase inequality (HOFFMANN, 1995).

The government also suffers from the inflation process, despite the seigniorage tax. Such difficulties can be observed by the considerable increase in public debt. Due to these difficulties,

[3]In addition to the Gini coefficient, the most used measures of inequality are the two measures proposed by Theil, known as the first index of Theil, the Theil-T; and the second index of Theil, or Theil-L.

[4]All variables are on the same scale, ranging from 0 to 1.

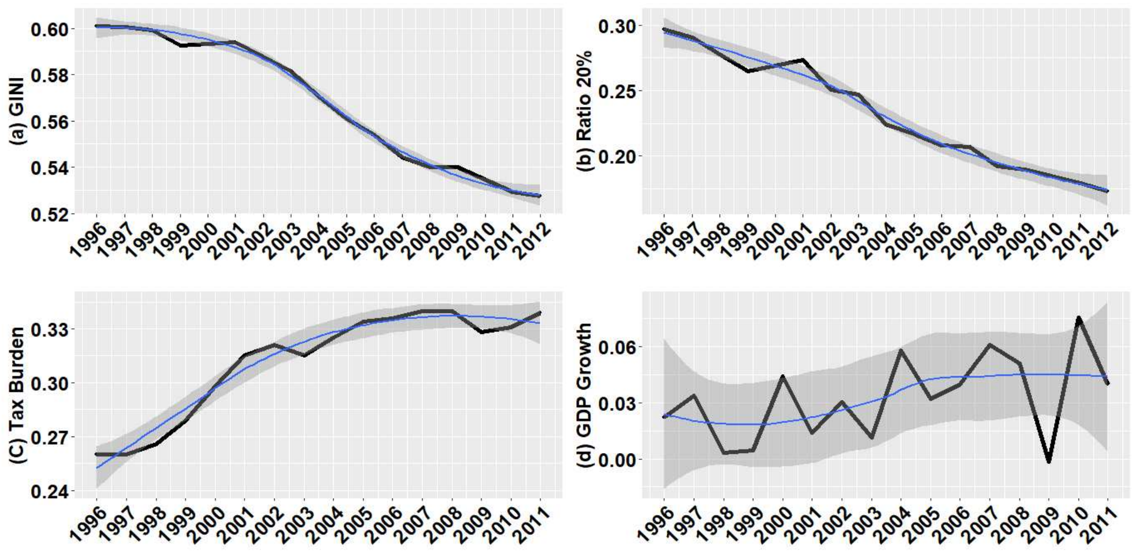

Figure 2.1: Evolution from 1996 to 2011 of Income Inequality, Tax Burden, Ratio 20% and GDP Growth of Brazil.
Source: Own' Elaboration.

the government has almost no fiscal policy of income redistribution. The combination of these factors makes fiscal policy have little impact on inequality in the first half of the 1990s. The low relevance of fiscal policy is also reflected in the Brazilian economic growth rates during this period. Due to the stability provided by the real plan (1994), it was possible to implement a fiscal policy of redistribution of income and an amplification of the effect of fiscal policy on income inequality and economic growth.

When we evaluate the Gini coefficient in state terms it is noted, besides the high income inequality, the disparities between the Brazilian states as we can see in Figure 2.2. Almost all states showed a decrease in the Gini coefficient between the years considered. Only Roraima (0.073) showed an increase in the coefficient between 1996 and 2011. The states that showed the greatest decrease in inequality are Tocantins (-0.115), Alagoas (-0.108) and Paraná (-0.106), respectively. The state of Amapá is the only one that does not present a change in the index of inequality. This result reflects a decrease in income inequality in Brazil during the period analyzed. This stylized fact has already been shown by other studies and indicates that the dynamics of income inequality in Brazil is given largely by the increase of the middle class.

The descriptive analysis of this indicator reflects, in general, a decrease in income inequality that contributes to reducing poverty and improving the living conditions of the poorest part of the population. Despite this decrease, inequality in the country remains extremely high. According to Barros, Foguel and Ulyssea (2006) and Medeiros, Souza and Castro (2015) some of the determinants of this decline would be changes in the demographic characteristics of the families such as an increase in the proportion of adults and homogenization within the same income groups, income transfers, greater access to the labor market and greater access to education, *etc*.

According to Medeiros, Souza and Castro (2015), the income inequality in Brazil, besides being high, is also stable. 1% of the adult population receives more than a quarter of all income in the country, and the 5% richest receive almost half of all income. Considering the period between 2006 and 2012, the authors emphasize that these levels of concentration remain practically the same. Another important point highlighted by this study is that when the incomes at the top of the distribution are adjusted taking into account the tax data, the reduction of inequality ceases to exist.

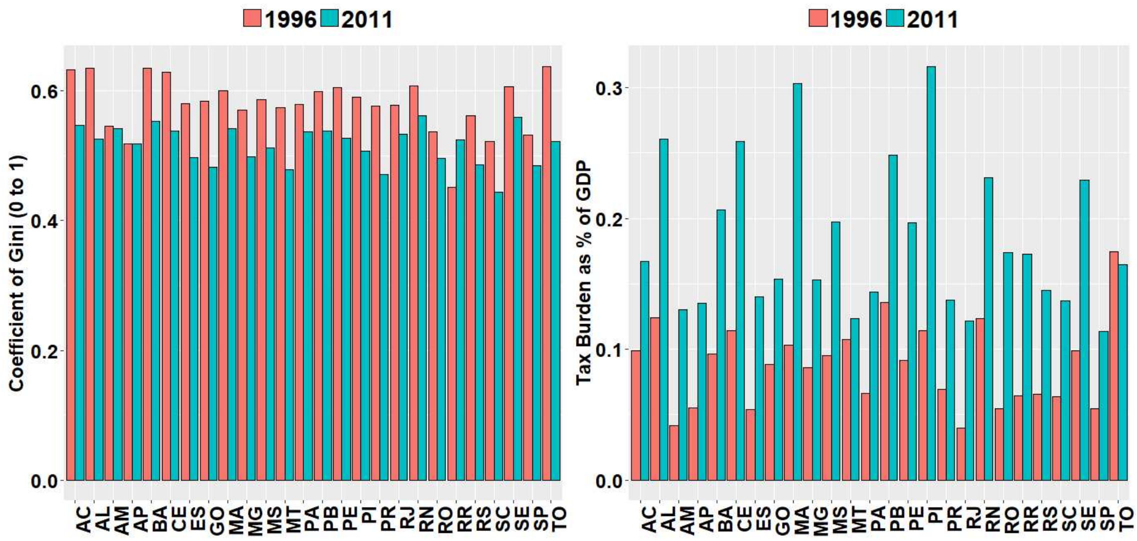

Figure 2.2: Comparison of the Gini coefficient and tax burden between 1996 and 2011 across Brazilian states.
Source: Own' Elaboration.

The evolution of the tax burden may be an indication that tax collection, fees and government contributions, and has shown stable behavior between 33% and 34% of GDP since 2005, as we can see in Figure 2.1 and in Figure 2.2 for each state. An important exception can be observed in the years 2009 and 2010. The decrease in the tax burden in these two years is a consequence of the subprime international financial crisis. We should also consider the policies of tax exemptions implemented by the federal government which culminated in the reduction of the tax burden to less than 33% of GDP.

Considering the entire period available for analysis (1996 to 2011), the increase in the tax burden occurred more intensively in two periods. The first one between 1997 and 2002, due to an intense fiscal adjustment due to the increase of tax collection to deal with the effects of the international crises of this period and enable the transition from the fixed exchange rate system to the inflation targeting system, with floating exchange rates and fiscal targets, which remains to this day.

In numbers, the tax burden increased from 26.1% of GDP in 1996 to 32.2% of GDP in 2002, an increase of 6.1 percentage points (pp) of GDP in six years. The second one is during the period from 2004 to 2005 when the government adopted a new fiscal adjustment to combat the effects of the strong exchange rate depreciation of 2002 and restore fiscal balance. During this period, the tax burden increased by 2.2 pp. of GDP in only two years, i.e. from 31.4% of GDP in 2003 to 33.6% of GDP in 2005.

Following the growth of the tax burden, Brazil also presented GDP growth during the period analyzed, although not as intense as the firsts of our series. Thus, Figure 2.1 shows that during the analyzed period the Brazilian economy was characterized by a decrease in income inequality, economic growth and an intense increase in the tax burden.

Another dimension of the dynamics of the Brazilian economy that we have to consider, given its importance, is the relationship between initial income concentration and economic growth. This is because the initial income distribution and the initial fiscal policy are fundamental in determining the path of economic growth. Thus, such relations were presented to the Brazilian states. The results are shown in Figure 2.3, where the growth rate is considered for the period from 1996 to 2011 and the Gini coefficient and tax burden of the year 1996 (initial) for the Brazilian states.

In general, Figure 2.3 shows a slight inverse relationship between the Gini coefficient and the growth rate. It is observed that even the states with the highest growth rates have a high level of income concentration. Thus, we highlight the ambiguous effect of inequality on economic growth across states. As pointed out by Goni, Lopez and Serven (2011), some studies show that inequality can be both harmful to the growth or stimulate it, depending on the region's development level.

Alesina and Perotti (1996), for example, argue that highly unequal societies provide incentives for individuals to engage in activities such as crimes, drugs, *etc*, accentuating existing social problems and increases the inequality. Besides, it would discourage capital accumulation because of the disturbances present and uncertainties about the future. Galor and Zeira (1993) also highlight the presence of credit restrictions as harmful to economic growth. According to these authors, poorer individuals would not have the finance conditions to invest in human capital, a fact that would inhibit growth.

In addition, among the arguments of those who consider that inequality can stimulate economic growth we have the effect of the differential in the propensity to save between rich and poor. Considering the hypothesis of Kaldor (1955), where the propensity to save of the rich is higher than the poorest, there would be a tendency for the investment rate to be higher in economies where inequality is more accentuated. This fact, in a way, would provide faster growth.

Finally, Figure 2.3 also highlights the relationship between the economic growth rate in the period from 1996 to 2011 and the logarithm of tax revenue for the Brazilian states in the year of 1996.

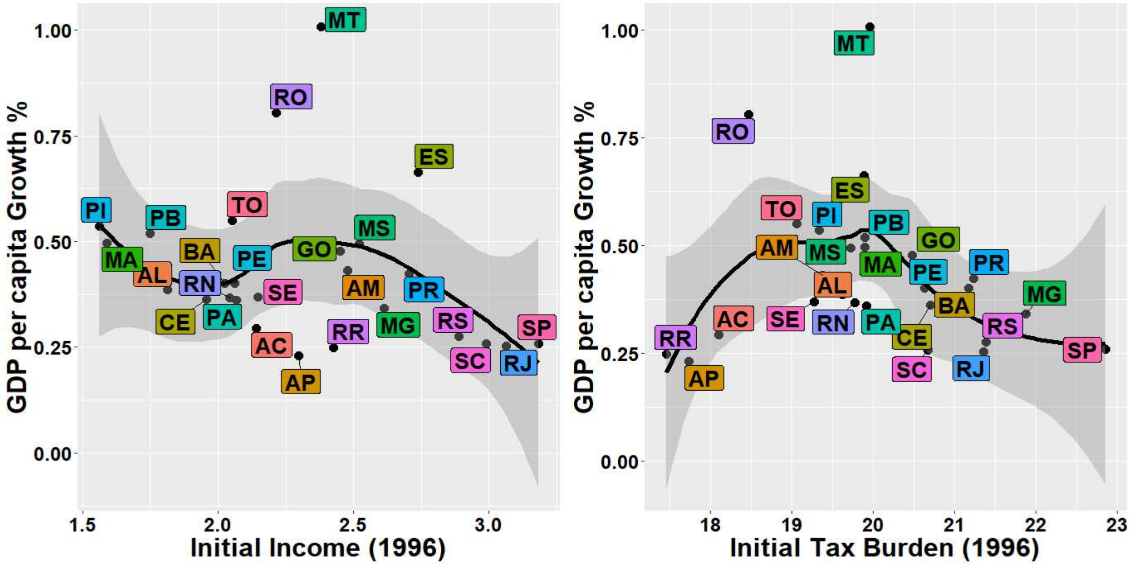

Figure 2.3: Relationship between the initial concentration of income, initial fiscal policy and economic growth for the Brazilian states.
Source: Own' Elaboration.

Figure 2.3 shows that higher state tax revenues have an ambiguous effect on economic growth. Similar results were found by Ferreira and Lledó (1997). Higher tax revenues would discourage growth by reducing the net rate of return on private investment. On the other hand, they would bring more public investments, particularly in infrastructure, making it possible to increase economic growth.

It can be inferred from Figure 2.3 that states such as São Paulo, Minas Gerais, Rio de Janeiro, Pará, Santa Catarina, among others could have grown at higher rates if their governments had opted in 1996 for lower taxation levels.

The information presented suggests, in a preliminary way, that tax and income distribution variables, could explain differences in growth rates observed among Brazilian states. However, without a more rigorous analysis in mathematical and statistical terms it is not possible to state the effects of fiscal policy on growth and inequality in Brazil. The next section presents the econometric estimates in panel data for a better understanding of the relationships presented in this section.

2.3 Data Base and Empirical Methodology

2.3.1 Data

In this section we present the data that will be used in the estimation of the models. We describe the sources of each variable and the manipulation made to create some of these variables. The selection of all variables follow the literature, therefore being corroborated by this.

Considering the fiscal consolidation of the states and financial stabilization with the implementation of the real plan, we chose to analyze the period for all variables that covers the years from 1996 to 2011, last year available, comprising 16 years for 26 of the 27 Brazilian states, totaling 416 observations per variable. We chose to exclude the Distrito Federal, because there are no municipalities in its territory and therefore is not classified as a standard Brazilian state. The Table 2.1 shows all variables used and their respective description and sources. We use three sources of data: Brazilian Institute of Geography and Statistics - IBGE, Applied Economic Institute - IPEA and National Treasury Secretariat -STN.

Table 2.1: Description of the variables used to estimate the models.

Variable	Description	Source
Economic Growth	Growth of GDP *per capita*	IBGE
Tax Burden	Budget Revenue ÷ GDP	STN
Gini	Gini Coefficient	IPEADATA
Theil	Theil's index	IPEADATA
Poor Income	Income per capita, average of the poor	IPEADATA
Rich Income	Income per capita 1% richer of the population	IPEADATA
Average Income	Average income of the population	IPEADATA
Population Growth	Rate of population Growth	IBGE
Illiterate	% of populations with no Education	IBGE

Source: Own' Elaboration.

We can consider fiscal policy as two major government economic activities, taxation and spending. Al-Shatti (2014) points out that fiscal policy influences the economy through the use of government revenue and government spending. It is carried out by the legislature of a country, state and municipalities. In Brazil, states are of great importance to fiscal policy due to decentralization in the 1980s. As shown by Guedes and Gasparini (2007), the federal constitution 1988 promoted several changes in Brazilian fiscal federalism creating, more independence of fiscal policy for states.

As a *proxy* for fiscal policy, we use government revenues in annual observations. The variable government revenues corresponds to the sum of budget revenue[5] with capital revenue[6], we also add the current transfers received by each state, all the data is provide by National Treasury Secretary (Secretaria Nacional do Tesouro - STN). From now on we will refer to this

[5]These are all operations for the maintenance and operation of public services, as well as those related to works of conservation, adaptation and maintenance of movable and immovable property, payment of staff, Purchase of consumer material, payment for services rendered by third parties, operation of schools and health centers, among others.

[6]They are those undertaken for the purpose of forming and/or acquiring real assets, covering, among other actions: planning and execution of construction, the purchase of installations, equipment, permanent materials, securities representing the capital of companies or entities of any nature, as well as debt repayments and loan concessions.

variable as Tax Burden. The variable Tax Burden is used considering the share of government revenues of Gross Domestic Product - GDP of their respective states. The series of state GDP is at constant prices and on the basis of R$ 2010 (thousand) from the Instituto Brasileiro de Geografia e Estatística - IBGE and was calculated by IPEA. In other words we divide tax revenue by GDP for each state. This procedure is commonly used in the economic growth literature such as in Barro (2000), Muinelo-Gallo and Roca-Sagales (2013).

In the economic growth model, we use the growth of GDP *per capita* for each state. The state GDP was used at constant prices (series calculated by IPEA) divided by the population. Based on this variable, we calculate the growth rate of GDP *per capita*. From now on we call this variable Economic Growth.

We also use one variable that measures the income inequality as an independent variable call Theil. The state Theil index is given by the napieriam logarithm of the ratio between the arithmetic and geometric means of the average per capita family income. If equal to zero, indicates the perfect by uniform distribution. The higher the value for Theil's index, the more unequal the distribution of income will be.

In the models of income inequality we use another measure of income inequality the coefficient of Gini. This coefficient measures the degree of inequality in the distribution of per capita household income among individuals. Their value can vary theoretically from 0, when there is no inequality (the incomes of all individuals have the same value) until 1, when inequality is maximum (one individual holds all income of the state and the income of all other individuals is zero).

The series of Gini coefficients for each state was calculated by IPEADATA from the responses to the National Survey by Household Sample (PNAD/IBGE). In the first model of inequality we also use the average household income per capita in reais (R$) of the population with real values at current prices in the last edition of the survey, updated according to the deflator for income of PNAD presented by IPEADATA. We refer to this variable as Average Income.

Despite the recent decline (see Figure 2.1), Brazil's income inequality remains extremely high. Hoffmann (1995) points out the appropriate income for the richest 1% of the population is equal to the income appropriated by the poorest 50%. In addition, the richest 10% earned more than 40% of the income, while the 40% poorer households account for less than 10% of total income. We tested these dynamics by estimating a model of inequality with variables that represent these income differences.

The variable Poor Income is the ratio between the sum of per capita income of all individuals considered poor and the total number of these individuals. The line of poverty considered here is double the extreme poverty line [7], the poverty line and different values are estimated

[7] An estimate of the value of a food basket with the minimum calories needed to adequately supply a person, based on recommendations from Food and Agriculture Organization - FAO and World Health Organization - WHO.

for 24 regions (defined by IPEA) of the country. The Rich Income - represents the average per capita household income of the individuals belonging to the richest (in terms of income) hundredth of the population. These two variables are based on the National Household Sample Survey and was calculated by IPEADATA.

To measure the impact of education on economic growth and income inequality, we use the variable Illiterate. This variable represent the percentage (%) of persons over 15 years of age who are illiterate. We collect this data from the IPEADATA. This variable is used as a *proxy* for the inverse of human capital. The reason for using this variable is to capture the effect of increased education on the poorest part of the population, where illiteracy is highest. So as the level of education increase, we expect that the level of education of the poor will also increase and the level of illiteracy will fall.

A very important control variable used in models of economic growth is the population growth. To construct this variable we use the number of people residing in each state, referred to by IBGE as population. We collect this variable from IBGE, the official institute that measures the Brazilian population. Using the population of each state for the years from 1995 to 2011, we were able to calculate the rate of population growth here in percentage.

2.3.2 Empirical Model

We can justify the use of a panel data model first because it is possible to describe the changes in fiscal policy, inequality and economic growth through time and across states. Second, multiple observations for each unit of the federation (N) over time (t) can provide better estimates when compared to the cross-sectional models or time series alone.

In addition to the previously mentioned advantages, Baltagi (2005) highlights other advantages of panel data estimation such as i) Increased sample size, ii) Produces more information, more variability, less collinearity, more degrees of freedom and more efficiency. Although panel data analysis has several advantages, there are some problems inherent to the time series and cross-section that the model can present, most commonly heteroskedasticity and autocorrelation.

Each individual has its own characteristics that may or may not influence the explanatory variables. So we can divide the estimates of panel data in two parts: fixed effect and random effect. When we use fixed effects we assume that something in the individual can bias or harm the explanatory power of the variables and we must therefore control this effect. The fixed effect model removes these time invariant characteristics of the explanatory variables so that the net effect of the variables can be analyzed. Another important assumption is that these individual characteristics are specific and are not correlated between individuals.

On the other hand, in the random effects model the individual-specific (in this case the states of Brazil) effect is a random variable that is uncorrelated with the explanatory variables (government spending and revenue). The reasoning is that when specifying the regression model, we fail to include relevant explanatory variables that do not change over time, and possibly others that change over time, but which have the same value for all cross-sectional units.

We can now describe the panel data model from a theoretical point of view, in order to understand the empirical estimation of the proposed theoretical model. Like Muinelo and Roca-Sagalés (2011) we estimate an individual equation to explain growth and two for inequality to determine the effect of fiscal policy on these variables. Further we subdivide the inequality model into two, one that considers only the average income of the households and other that considers the average income of the rich and the poor.

Given the empirical evidence of a positive and negative relationship between the tax burden and economic growth as shown by the different relationship presented in sections above, one possible explanation is the existence of a curved relationship where the two different types of relationships occur at different periods of time. As argued by Al-Shatti (2014), the income elasticity of fiscal policy is most of the time more than one, so there is a positive relationship between the per capita income and fiscal policy, but this relationship is not certain because there are periods when the elasticity of fiscal policy is less than one (inelastic), especially in times of recession.

So based on this evidence we can make the assumption that the relationship between fiscal policy (Tax Burden) - economic growth and fiscal policy (Tax Burden) - Income Inequality is not linear. So we adopt the hypothesis that in both cases the relation follows a parabola. We estimate all models based on a square specification for fiscal policy.

We can first describe the Pooled model, this is the simplest way to estimate a panel data[8] model where the data is just stacked and is estimate a by the standard method Ordinary Least Square - OLS. For the model of economic *Growth* we have the following equation:

$$\begin{aligned} Growth_{it} = \; & \alpha + \beta_1 \text{Theil}_{it} + \beta_{2t} \text{Tax Burden}_{it} + \beta_{3t} \text{Tax Burden}_{it}^2 \\ & + \beta_{4t} \text{Population Growth}_{it} + \beta_{5t} \text{Illiterate}_{it} + u_{it} \qquad (2.1) \\ & i = 1, \ldots, 26 \\ & t = 1996, \ldots, 2011 \end{aligned}$$

Where the variable $Growth_{it}$ is the GDP growth rates of the states, β_{it} is the parameters to be estimated for all independent variables which in this case are Theil, Tax burden, Tax burden^2, Population Growth and Illiterate; u_{it} is a random disturbance term with mean 0 and variance σ. In our case, we have a balanced data, in which all units i (26) have the same number of observations T (16). From now on we will refer to this model as a model of economic growth.

[8]For a more detailed reading it is recommended to read Baltagi (2005).

For income inequality, we estimate two types of model with two different sets of independent variables X_{it}. First we analyze the impact of Average Income, Tax Burden, Economic Growth and Illiterate on Income Inequality (GINI). Second, we analyze how the extremes of the income distribution affects income distribution in the Brazilian states using as explanatory variables the Tax Burden, Rich Income, Poor Income, Economic Growth and Illiterate. In both models we use the *Gini* coefficient as our dependent variable. The first model can be mathematically described as:

$$\begin{aligned} Gini_{it} = \quad & \alpha + \beta_1 \text{Average Income}_{it} + \beta_{2t} \text{Tax Burden}_{it} + \beta_{3t} \text{Tax Burden}_{it}^2 \\ & + \beta_{4t} \text{Economic Growth}_{it} + \beta_{5t} \text{Illiterate}_{it} + u_{it} \\ & i = 1, \ldots, 26 \\ & t = 1996, \ldots, 2011 \end{aligned} \tag{2.2}$$

Where the *Gini* coefficient is used as a measure of income inequality, u_{it} a random disturbance term with mean 0 and variance σ. For this model we also have balanced data, in which all units i (26) have the same number of observations T (16). From now on we will refer to this model as Inequality Model I.

As argued in section 2.3, the income dynamics between rich and poor can explain to a large extent the decrease in income inequality in Brazil, so it is important to include some measure that captures this effect. As a way of analyzing the dynamics between the extremes of income distribution in Brazil, we estimate a second model where X_{it} contains two different variables when compared with the first model of inequality, Poor Income and Rich Income. Thus we can described the second model mathematically as:

$$\begin{aligned} Gini_{it} = \quad & \alpha + \beta_1 \text{Poor Income}_{it} + \beta_{2t} \text{Rich Income}_{it} + \beta_{3t} \text{Tax Burden}_{it} \\ & + \beta_{4t} \text{Tax Burden}_{it}^2 + \beta_{5t} \text{Population Growth}_{it} + \beta_{6t} \text{Illiterate}_{it} + u_{it} \\ & i = 1, \ldots, 26 \\ & t = 1996, \ldots, 2011 \end{aligned} \tag{2.3}$$

As in the previous model the *Gini* coefficient is used as a measure of income inequality, u_{it} is a random disturbance term with mean 0 and variance σ, and the panel data is also strongly balanced. From now on we will refer to this model as Inequality Model II.

One of the limitations of the pooled models represented by equations (2.1), (2.2) and (2.3) is that they camouflage the heterogeneity present in the data. A very common way of controlling the heterogeneity present between states is the use of dummy variables, allowing each state to have its own intercept term so that we have:

$$y_{it} = \alpha_i + \beta X_{it} + u_{it}. \quad (2.4)$$

We notice that the subscript i in term of intercept α_i makes each state have a different time-invariant intercept $(\alpha_i + u_i)$. We are usually interested in the estimated value of β, but not in any α_i. The intercepts estimated for each state are therefore capable of capturing the intrinsic characteristics of each unit, allowing us to analyze more directly the effects of the explanatory variables on the dependent variable.

The form that we used for the estimation of the fixed effects model, and also the most commonly adopted, is to remove the fixed effect α_i by time demeaning each variable (the so called within estimator). So that the equations (2.1), (2.2) and (2.3) becomes:

$$(y_{it} - \overline{y}_i) = \beta(X_{it} - \overline{X}_i) + \varepsilon_{it} \quad (2.5)$$

In which $\overline{y}_i$ and $\overline{X}_i$ represent the sample mean values of the variables of the *ith* states. Thus for each state the mean of each variable is calculated and subtracted from the individual values of these variables. The resulting values are called corrected for the mean. After combining the resulting values, we estimated a regression by OLS. We can show that the estimators *within* produces consistent estimates of the angular coefficients while regression for stacked data may not produce. Finally, we add that the *within*, although consistent, are inefficient (they have larger variances) compared to the regression results with ordinary stacked data[9]. It is also noted that due to the subtraction of the mean we can not directly estimate the intercept term of the equation of each state.

If the dummy variables represent in fact the lack of knowledge about the true of the model why not express it by means of the error term? The random effects model does just that. A random effects model has the same mean equation that equation (2.4), but imposes the additional restriction that the individual specific effect is uncorrelated with the explanatory variables X_{it}, which can be formally represented by $E[u_i X_{it}] = 0$. From an econometric point of view, this is a more restrictive version of the fixed effects estimator, and this allows the analysis to arbitrary correlation between the effect and exogenous variables.

Each state is different and therefore the error term and the constant, which captures the individual characteristics, should not be correlated with those of the other states. If the error

[9]When we express variables as deviations from their mean values, the variation in these corrected values for the mean will be much smaller than the variation in the original values of the variables. In this case the variation in the error term u_{it} can be relatively large, leading to larger standard errors of the estimated coefficients.

terms are correlated then we can not adopt fixed effects, and this relationship will have to be modeled probably using random effects. The above characteristics are the rationale behind of the Hausman test, fundamental in choosing the model most appropriate to the data.

We faced a few problems when estimating panel data: serial correlation and heterogeneity, this is because when combine cross-sectional and time series. In econometric terms we have the presence of non-spherical disorders. In this way we use a strategy to deal with each of these problems. There are two methods widely used in the literature, first we use the robust estimators of the covariance matrix of coefficients usually provided by Wald-type tests, second we also present the estimation by the method of Parks (1967), which uses Feasible General Least Square - FGLS.

Considering that we work with a short panel ($T < N$) the problem of heterogeneity requires a special attention. There are several ways to estimate robust standard errors in **R** and use a robust covariance matrix to reduce the heterogeneity. The `sandwich` package by Zeileis (2004) can be used for reestimating the variance-covariance matrix, and the `lmtest` package by Zeileis and Hothorn (2002) for easily performing summarizing tests on the coefficients using the corrected variance estimation.

The main objective in correcting heteroskedasticity is to get at cluster robust stand errors, as we would expect errors to be correlated for a given state. For models with fixed effects we use the method of Arellano and Bond (1991) and a cluster of type that gives less weight to influential observations, with cluster errors in group. For the random effects models we use the restricted White method.

As pointed out by Croissant and Millo (2008) the FGLS estimators are empirically made by a strategy in two steps. First an OLS (pooling) model is estimated, then its residuals $\hat{u}_{it}$ are used to estimate an error covariance matrix more general than the random effects one for use in a FGLS analysis. Econometrically the covariance matrix is $\hat{V} = I_n \otimes \hat{\Omega}$ using $\hat{\Omega}$ as:

$$\hat{\Omega} = \sum_{i=1}^{n} \frac{\hat{u}_{it}\hat{u}_{it}^T}{n} \tag{2.6}$$

This type of estimation allows the error covariance structure inside every group (state) of observation to be fully unrestricted which makes the estimates robust against any type of intra-group heteroskedasticity and serial correlation (WOOLDRIDGE, 2002). This approach is specially important because it correct not only correlation, but also heteroskedasticity across panels.

Another important problem we must consider that makes estimates by OLS inconsistent is the endogeneity. We followed the solution suggested by Wooldridge (2002) and estimate a model by Instrumental Variables - IV, which according to this author provides consistent estimates even when the hypothesis of strict exogenity is not valid. If we consider W_{it} as a variable that is endogenous $Cov(W_{it}, u_{it}) \neq 0$, this variable being contained in X_{it} so we can write:

$$y_{it} = \beta X_{it} + \gamma W_{it} + u_{it} \tag{2.7}$$

When estimated by OLS β and γ are inconsistent. To use the IV method it is necessary an observable variable Z_{it} that serves as an instrument and is not contained in equation (2.7), so Z_{it} must be $Cov(Z_{it}, u_{it}) = 0$. Furthermore in a linear projection of W_{it} in all exogenous variables, must follow:

$$W_{it} = \delta X_{it} + \theta Z_{it} + \eta_{it} \tag{2.8}$$

Where the coefficient of Z_{it} must be $\theta \neq 0$, this condition can be extended in a not so strict way like $Cov(W_{it}, Z_{it}) \neq 0$, which means that the instrumental variable should be correlated with the endogenous variable. In practice we use lagged observations as the instruments variables. The IV model can not be estimated by OLS, so normally a two-stage estimation is made by General Two-Stage Least Squares - G2SLS. In particular we use the implementation of G2SLS proposed by Balestra and Varadharajan-Krishnakumar (1987).

Studies such as Arellano (2003) and Leon-Gonzalez and Montolio (2015) also use lagged observations as instrumental variables to deal with the problem of endogeneity in models of panel data. Arellano (2003) proposes a framework for the modelling of optimal instruments in the analysis of panel data in which the properties of the estimators are not fundamentally affected by relative dimensions of T and N. Leon-Gonzalez and Montolio (2015) apply this same method to verify the impact of foreign aid and macroeconomics policies have on economic growth. The results indicate that the macroeconomics policies implemented has great potential to explain GDP growth rates in developing countries, while foreign aid has no impact when interacted with good policies.

According to both studies, the Generalized Method of Moments - GMM of the standard panel requires many instruments so that sample properties are acceptable in finite or large samples when the dimension of the time series is not determined. The use of lagged instrumental variables, on the contrary, besides not being specifically for a certain dimension of T and N, allows the optimal instruments to be parameterized using a fixed number of coefficients for any T. In this way, Arellano (2003) establishes a parsimonious modelling of instruments in the context of panel data, bypassing the problem of overparameterization and providing estimators with the same robustness characteristics as the popular GMM methods.

In this work we use the same approach developed by Arellano (2003), but we use FGLS and G2SLS as an estimators, not GMM, because theses estimators allows us to deal with the problem of endogenity and heterogeneity at the same time. Furthermore, as previously mentioned, this aprrocah without compromising the validity of estimates and instruments.

The models are estimated in the software **R** Core Team using the `plm` package developed by Croissant and Millo (2008), which deals with linear models of panel data. The package `plm` provides functions to estimate a wide variety of models and to make (robust) inference. We estimate the models (2.1), (2.2) and (2.3) by pooling, fixed effect (within) and random effects, all corrected by Cluster-robust estimation, then the models are estimates by pooling-FGLS, fixed effect-FGLS and also by IV with Cluster-robust estimation, the main results are presented in the next section.

2.4 Estimation and Analysis of Empirical Models

We analyze the relationship between fiscal policy, inequality and growth separately. First, we analyze how fiscal policy affects the economic growth of the states by estimating the economic growth model. We estimate a pooling model, fixed effect model, and random effect model. Since the Hausnam (GREENE, 2003) test indicates the use of the fixed-effect model, we only report this model. Following the same procedure we only report the fixed-effect FGLS model, and also estimates by the IV model. Table 2.2 presents the results of these estimated models considering economic growth as the dependent variable.

We can observe in Table 2.2 that in the estimation of all the models the signs of the coefficient are exactly the same. The signs presented by the estimated parameters corroborate the empirical evidence presented by previous studies and are in accordance with economic growth models found in economic theory. Where as inequality (Theil) diminishes economic growth, Tax Burden first increases economic growth, but in a second moment decreases (Tax Burden^2). The Population Growth and number of people without education (Illiterate) also decrease economic growth.

It is possible to find in the literature theoretical explanations that clarify both negative and positive impacts of fiscal policy on economic growth and the empirical evidence corroborates both the conclusions. The result we found seems to confirm our initial hypothesis that fiscal policy has a non-linear relationship with economic growth, since in all models Tax Burden has a positive sign and Tax Burden^2 has a negative sing, and all statistically significant. A possible explanation for conflicting results, found in the literature is that the sign of the relationship between tax burden and economic growth (if positive or negative) depends on which part of the curve the level of taxation is on.

The results presented in Table 2.2 are in agreement with several empirical studies pre-

Table 2.2: Economic Growth Model Estimates.

	Dependent variable: Economic Growth		
	Fixed-Effect	Fixed-FGLS	IV
Theil	-0.0431 **	-0.03441***	-0.0331*
	(0.0167)	(0.00994)	(0.0191)
Tax Burden	0.2793*	0.28810***	0.6985***
	(0.1667)	(0.05057)	(0.1593)
Tax Burden^2	-0.5266	-0.62543***	-1.5149***
	(0.3992)	(0.15367)	(0.3693)
Population Growth	-0.9737***	-0.96964***	-0.9377***
	(0.1128)	(0.04944)	(0.1094)
Illiterate	-0.1023	-0.16939 ***	-0.2475***
	(0.173)	(0.04633)	(0.0879)
N	416	416	364
n	26	26	26
T	16	16	14
R^2	0.267	0.305	0.313
Hausman	$\chi^2 = 11^*$		

Source: Own' Elaboration.
Note: Standard errors in parentheses.
Indicates significance at *p<0.1; **p<0.05; ***p<0.01

sented in the literature such as Baisalbayeva (2013) and Nantob (2014). Gemmell, Kneller and Sanz (2013) find that revenue decentralization, measured by the tax burden of states, has affected economic growth in an positive way in OECD countries, a fact that authors attributed to decentralization of tax revenue. So that in OECD countries, therefore, it would appear that, *ceteris paribus*, their growth rates have been attribute by the use of increased tax revenues, in preference to financing a higher fraction of subnational expenditures with revenues at the subnational level.

Guedes and Gasparini (2007) had already drawn attention to fiscal decentralization in Brazil and its effects on the Brazilian economy, resulting in a significant increase in the state tax burden. The positive relationship between fiscal policy and economic growth is mainly supported by Keynesian macroeconomic models. We can also argue that a positive relationship is due to indirect effects of government expenditures on aggregate demand. Another channel is suggest by the endogenous growth model, in which a public good is considered as an input and can provide an increase in economic growth.

On the other, hand many studies in macroeconomic theory have indicated that an increase in the tax rate reduces the level of economic activity and the rate of economic growth, which at first can contradict the result found in this work. However, we hypothesize that in fact fiscal policy has an initially positive impact on economic growth, but that when it exceeds a maximum point this relation becomes negative. This hypothesis is empirically tested in the section 2.6.

The negative effect of fiscal policy on economic growth is due to the distortions in choices and effects of discouragement on the accumulation of factors of production such as physical and human capital. As highlighted by Nantob (2014), the relationship between fiscal policy and economic growth is not trivial, because an increase in the tax rate over the optimum reduces the growth rate. Thus, a tax policy that increases the share of the tax burden in relation to GDP means lower economic growth rates. The reason for this is that taxation creates distortions in the economy because they change the behavior of economic agents.

Inequality, as expected, has a negative impact on the economic growth of the states. This result is in line with the studies mentioned in previous sections, which show that in developing countries this relationship is negative. Early economic models already point out this relationship, such as Alesina and Rodrik (1994), Persson and Tabellini (1991), Muinelo-Gallo and Roca-Sagales (2013).

For estimating the first inequality model, we follow the same procedure as for the economic growth model. We estimate a pooling model, fixed effects model, and random effects model. Since the Hausman test indicates the use of the fixed-effects model, we only report this model. Following the same reasoning we only report the fixed-effect FGLS model, and we also estimated a model by IV. Remembering that, in this case, inequality is measured by the Gini coefficient, the results are all presented in Table 2.3.

The analysis of the relationship between fiscal policy and inequality confirms the hypothesis initially raised in this study that fiscal policy in Brazilian states is capable of reducing income inequality up to a certain degree, then an increase in Tax Burden[2] (fiscal policy) increases income inequality. The increase of Average Income seems to contribute to a lower level of inequality. The control variables in the estimations also presented the expect signals. The Economic Growth contributes to decrease income inequality, on the other had the increase of people without education (Illiterate) increase inequality across states, showing the importance of human capital to decrease income inequality, as postulated by economic theory.

In the case of Average Income the possible explanation is that the increase occurred in the lower part of the income distribution, meaning that the increase of income is more beneficial to the poor. On other hand, the increase in the number of people without education could represent the growth of the poor part of the population, which contributes to more inequality and lower economic growth.

The finding in this study show that fiscal policy can affect both economic growth and inequality across states. These results are in agreement with those found by Barro (2000), Muinelo and Roca-Sagalés (2011), Muinelo-Gallo and Roca-Sagales (2013). There was also a negative relationship between fiscal policy and inequality, such as, shown by Galor and Zeira (1993), Alesina and Perotti (1996).

Table 2.3: Estimate Model for Inequality I - Average Income - Alternative.

	Dependent variable: Gini		
	Fixed-Effect	Fixed-FGLS	IV
Average Income	-0.0468***	-0.01820**	-0.0372***
	(0.0139)	(0.00710)	(0.0143)
Tax Burden	-0.4854***	-0.32055***	-0.7416***
	(0.1809)	(0.04174)	(0.1715)
Tax Burden^2	1.3522***	0.88599***	1.8747***
	(0.3685)	(0.12100)	(0.3491)
Economic Growth	-0.0555***	-0.04215***	-0.0372*
	(0.0180)	(0.00731)	(0.0192)
Illiterate	0.5055***	0.54279***	0.5382***
	(0.0809)	(0.04594)	(0.0757)
N	416	416	364
n	26	26	26
T	16	16	14
R^2	0.497	0.732	0.486
Hausman	$\chi^2 = 40^{***}$		

Source: Own' Elaboration.
Note: Standard errors in parentheses.
Indicates significance at *p<0.1; **p<0.05; ***p<0.01

The results presented in Table 2.3 seem to confirm the economic theory that argues that in developing countries such as Brazil income inequality slows economic growth, as pointed out by Shin (2012). The empirical evidence shows that even in developed countries, this relationship is present, as indicated by the study of Davtyan (2014) in which inequality decreases economic growth even in Anglo-Saxon countries belonging to the United Kingdom.

Muinelo-Gallo and Roca-Sagales (2013) argue that due to the process of capital accumulation in an economy with imperfect asset markets, redistributing income from the rich to the poor could increase economic growth, due to the differences between the marginal productivity of investment which is higher for the poor than for the rich. So we need to test the effect of average income of the poor and rich on income inequality in Brazil.

Based on the empirical evidence, we assume that the hypothesis that the Average Income of the poor has grown at rates higher than the Average Income of the rich is true. Thus, we finally estimate the impacts of Tax Burden (fiscal policy) on income inequality and control for the rich and poor portions of the population. For this purpose, we consider the average income of the richest 1% of the population and the average income of the poorest part of the population of each state.

We use the same econometric approach as for the economic growth model and income inequality model I (see Table 2.3). We estimate a pooling model, a fixed effects model, and a random effects model. Since the Hausman test indicates the use of the fixed-effects model, we only report this model. Following the same procedure, we only report the fixed-effect FGLS model, and we also estimate a model by IV. Table 2.4 presents the main results of the estimated models for inequality with differentiates average income.

Table 2.4: Estimate Model for Inequality II - rich and poor - Alternative.

	Dependent variable: Gini		
	Fixed-Effect	Fixed-FGLS	IV
Poor Income	-0.18052 ***	-0.16887***	-0.16760***
	(0.02930)	(0.00853)	(0.04753)
Rich Income	0.05966 ***	0.06664***	0.05637***
	(0.00881)	(0.00234)	(0.00741)
Tax Burden	-0.27615	-0.20821***	-0.49586**
	(0.19637)	(0.02290)	(0.2112)
Tax Burden^2	0.82703*	0.61157***	1.23620**
	(0.44120)	(0.06785)	(0.49186)
Economic Growth	-0.04893**	-0.04258***	-0.03279**
	(0.02363)	(0.00406)	(0.0226)
Illiterate	0.52679***	0.44503***	0.56526***
	(0.11939)	(0.02939)	(0.11976)
N	416	416	364
n	26	26	26
T	16	16	14
R^2	0.621	0.799	0.606
Hausman	$\chi^2 = 43^{***}$		

Source: Own' Elaboration.
Note: Standard errors in parentheses.
Indicates significance at *p<0.1; **p<0.05; ***p<0.01

As we can see, all coefficients estimated are statistically significant and have the expected signs. The results of Tax Burden, Tax Burden^2, Economic Growth and Illiterate are the same as in the previous model, confirming the consistency of our estimations. When we analyze specifically fiscal policy (Tax Burden) it can be observed that it first has a negative effect on inequality and in a second moment any increase in fiscal policy (Tax Burden^2) also increase income inequality. In this way, a greater fiscal policy could reduce the inequality in the Brazilian states, but not in any amount. This result is expected for a developing country such as Brazil, as assumed by the theory of economic growth.

As postulated by economic theory, the average income of the rich and the poor impacts inequality in different ways. An increase in the average income of the rich indicates an increase in the concentration of income and therefore an increase in inequality. On the other hand, an increase in the average income of the poor indicates a lower concentration of income and therefore, the inequality decreases, as shown by the estimated negative coefficients.

The results are similar to Barros, Foguel and Ulyssea (2006) who attribute the recent decrease in inequality in Brazil to changes in income distribution, showing the importance of per capita income in economic growth and in the reduction of inequality, although there are other important factors in reducing inequality, such as falling inflation rates and demographic changes. Barros, Foguel and Ulyssea (2006) also argue that, in Brazil, the growth of social welfare corresponds to a pro-poor economic policy.

Economic growth increases if the benefits of growth reach proportionately more those of lower income than the other social segments. This results seems even more consistent, since Economic Growth decreases income inequality in all estimated models.

In this context we can highlight that increasing the income of the poor allows them to invest more in human capital, so that aggregate income inequality is a declining function of the rate of income distribution between rich and poor. This result seems to corroborate the idea that pro-poor growth is associated with a fall in inequality, while the anti-poor increases inequality.

The relations between fiscal policy, inequality and growth are quite complex and there are probably other factors that interact with these variables. This study presents evidence that there are a strong relationship between these variables at the state level in Brazil, and these relationship seems to follow those report in the international literature.

Many studies in economic theory have argued that differences in the level of tax burden can be important determinants of the level of output, but are unlikely to have a significant permanent effect on the economic growth rate. An important question that we should ask then is: What is the effect of fiscal policy on economic growth and inequality in the long run?

As highlighted by Muinelo and Roca-Sagalés (2011), investment in human and physical capital can affect the rate of growth in the steady state, and as a consequence there is a greater scope for fiscal policy to affect the growth trajectory in the of long-run. So the short term impact of fiscal policy can then become a permanent effect. Al-Shatti (2014) also points out that many researchers find empirical evidence of the importance of the effects of fiscal policy on economic growth in the short term, but that the tax burden can have permanent long-term effects on consumption and aggregate demand, inequality and economic growth.

The positive effect of fiscal policy on economic growth are also find for the long-run, which seems corroborates by previous studies. Nantob (2014) find empirical support for a non-linear relationship between taxation and economic growth using an average of four years for a set of African countries, demonstrating the positive effect of fiscal policy on economic growth. Using a panel of 23 OECD countries, in an average of 10 years Padovano and Galli (2001) found that a one (p.p) unit increase in the marginal tax rate results in 0.011 unit increase of GDP growth rate in average.

2.4.1 Intraregional Differences

We deepen our analysis a little more and analyze the difference in the effects of fiscal policy among Brazilian states. A large number of papers explore the possibility that the effects of fiscal policy depend on the level of income of each region. Few examples of this approach can be seen in Barro (2000) and Shin (2012). The hypothesis raised by these studies is that the level of initial income is important to how the fiscal policy will be conducted.

First, we investigate the role of the level of initial income on the average tax burden and inequality in a more disaggregated form, allowing an analysis of each state in its own context. This type of analysis is necessary because there are specific aspects of each state that we can not visualize with in the global analysis. In this way, this analysis allows us to understand the idiosyncratic effects of fiscal policy across Brazilian states.

Figure 2.4 shows the relationship between income measured by the initial GDP per capita and average tax burden for the period analyzed and the initial income and inequality represented by the Gini coefficient in the year 2011. We split each graph of Figure 2.4 into four quadrants, representing the states in four different patterns of the relationship between the variables studied here. We use the averages of each variable to divide the graph, the dashed lines in blue represent the respective average of variables.

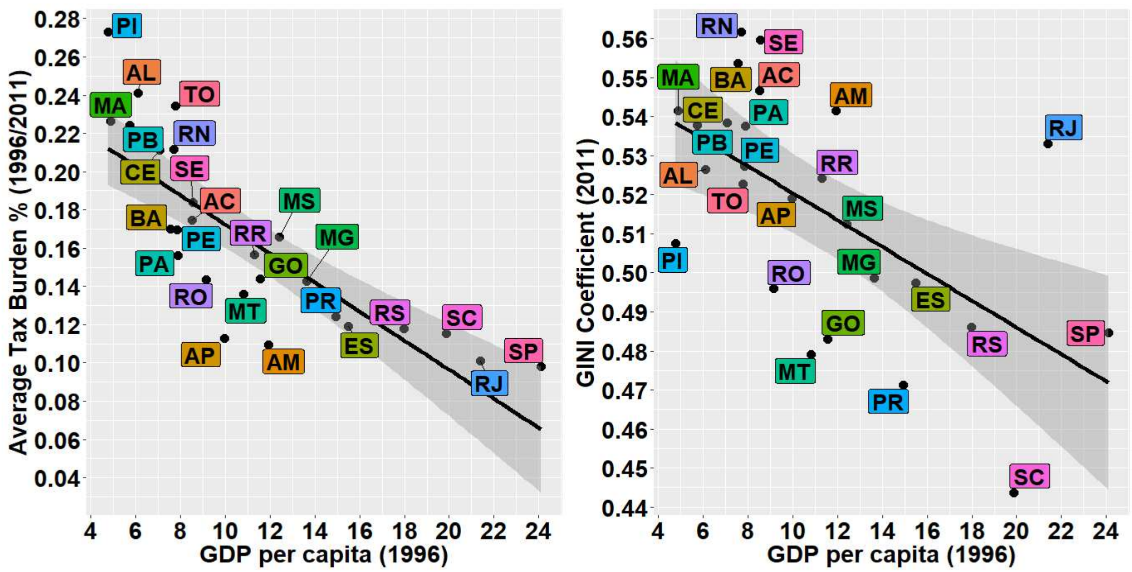

Figure 2.4: Relationship between initial income, income inequality and tax burden.
Source: Own' Elaboration.

In the left of Figure 2.4 we present the initial income of each state and their respective average tax burdens. As can be observed the states with lower level of initial income are also those with higher level of average tax burden, as suggested by the trend line (black line)[10] with negative slope. As an example of this combination we can mention the state of Piauí (PI), the

[10]We used a linear estimation to plot the trend line.

extreme point in the fourth quadrant, which presents a low level of initial per capita income leading to (as a result) a large average tax burden throughout the analyzed period, although this has not slowed down its economic growth (see Figure 2.5).

This relationship can be explained in theory by the choice of the level of fiscal policy made by the median voter. Muinelo-Gallo and Roca-Sagales (2011), for example, show that in regions with high income inequality, greater is the demand for redistribution of income through fiscal policy, this financed by taxes, which leads to such regions having higher tax burdens. As the effects of fiscal policy on the economy are ambiguous, on the one hand this reduces investment, but also contributes to greater social stability and therefore a greater accumulation of capital.

The largest group consists of those states with low initial income level and with high average tax burden, concentrated in the fourth quadrant. The feature that is less likely, are states with high levels of initial income and who have a high average tax burden. The only state to present such a circumstance is Mato Grosso (MT) in the first quadrant.

Expanding our analysis, we investigate whether data individually, that is each state, shows the relationships shown by the estimates of section 2.4. We focus primarily on the impact that fiscal policy has on the dependent variables of the models estimated in Tables 2.2, 2.3 and 2.4. Figure 2.5 show these relationships.

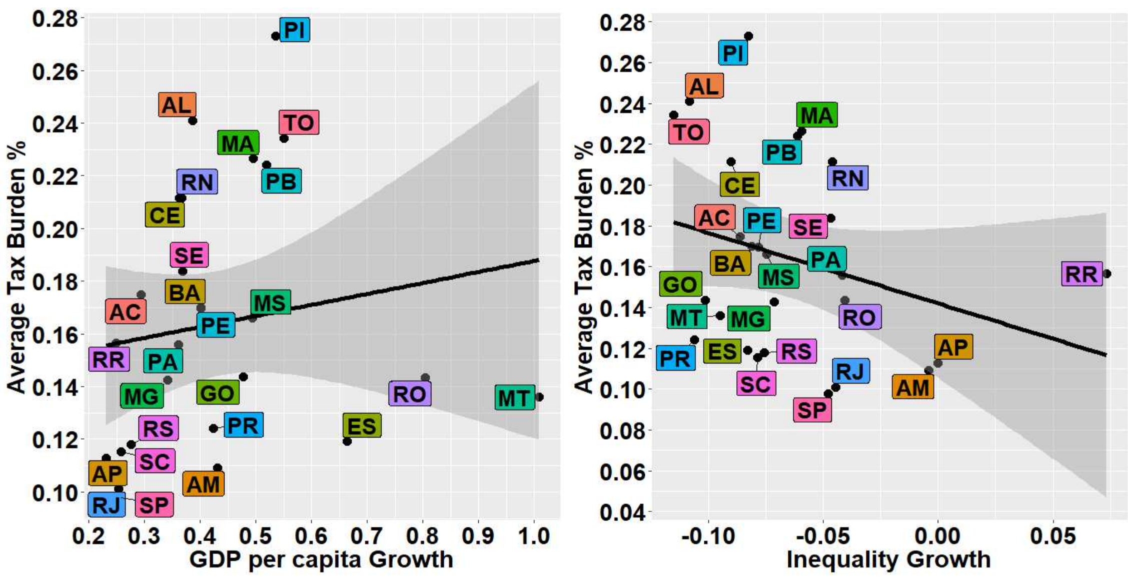

Figure 2.5: Relationship between tax burden, income inequality and economic growth.
Source: Own' Elaboration.

We can interpret the results in a similar way to previous cases. The relationship between the variables seems to confirm the results of the models estimated in the section 2.4. The relation between economic growth and tax burden is positive, as shown by the slope of the trend curve. When it comes to the relation of tax burden and income inequality, the relation is negative as previously argued and shown by the slope of the trend curve of the Figure 2.5.

Most of the states presented low economic growth and low tax burden, perhaps there is still room for the effect of fiscal policy in those regions. There are also states where economic growth has been below average and at the same time an tax burden above average, a higher growth could be achieved by these states with the reduction of fiscal policy.

With regard to the reduction of income inequality, the majority of states presented a decrease in the income inequality associated with a relatively high tax burden. The states in the second quadrant are those in which the reduction of inequality was not so accentuated, being below average, and the tax burden was below average also. Following the models of section 2.4, a further reduction of inequality could be achieved by those states with a higher tax burden.

This section presents the idea that although fiscal policy can positively impact economic growth and can also reduce income inequality, it is necessary to evaluate intrinsic aspects of each state in determining fiscal policy. In this way the fiscal policy is conditioned to the level of income and the level of income inequality of each region, these being the basic elements for the establishment of the path of fiscal policy.

2.4.2 Optimal tax burden

Based on the empirical evidence, we can raise the hypothesis of existence of an optimal fiscal policy (Tax Burden). The economic theory that supports this hypothesis can be found in authors such as Kaldor (1961). This author argues that if a country wishes to become developed it must maintain its tax burden around 25 and 30 percent of GDP, rather than the 10 and 15 percent found in many developing countries.

Using a similar idea, although a few years earlier, Lewis argued that the government of an under-developed country needs a Tax Burden of around 17 to 19 percent of GDP in order to give a not better than average standard of services, which could lead to an increase in economic growth. This assumption implies the existence of an optimal tax burden, in which the positive effect of fiscal policy on growth would be maximized.

Bird and Martinez-Vazquez (2008) explains this theory by stating that in many developing countries there is a need to spend more on public infrastructure, education, health services *etc*. These countries need to increase their tax revenue as a percentage of GDP. This effort is necessary for these countries to overcome the limitations and grow.

In this section we present an important characteristic of the estimations of section 2.4, shown in detail in Tables 2.2, 2.3 and 2.4: the non-linearity of fiscal policy. We used the parameters estimated by the instrumental variables (IV) model to determine the optimal tax burden. We used only the estimated parameters Tax Burden and Tax Burden^2 of each model, as a way of isolating the effect of fiscal policy on economic growth and income inequality.

With these equations, we are be able to estimate the maximum point for economic growth and the minimum point for income inequality. To calculate the maximum point of the growth equation (see Table 2.2) we use the expression max{0.47558(Tax Burden)-1.0129(Tax Burden)^2}. To calculate the minimum point of the inequality equations (see Table 2.3 and 2.4) we use the expressions min{0.4755(Tax Burden) -1.0129(Tax Burden)^2} and min{0.9852(Tax Burden) -0.3664(Tax Burden)^2}, respectively. We plot the the three equations (parabolas) and the result is shown in Figure 2.6.

Figure 2.6 presents the trajectory of maximization of the fiscal policy of Brazil's states, showing that at the first moment an increase in fiscal policy has a positive effect on the economic growth, but when it exceeds the global maximum point, this relationship becomes negative.

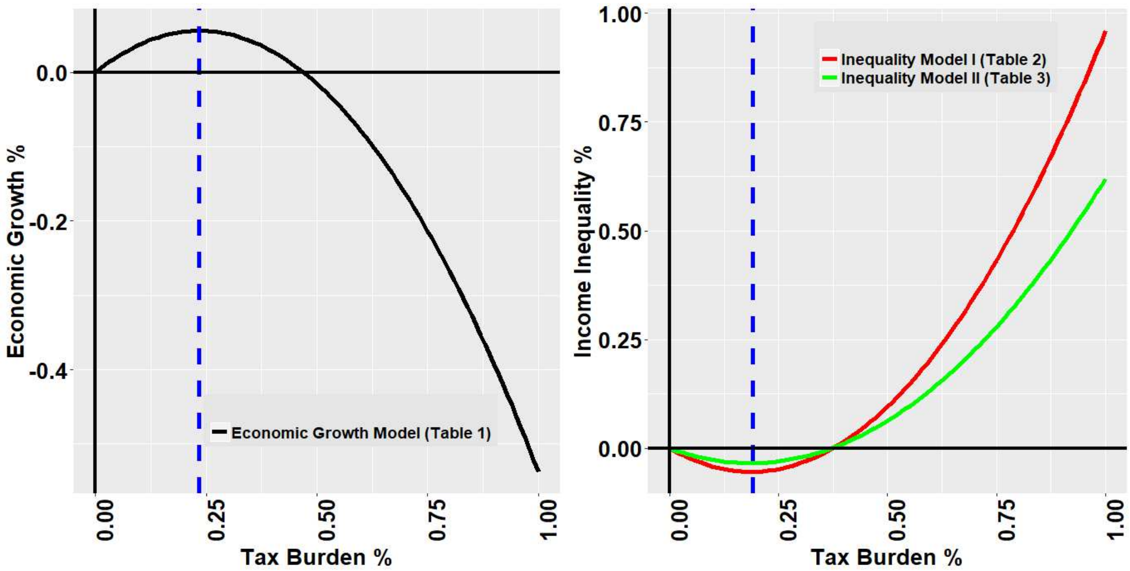

Figure 2.6: The relationship between fiscal policy and economic growth and inequality.
Source: Own' Elaboration.

The calculation indicates that fiscal policy increases economic growth to the point that the tax burden corresponds to 23.47% of GDP. The maximum point is shown by the vertical dashed blue line in left side of Figure 2.6. When the tax burden accounts for a share greater than 23.47%, then any increase in the tax burden decreases economic growth. If we calculate the average share of the tax burden of GDP during the analyzed years and among the states in our sample, we find an average of 16.39%, which explains the positive relationship found in all models analyzed in this study. In other words, on average, the Brazilian states have not yet reached the turning point in which the increase of tax burden causes diminished economic growth.

This condition becomes even clearer when we analyze the initial and final periods of our sample, in 1996 the tax burden corresponded to 8.21% of GDP, while in the year 2011 this share had increased to 16.39% of GDP. Although there has been a considerable increase in the tax burden during the years analyzed here, in fact the share doubled in size, all this increase occurred in the positive part of the curve shown in Figure 2.6, indicating the effects of fiscal policy (Tax Burden) on growth during this period was positive. However, as we previously argued, there is a limit to the positive relationship (first vertical blue line), and some states have already begun to show signs that an increase in fiscal policy has slowed economic growth.

When we analyze the relationship between fiscal policy (Tax Burden) and inequality, we realize that the relationship although at first may be negative, in the sense that an increase in fiscal policy decreases inequality, given a certain high level of tax burden, this relationship becomes positive. In other words, we can say that there is a limit to the re-distributive effects of fiscal policy. When exceeding the value of the minimum point, the increase of fiscal policy (Tax Burden) causes inequality to increase.

We present curves of the equations for inequality on the right side of Figure 2.6. Both equations have the same minimum point, 19%, as shown by the vertical dashed blue line on the right side of Figure 2.6. This result shows that beyond the turning point (19%) the relationship between tax burden and income inequality stops being negative and becomes positive. This result seems to be corroborated by the data: while in 1996 the Gini coefficient was 0.579, in the year 2011 it had dropped to 0.519, representing a considerable reduction in income inequality, accompanied by an increase in the tax burden as we had previously shown. However, there seems to be space for fiscal policy to still reduce inequality, since the tax burden has not yet reached the turning point.

We can conclude from the evidence presented above, that there is no conflicting interest between using tax burden as an instrument of fiscal policy, since such a policy can be used to maximize economic growth and at the same time to minimize inequality. This is evident when we compare the points that maximize economic growth (23%) and minimize inequality (19%), the first being bigger than the second.

As can observed in Figure 2.6, there is a region between the points of minimum income inequality and maximum economic growth in which the tax burden should operate. This region corresponds to the region between the vertical blue lines and the three curves. So we can consider that a tax burden has a lower limit (19%) and a upper limit (23%). Any tax burden outside this range would not be ideal.

However once the tax burden reaches 19% of the GDP (minimum income inequality), an increase in tax burden can still increase economic growth. Thus the states may want to raise the tax burden until 23% to maximize the growth. Any tax burden above 23% would simultaneously lead to a decrease in economic growth and an increase in income inequality and therefore would not be desired. When we look to the data the state of Tocantins (TO), presenting an average tax burden of 23.42% during the period analyzed, at the same time the Gini coefficient drop from 0.6379 in 1996 to 0.5261 in 2011, being the biggest decrease of the inequality among the Brazilian states, and the state also has the fourth biggest economic growth.

This result is also supported by the well know Laffer curve theory, which determines the rate of taxation which will raise the maximum revenue, which can be understood as the optimization of revenue collection. However, the revenue maximizing rate in the Laffer curve should not be confused with the optimal tax rate for economic growth. The economic theory uses Laffer curve to describe a fiscal policy which raises a given amount of revenue with the least distortions to the economy.

The Laffer curve is typically represented by a stylized parabola (similar to Figure 2.6) which starts at 0%, increases to a maximum value at a certain intermediate rate, and then falls again to 0 at a rate of 100% tax. A potential result of the Laffer curve is that increasing the rates beyond a certain point become unproductive, as revenue also decreases.

The empirical evidence seems to show that the result found by us that optimal level of tax burden should be something between 19% and 23% of GDP. A draft paper by Hsing (1996) showed that for the economy of United States economy between 1959 and 1991 placed the optimal tax burden between 32.67% and 35.21%. More recently a well known example is presented by Romer and Romer (2010) who analyze the consequences of changes in the level of taxation in United States and found that the maximum point corresponds to a rate 33% of taxation.

2.4.3 Kuznets Curve

In the previous sections, we pointed out the existence of a relationship between economic growth and income inequality. The theoretical explanation is provided mainly in section 2.2.1. In this section, we focus on this analyses as a way of finding empirical evidence for the theoretical point of view. We take as basis the Kuznets theory to verify the relationship between these two variables in Brazil.

This analysis is done by a model of inequality, considering as explanatory variable the level of per capita income measured by GDP *per capita* and GDP *per capita*^{2}, so that the estimated equation has a quadratic form in the inverted U-shape as postulated by Kuznets' hypothesis. We also use the Illiterate rate as explanatory variable. The results of the estimated model are presented in Table 2.5.

The regression was estimated by FGLS, since this was the method with the best fit. The estimates show that all coefficients are statistically significant and have a high explanatory capacity. This result has already been found by several other works in the national literature (see Bagolin, Gabe and Ribeiro (2004), Taques and Mazzutti (2010), Figueiredo, Junior and Jacinto (2011)). We represent the results of this regression in Figure 2.7, as a way to better understand the inverted U-shape format.

As we can observe, our result seems to corroborate the hypothesis of Kuznets for the Brazilian states. Barro (2000) had already shown that the relationship between the level of income per capita and income inequality, in fact, depends on the level of development of the

Table 2.5: Model Estimated for the Kuznets curve in Brazil.

	Dependent variable: Gini			
Coefficients	Estimate	Std. Error	z-value	$Pr(>\|z\|)$
GDP *per capita*	0.09839	0.03196	3.08	0.0021**
GDP *per capita*^{2}	-0.03891	0.00634	-6.13	0.0000***
Illiterate	0.40586	0.03630	11.18	0.0000***
N	416			
n	26			
T	16			
R^2	0.793			

Source: Own' Elaboration.
Note: Standard errors in parentheses.
Indicates significance at *p<0.1; **p<0.05; ***p<0.01

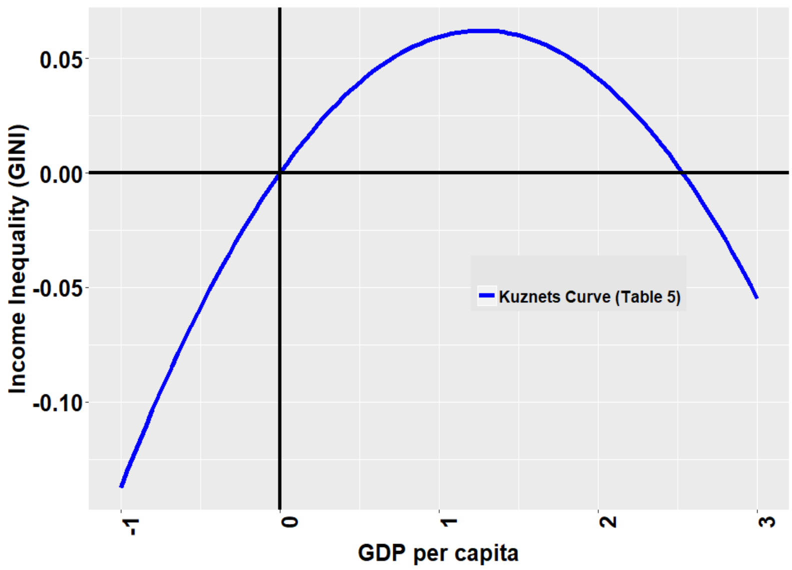

Figure 2.7: The relationship between GDP *per capita* and income inequality.
Source: Own' Elaboration.

region analyzed. This result has important implications for Brazilian economic policies of income redistribution.

More recently, Linhares *et al.* (2012) estimated this model for Brazil and showed that economic growth policies do not necessarily improve the income distribution of all Brazilian states. The non-linearity apparent in the relationship between income inequality and economic growth suggests that fiscal policies at national level do not produce similar results for all regions of the country.

2.5 Conclusions and Remarks

This study aimed to show the effects of fiscal policy (Tax Burden) on economic growth and income inequality in the Brazilian states. Based on panel data models, we present evidence that the Tax Burden has important effects on economic growth and income inequality, confirming our hypothesis. The models also show more than this: they showed the relationship between Tax Burden and Economic Growth and Income Inequality is not linear.

It has been shown that fiscal policy (Tax Burden) has a positive effect on economic growth and also can diminish income inequality. However, in a second moment, the effect of fiscal policy can be harmful for both economic growth and income inequality. We also find evidence that income inequality has a negative impact on economic growth.

The analysis of the inequality in the models one and two showed that fiscal policy is capable of reducing inequality as well as the Average Income. If we consider the average income of the rich and the average income of the poor, we observed that while an increase in the income of the rich implies a greater concentration of income, an increase in the income of the poor will result in less inequality.

The Population Growth seems to slow down the Economic Growth. The number of people without education shows the importance of human capital to economic growth and inequality, since an increase in this variable reduces economic growth and increases income inequality. In both models of inequality, the Economic Growth reduces inequality. There is also evidence for the Kuznets Curve for Brazilian states.

Finally, we analyze the possibility of an optimal fiscal policy that at the same time increases economic growth and reduces inequality. We show that when the Tax Burden corresponds to 23% of GDP the economic growth is maximum and when the Tax Burden corresponds to 19% of GDP the inequality is minimal. By exceeding these limits the relations between fiscal policy and economic growth and income inequality are reversed. Thus we can affirm that there is an interval at which fiscal policy should operate to maximize economic growth and minimize income inequality among the Brazilian states.

Appendix

2.A Statistics across states

Figure 2.A.1 presents some descriptive statistics of the variables used in our study. The boxplot is a graph based on the summary of the five numbers, consisting of Minimum value, first quartile (Q1), median (second quartile Q2), Third quartile (Q3) and Maximum value. The points represent discrepant observations.

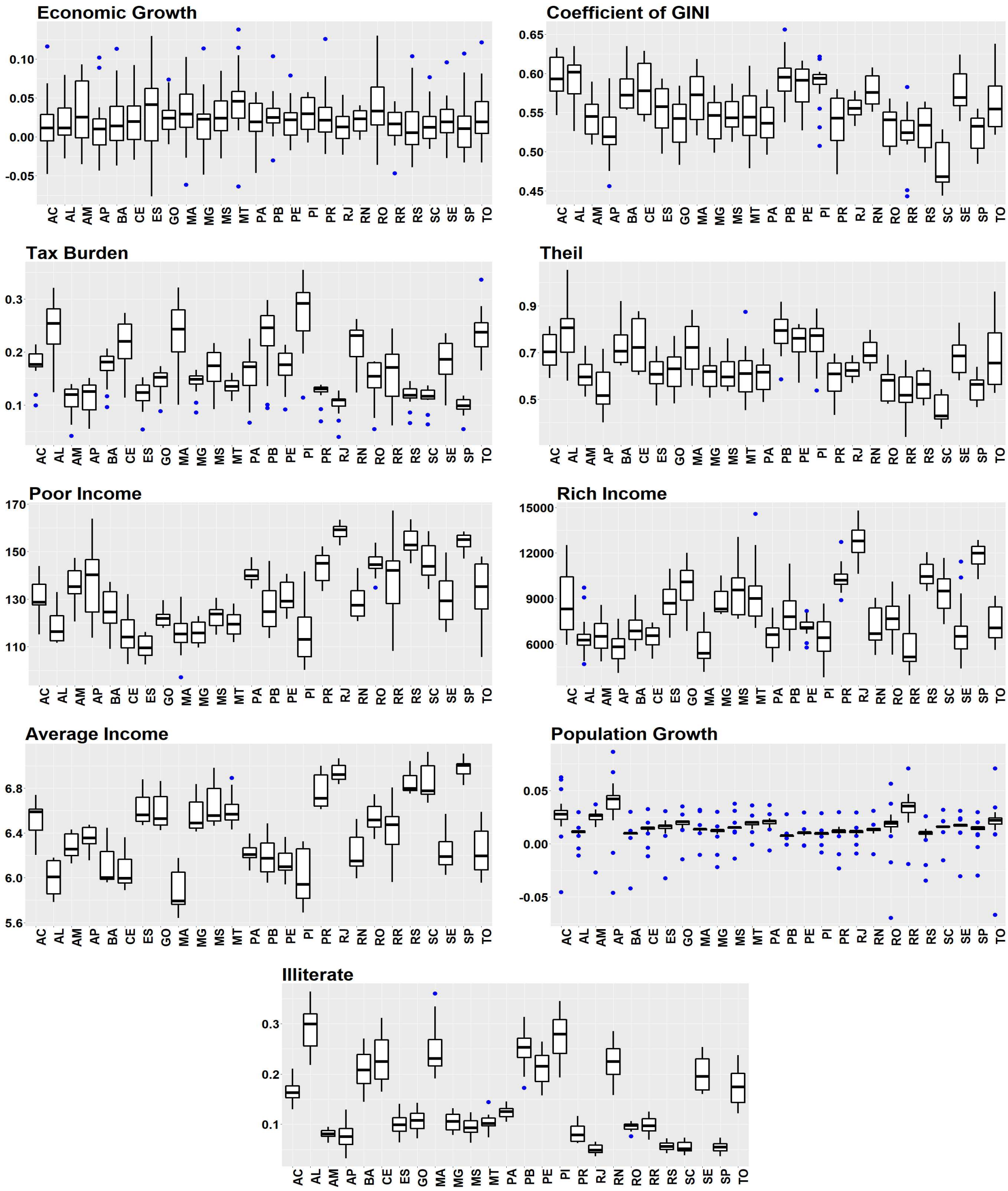

Figure 2.A.1: Descriptive statistics of the variables used for each state for the period evaluated.

The values were calculated for each state considering the sample years. Thus the median value presented in Figure 2.A.1 is the median of the years for each state and each variable, the same is true for the other measures.

2.B Others Results

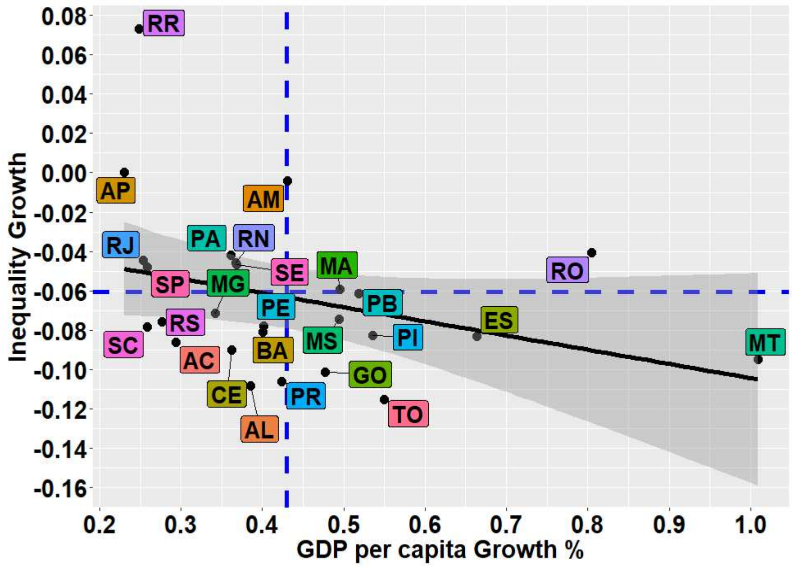

Figure 2.B.1: Relationship between income inequality and economic growth.
Source: Own' Elaboration.

2.C Names of States in Brazil

Table 2.C.1: Names and abbreviations of the Brazilian states

Abbreviations	Names
AC	Acre
AL	Alagoas
AM	Amazonas
AP	Amapá
BA	Bahia
CE	Ceará
DF	Distrito Federal
ES	Espírito Santo
GO	Goiás
MA	Maranhão
MG	Minas Gerais
MS	Mato Grosso do Sul
MT	Mato Grosso
PA	Pará
PB	Paraíba
PE	Pernambuco
PI	Piauí
PR	Paraná
RJ	Rio de Janeiro
RN	Rio Grande do Norte
RO	Rondônia
RR	Roraima
RS	Rio Grande do Sul
SC	Santa Catarina
SE	Sergipe
SP	São Paulo
TO	Tocantins

Source: Own' Elaboration based on Instituto Brasileiro de Geografia e Estatística - IBGE

3. Optimal fiscal policy and income inequality: a dynamics analysis for Brazil.

Abstract

The importance of fiscal policy lies in the fact that it simultaneously impacts both economic growth and income inequality, in addition to other macroeconomic variables. Our objective is to construct a model and analyze the effects of a fiscal policy of redistribution of income for the Brazilian economy. Specifically we try to show the effects of an income transfer for the poorest part of the population. We build a dynamic stochastic model, calibrated with Brazil parameters. After finding the solution for the steady state, we simulated two types of shock, productivity shock and government spending. The results show the optimal taxes on capital income and labor income move in opposite ways for both shocks. The wealthy agent can take advantage of both shocks, while the poor agent is only better off after a productivity shock. The composition of the government budget changes according to the favoritism to the poor. Specifically the high level of income inequality in Brazil causes a pro-poor government to perform large income transfers. The simulations show that the existence of income inequality changes the optimal level of taxes and the reactions to supply and demand shocks, although the fiscal policy has limits. A fiscal policy that favors the poor must be accompanied by a high level of tax on labor income and subsidies on capital stock to maintain the economic growth. Also we presenting evidence that reducing poverty can increase output, eliminate the necessity of transfers and considerable reduce the fluctuations on taxes.

keywords: Inequality, Fiscal Policy, Fluctuations.

JEL Codes: D63, E62, O40

3.1 Introduction

Fiscal policy is an important macroeconomic policy instrument, especially for the purposes of income redistribution. This subject has recently gained importance in the economic literature, especially considering developing countries. In Brazil, income transfer policies have been suggested as one of the sources of the recent decrease in income inequality.

The importance of fiscal policy lies in the fact that it simultaneously impacts both economic growth and income inequality, in addition to other macroeconomic variables. The way fiscal policy impacts growth and income inequality depends, to a large extent, on the degree of development of the country.

Empirical studies have shown that a fiscal policy of re-distributive taxes and government expenditure is capable of reducing inequality in developing countries, whereas it increases inequality in developed countries. For example we can cite the studies of Barro (2000), Chen and Turnovsky (2010) and Shin (2012). Thus, it is more likely that in countries like Brazil inequality impacts negatively on economic growth.

In the Brazilian literature, the majority of studies have mainly focused on the analysis of the conditions of income distribution, showing whether the income inequality is increasing or decreasing. However, these studies do not investigate the causes of inequality, nor how to reduce it. The studies on the impacts of fiscal policy on inequality are still incipient, and we can mention, for instance, the works of Fochezatto and Bagolin (2006), Costa-Junior, Sampaio and Gonçalves (2012), Carvalho and Valli (2011) and Cavalcanti and Vereda (2014).

In this context, we ask: what is the effect of an optimal fiscal policy on income inequality in Brazil? Understanding the dynamic effects of fiscal policy is important, since it highlights how policymakers can implement taxation in the presence of the shocks typically experienced by the Brazilian economy. The use of *Dynamic Stochastic General Equilibrium* (DSGE) models is therefore important to understand the dynamics of the interactions between fiscal policy and macroeconomic aggregates.

In recent years there has been an increasing number of studies that use the DSGE methodology to analyze the Brazilian economy. This increase is partly due to the increasing computational capacity available, and in part because these models have micro fundamentals and are based on the preferences of decision makers, and as such feature a natural benchmark for evaluating the welfare effects of policy changes in this case, fiscal policy changes.

The DSGE models applied to Brazil are of varied themes and assumptions, for example we can cite the work on inflation targeting and monetary policy conducted by Santana (2014), and the most recent works of Linardi (2016) and Souza (2016). There are some studies on interaction between fiscal and monetary policy such as Nunes and Portugal (2009), Lobato (2011) and Santos (2016). The credit channel of monetary policy was studied by Fonseca (2014). We can also mention studies on monetary policy by Furlani (2014) and Matni *et al.* (2016).

In the case of fiscal policy specifically, Brazilian studies which use DSGE, focus mainly on the impact of fiscal shocks on macroeconomic aggregates such as in Silva (2010) and Fantinatti (2015). A model of fiscal policy was built by Valli and Carvalho (2010), with the purpose of analyzing primary balance targets and the debt/GDP ratio. Carvalho and Valli (2011), on the other hand, using an open economy model, analyze the cyclical expenditures, public investment and distributive taxes.

The effects of fiscal policy on welfare in Brazil were studied by Mussolini and Kanczuk (2011) including tax shocks, consumption and investment. Junior and Portugal (2014) analyze fiscal policy in various forms such as taxation on consumption, salary, capital, government expenditures and their impact on the exchange rate.

Considering what has been presented previously about the Brazilian literature, this work differs by proposing a DSGE model with heterogeneous agents. Furthermore our model distinguishes itself by exploring a relationship that has been little analyzed in fiscal policy studies. We do this by analyze the interaction between optimal fiscal policy such as tax on labor and capital income, as well as income transfers, in a model where income inequality is endogenous.

Our objective is to construct a dynamic model for analyzing the effects of a fiscal policy of income redistribution for Brazilian economy. Specifically, we try to show the effects of an income transfer policy for the poorest part of the population. This study intends to contribute to the literature by constructing an optimal fiscal policy model with the presence of heterogeneous agents and inequality. We also analyze the effects of initial inequality on the long-run equilibrium, as well as the effects of short-term shocks on inequality.

3.2 Model with Heterogeneous Agents

Usually, studies on optimal fiscal policy utilize representative agent modeling, following in the tradition of the work of Ramsey (1927), which considers the problem of a representative agent choosing an optimal tax when only flat-rate taxes are implemented. This approach also forms the foundation of the works by Kydland and Prescott (1982) and Lucas and Stokey (1983)[1].

The studies of optimal fiscal policy use variants of neoclassical growth models, and the analysis often leads to a few common conclusions. Most notably that the tax rate on capital income should be zero in the steady state. In fact, this celebrated result is found in a great number of studies, although perhaps most famously a study by Chamley (1986). This result holds even in models with human capital, such as in Jones, Manuelli and Rossi (1997), who show that the tax on physical and human capital in state steady should be zero.

Another common result in the literature of public finance, is that tax rates on labor and consumption should be constant. This result is based on the fact that the fiscal policy is then able to smooth the effects of tax distortions over time. Consequently, this implies that capital income tax rates should be zero, whereas labor and consumption tax rates should be constant (LUCAS;

[1] An earlier contribution can be seen in the works of Cass and Koopmans.

STOKEY, 1983). Chari and Kehoe (1999) also highlighted that Ramsey policies imply that it is optimal to apply heavy taxation to inelastically supplied inputs.

The most common method of finding a solution to the optimal taxation problem is known as the primal approach, which basically consists of finding optimal wedges between the marginal rates of substitution and the marginal rates of transformation. However, the optimal taxes depend on the particulars of the tax system implemented or what goods are chosen to be taxed.

In an economy with taxes on both capital and labor income, in order to set the intertemporal marginal rates of substitution equal to the intertemporal marginal rates of transformation in the long run, the government must set the tax rate on capital income equal to zero. When the set of taxes includes taxes on labor and consumption, the government must keep the tax on consumption constant over time. The Ramsey allocation can thus be implemented by choosing zero tax on capital income and a constant tax on consumption.

However, if the tax system is incomplete, then the optimal capital income tax can be different from zero. This possibility is analyzed by Aiyagari (1995), who presents a model with incomplete insurance markets and borrowing constraints. The study demonstrates that, under these conditions, the optimal capital tax can be positive in the long run.

A limitation of these models is that income inequality does not appear endogenously in the model, as they only consider a single representative agent. Therefore, in order to study income transfer policies, it is necessary to build a model that incorporates a source of inequality, and this is usually done by considering heterogeneous agents.

There are several models in the macroeconomic literature that consider heterogeneous agents, which are largely based on stylized facts about income distribution. An extreme case of heterogeneity is studied by Judd (1985), who creates a model with two types of agents, workers and capitalists. The workers only supply their labor and do not own physical capital, whereas the capitalist do not work and only receive income from renting out the capital they own. The main conclusion of the study remains the same: if there is a steady state, the optimal tax on capital income should be zero.

The difference between the agents is based largely on the net present value of wealth, and can only be explained by the existence of capital market imperfections. Galor and Zeira (1993), Banerjee and Newman (1993) and Aghion (1997) found that poor people can not make productive investments in physical or human capital. This factor represents a market failure, implying that this economy does not operate optimally. This also implies that the greater the inequality, the more individuals face credit constraints.

The model with heterogeneous agents can be generalized to N number of agents, as shown by Sargent and Ljungvist (2000), who demonstrate that – even in an economy with a finite number of different types of agents – if there is a steady-state condition, we can once again deduce that the limiting capital income tax must be zero in any convergent, Pareto efficient tax system.

The model presented here is largely based on those built by Mankiw (2000) and Swarbrick (2012), which introduce a second type of agent who holds no wealth and consumes all their income each period. This behavior of the households is largely modeled according to empirical evidence. On the other hand, the wealthy agents own all the capital and are able to smooth their consumption path, as standard Ricardian agents.

3.2.1 Poor Households

The representative poor household maximizes its utility, which is increasing in consumption and leisure, by choosing the level of consumption and the number of hours worked each period:

$$\max_{\{C_t^P, H_t^P\}} \sum_{t=0}^{\infty} U_t^P(C_t^P, H_t^P) \tag{3.1}$$
$$C_t^P \geq 0, \quad H_t^P \in [0,1].$$

We assume that U_t^P is increasing, strictly concave, and three times continuously differentiable in consumption and leisure, C_t^P is the consumption and H_t are the hours worked, both in period t. The poor household is endowed with one unit of time that can be used either for leisure l_t^P or labor H_t^P, satisfying the relation $1 = l_t^P + H_t^P$. Furthermore, we assume that the utility function has the following functional form: $U_t^P(C_t^P, H_t^P) = \ln(C_t^P) + \xi \ln(1 - H_t^P)$. The poor agent receives income from supplying labor and a transfer from the government, thus the agent maximizes its inter-temporal utility (3.1) subject to the following budget constraint:

$$C_t^P = (1 - \tau_t^H)\omega_t H_t^P + T_t^P, \tag{3.2}$$

where T_t^P is government transfers to the poor agent in period t and τ^H is the tax on the labor income, ω_t.

Using the first order conditions of the maximization problem we can find the equation that shows the trade-off between labor supply and consumption in the optimum:

$$\omega_t(1 - \tau_t^H) = \xi \frac{C_t^P}{(1 - H_t^P)}. \tag{3.3}$$

The ratio between consumption and leisure depends on the after-tax wage $\omega_t(1 - \tau_t^H)$, and the relative preference between consumption and leisure ξ. We note that, as one would expect, the tax on labor income reduces the labor supply of the poor agent.

3.2.2 Wealthy Households

The representative wealthy household also maximizes its utility, which is increasing in consumption and leisure. In addition to choosing the number of hours worked and the level of consumption in each period, the wealthy household chooses how much to invest, and thereby determines the stock of capital for the next period. The wealthy household receives income from providing both labor and capital to the firm. Thus, the expected utility of the wealthy agent is given by:

$$\max_{\{C_t^W, H_t^W, I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t^W(C_t^W, H_t^W) \qquad 0 < \beta < 1 \tag{3.4}$$

$$C_t^W \geq 0, \quad K_{t+1}^W \geq 0, \quad H_t^W \in [0,1]$$

$$\text{Given} \quad K_0^W.$$

U_t^W is assumed to be increasing, strictly concave, and three times continuously differentiable in consumption and leisure, C_t^W and H_t^w are household consumption and labor provided at time t, β is the inter-temporal discount factor on utility and represents the degree of impatience and the willingness of the agent in exchanging consumption in the present for consumption in the future.

The wealthy household is also endowed with one unit of time per period that can be used for leisure l_t^W or labor H_t^W, satisfying the relation $1 = l_t^W + H_t^W$. Similar to the poor agent, the utility function of the wealthy agent has the following functional form: $U_t^W(C_t^W, H_t^W) = \ln(C_t^W) + \xi \ln(1 - H_t^W)$.

The wealthy agent is faced with the following budget constraint:

$$C_t^W + K_{t+1}^W = (1 - \tau_t^{H,W})\omega_t H_t^W + \Big[1 + (1 - \tau^K)R_t - \delta\Big] K_t^W, \tag{3.5}$$

where C_t^W and H_t^W are the household consumption and the labor provided at time t, ω_t is the wage rate, R_t is the rental rate of capital, δ is the capital depreciation rate, τ^K is the tax on capital income, such that K_t^W and K_{t+1}^W are the capital stock in periods t and $t+1$ respectively. Unlike the poor agent, the wealthy agent owns capital and determines the development of the capital stock of the economy. The capital stock evolves according to the law of motion: $K_{t+1}^W = I_t + (1 - \delta)K_t^W$.

From the first order conditions, we can determine the labor supply equation of the wealthy agent:

$$\omega_t(1 - \tau_t^H) = \xi \frac{C_t^W}{(1 - H_t^W)}. \tag{3.6}$$

As we can see, the wealthy agent's condition on the trade-off between labor supply and consumption is similar to that of the poor agent, and can be interpreted in the same way. In addition to the trade-ff between labor supply and consumption, the maximization problem for the wealthy agent also results in the following Euler equation:

$$\frac{1}{C_t^W} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}^W} \left(1 + (1 - \tau_t^K) R_{t+1} - \delta \right) \right]. \tag{3.7}$$

The Euler equation represents the intertemporal optimality condition for how the household determines the balance between consumption in period t and in period $t+1$.

3.2.3 The Representative Firm

A representative firm rents capital from the wealthy agent and employs labor from both agents to produce consumption goods. We assume a technology represented by a Cobb-Douglas production function:

$$Y_t = A^{a_t} H_t^{\alpha} K_t^{(1-\alpha)} \tag{3.8}$$

where A is a stochastic productivity factor. With all prices normalised in terms of the consumption good, the profit of the firm is given by:

$$\Pi = A^{a_t} F(K_t, H_t) - R_t K_t - \omega_t H_t, \tag{3.9}$$

where Π is the profit of the firm, ω_t is the wage rate paid by the firm to both agents, and R_t is the rental rate of capital paid to the wealthy agent. In each period, the representative firm employs the inputs from households in order to maximize profits as in equation (3.9).

We assume the firm earns no profit, such that capital and labor are remunerated at their marginal products. Thus, we have the following specification for the first order conditions of the profit maximization problem:

$$\omega_t = \alpha A^{a_t} \left(\frac{H_t}{K_t} \right)^{(1-\alpha)} \tag{3.10}$$

$$R_t = (1-\alpha) A^{a_t} \left(\frac{H_t}{K_t} \right)^{\alpha}. \tag{3.11}$$

The first order conditions imply that the firm should employ the inputs (K_t, H_t) until the marginal product is equal to its rental price (R_t, ω_t). If the production function has constant returns to scale, we get the standard result that firm profits are zero.

In order to analyse the effects of productivity or supply shocks, we can include a factor representing labor-augmented productivity a_t, and assume that technical progress evolves according to an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_{a,t}$.

3.2.4 Government

The government collects taxes on household income from both labor and capital. Part of the revenue is used in government spending and another part is transferred to the poor agents, as shown in equation (3.2). The government budget constraint can then be expressed as:

$$e^{g_t} G_t + T_t^P = \tau_t^H \omega_t H_t + \tau_t^K R_t K_t, \tag{3.12}$$

where G_t is the government spending and is determined exogenously, T_t^P is the lump-sum income transfer to the poor household. The right-hand side of equation (3.12) represents government revenues obtained through flat-rate taxation of capital and labor income. It is assumed that the government must operate with a balanced budget.

Similar to the labor-augmented productivity shock, we also test the effects of a government spending shock g_t that evolves according to an autoregressive AR(1) process: $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$.

3.3 Dynamics of fiscal policy

The single good is produced using both labor and capital. The output Y_t can be consumed by households, used by the government, or used by the wealthy agent to augment the capital stock. Given the behavior of the households, firm and government, the economy is bound by the resource constraint in every period, so we have:

$$Y_t = C_t + I_t + G_t \tag{3.13}$$

Remember that since we have two types of agents, the variables related to consumption (C_t) and labor (H_t), correspond to the sum of the portions of the respective variables held by poor and wealthy agents. The aggregates variables K_t, H_t and C_t can be summarized as follows

$$C_t = \phi C_t^P + (1-\phi) C_t^W \tag{3.14}$$
$$H_t = \phi H_t^P + (1-\phi) H_t^W \tag{3.15}$$
$$K_t = (1-\phi) K_t^W \tag{3.16}$$

where ϕ is part of the population that is poor and $(1-\phi)$ corresponds to a portion of the population considered wealthy. Taking into account the equation (3.12), we must have that $T_t = \phi T_t^P$.

Definition 3.3.1. A competitive equilibrium is a government policy $\{\tau^K, \tau^H, T^P\}_{t=0}^{\infty}$, allocations $\{H^P, H^W, C^P, C^W, K^W\}_{t=0}^{\infty}$ and prices $\{R, \omega\}_{t=0}^{\infty}$ such that the following conditions hold: I - The households maximize their utilities (3.1) and (3.4) subject to (3.2) and (3.5), respectively. II - The firms maximize profits (3.8) such that (3.11) and (3.10) hold. III - The government has a balanced budget (3.12). IV - The resource constraint of the economy is satisfied (equation 3.13).

3.3.1 Ramsey Policy

Following the public finance tradition, we implemented the Ramsey policy to show how fiscal policy should be set over the business cycle using the primal approach. In this way, we can verify how the optimal fiscal policy impacts income inequality (ϕ).

The government solves the Ramsey problem by finding allocations, prices and policies that maximize the social welfare function. The Ramsey problem can be presented in the Lagrangian form, where the fiscal policy is constrained by the households' first order conditions, the households' budget constraints and the government budget constraint.

The primal approach identifies the set of allocations that can be implemented as a competitive equilibrium with flat-rate taxes, by following two conditions. First, the resource constraint must be binding. Second, the agents' budget constraints and the firms' first order conditions are used to substitute prices and polices – this is the implementability constraint. These conditions make both constraints depend only on allocations, which implies that optimal allocations are solutions to a simple programming problem, as shown in appendix 3.C. We refer to solutions as the Ramsey allocations.

Following Chamley (1986), Sargent and Ljungvist (2000) and Swarbrick (2012) we do not include the resource constraint (equation 3.13), because as the government and household budget constraints are satisfied, then tacitly the resource constraint also holds. Therefore, we do not need to include this equation in the Lagrangian of the Ramsey problem.

Mathematically, we change the equation of the government budget constraint, such that it can be expressed in terms of the agents' allocations as:

$$\begin{aligned} G_t + T_t^P = F(H_t^P, H_t^W, K_t^W) \quad &- \quad (1-\tau_t^K) R_t (1-\phi) K_t^W \\ &- \quad (1-\tau_t^H) \left[\phi H_t^P + (1-\phi) H_t^W \right] \omega_t. \end{aligned} \tag{3.17}$$

Although the production function presented here may look different from equation (3.8), both equation are the same, considering that $H_t = H_t^P + H_t^W$. We use the allocations of the agents to solve the Ramsey problem. Thus, the fiscal policy is constrained by the households' first order conditions, the households' budget constraints and the government budget constraint:

$$\begin{aligned}
\mathscr{L} = \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ Z_t(U_t^W, U_t^P) \\
+ \quad & \Psi_t \Big[F(K_t, H_t) - (1-\tau_t^K) R_t K_t - (1-\tau_t^H)\omega_t H_t - G_t - T_t^P \Big] \\
+ \quad & \lambda_t^P \Big[(1-\tau_t^H)\omega_t H_t^P + T_t^P - C_t^P \Big] \\
+ \quad & \lambda_t^W \Big[C_t^W + K_{t+1}^W - (1-\tau_t^H)\omega_t H_t^W + (1 + R_t(1-\tau_t^K) - \delta) K_t^W \Big] \\
+ \quad & \Phi_t^P \Big[U_{H_t}^P - U_{C_t}^P (1-\tau_t^H)\omega_t \Big] \\
+ \quad & \Phi_t^W \Big[U_{H_t}^W - U_{C_t}^W (1-\tau_t^H)\omega_t \Big] \\
+ \quad & \psi_t \Big[U_{C_t}^W - \beta U_{C_{t+1}}^W \Big((1-\tau_t^K) R_t + 1 - \delta \Big) \Big] \Bigg\}. \qquad (3.18)
\end{aligned}$$

For simplicity we use U_{C_t} as marginal utility of consumption, U_{H_t} as the marginal utility of leisure and $F(K_t, H_t)$ is the production function. When solving the Ramsey problem, the utilities of both agents must be considered together, so we must sum the equations (3.1) and (3.4), and therefore:

$$\begin{aligned}
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Z_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \quad & \Big[\Theta \big[ln(C_t^P) + \xi ln(1 - H_t^P) \big] \\
& + (1-\Theta) \big[ln(C_t^W) + \xi ln(1 - H_t^W) \big] \Big]. \qquad (3.19)
\end{aligned}$$

We use the parameter Θ to indicate how fiscal policy can favor the wealthy or poor agent. Thus, if Θ is close to zero, the government conducts a fiscal policy that is highly beneficial to the wealthy agent. When Θ is close to one, the fiscal policy is highly beneficial to the poor agent, characterizing an extreme income re-distribution policy. Now we can solve the first order conditions for the Lagrangian of the Ramsey problem for $t = 0$ and $t \geq 1$.

One of the most common analytical results, is that the tax on capital income should be zero in the steady state. This result can easily be demonstrated from the first order conditions of the Lagrangian of the Ramsey problem. Taking the derivative with respect to K_{t+1} in the steady state:

$$\beta \Big[\Psi \Big((1-\phi)R - (1-\phi)(1-\tau^K)R \Big) + \lambda^W \Big(1 + (1-\tau^K)R - \delta \Big) \Big] = \lambda^W \qquad (3.20)$$

We can further simplify this equation by substituting the steady state Euler equation of the wealthy agent into the equation, so that we have:

$$\Psi\left[(1-\phi)R-(1-\phi)(1-\tau^K)R\right]=0 \tag{3.21}$$

The marginal social value of reducing government taxes Ψ is nonnegative, which leads to $\tau^K = 0$. This solution shows that if there exists a steady state for the Ramsey allocation, then the associated limiting tax rate on capital income is zero. This result shows that the proposed model is in agreement with the optimal taxation theory and that the Ramsey problem can be solved by finding the optimal allocations.

3.4 Calibration

The model will be calibrated to match Brazilian data. We will implement and run the model in *Dynare* (ADJEMIAN *et al.*, 2011) in order to demonstrate the optimal fiscal policy and analyze how government spending and transfers will be financed by taxing labor income and capital income. This analysis will help determine how fiscal policy changes impact the economic fluctuations over the business cycle. All calibrated parameters are presented in Table 3.1. The strategy used for the calibration of the model is similar to that used by Sargent and Ljungvist (2000) and Swarbrick (2012).

Table 3.1: Calibrated parameters for the Brazilian economy

Parameter	Value	Description	Source
β	0.9776	Discount Factor	BACEN
α	0.6547	Labor Share	National Accounts
δ	0.0250	Depreciation Rate	Literature
G	0.3267	Government Spending	STN
ϕ	0.8998	Proportion of population that is poor	PNAB (IBGE)
ρ_{g_t}	0.7597	Autocorrelation of government shock	STN
σ_{g_t}	0.0583	Variance of shocks government	STN
ρ_{a_t}	0.8727	Autocorrelation of techology shock	IBGE
σ_{a_t}	0.0302	Variance of techology shock	IBGE

Source: Research results.

There is a significant difference in the values of the β parameter used in national and international studies, mainly due to the great difference between the interest rate in Brazil and other countries. The macroeconomic changes caused by the creation of the *plano real* led to high interest rates in the 1990s. However, after the adoption of a floating exchange rate and with a lower inflation rate, the interest rate began to operate at a lower level in the decade 2000-2010. These factors cause some difficulties for the calibration of this parameter for the

Brazilian economy, due to the substantial economic differences between the two decades. The decade 2000-2010 is more commonly used as a sample, because it presents more economic stability. For example Smets and Wouters (2003) and Juillard *et al.* (2006) use $\beta = 0.99$, whereas Christiano, Eichenbaum and Evans (2005) use $\beta = 0.9926$.

In studies for Brazil, Ellery-Jr, Gomes and Sachsida (2002) find a value of $\beta = 0.971$, which is very similar to the values found by Kanczuk (2002) in a model adjusted to quarterly data of $\beta = 0.98$, and the same is used by Carvalho and Valli (2011). The calibration of the parameter β for the Brazilian economy shows that the agents value future consumption less than present consumption, a behavior caused by high interest rates.

The intertemporal discount factor β was calibrated using the interest rate (Selic) for the period 2000 to 2014, discounting the inflation rate for the same period, measured by the Extended National Consumer Price Index. The value found for the discount rate was 6.78%, which is in agreement with the national literature.

The capital depreciation rate was calibrated based on the literature, which uses the value of 2.5% (per quarter) as we can find in Cavalcanti and Vereda (2011), who argue that it is difficult to find in the international literature any DSGE model that incorporates capital into the production function used by firms and does not assume, from the outset, that its depreciation rate is equal to 2.5%. In fact, as an example, we can mention the studies of Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005) that use this value, and in Brazil we can highlight the study by Carvalho and Valli (2011).

The parameter that measures the share of capital $(1-\alpha)$ in the production function of the firm was calibrated by calculating the ratio between Gross Operating Surplus and GDP, obtained from the data series of the IBGE National Accounts System. In the national accounts, the Gross Operating Surplus is the residual value of the income, which comprises the remuneration of the capital, interest, profits and rents. We find the value of $\alpha = 0.65$, which seems to be largely in agreement with other studies on Brazil, such as, for instance, that by Bender-Filho (2011). Ellery-Jr, Gomes and Sachsida (2002) use a value of $(1-\alpha) = 0.49$, and Carvalho and Valli (2011) adopted an exponent of 0.30 for capital, whereas Kanczuk (2004) find that the share of capital in the production function is 0.40.

To calibrate the parameter G, we use the series of tax burden as a proportion of GDP for the period from 1947 to 2015, published by IBGE. Although it presented a very strong growth in the decade of 2010, the average for the entire period was 32.66%.

For calibrating the proportion of poor households in the economy, ϕ, we will use data on income distribution from the national household Sample Survey (PNAD) of 2015. We can use this survey to calibrate the parameter ϕ by following the procedure used by Carvalho and Valli (2011) based on a PNAD survey: divide the population, considering the percentage of population that receives less than 3 minimum wages as poor. The same procedure is used by

Castro *et al.* (2011), assuming that workers that receive up to 2.5 minimum wages in Brazil behave as rule-of-thumb consumers.

The value calibrated for the parameter ϕ was 0.89, that is, 89% of the population is considered poor, demonstrating the great income inequality present in the Brazilian economy. The value found in this study differs significantly from those used by Mankiw (2000) and Gali, Salido and Valles (2007), who adopt the value of 0.50.

In the literature the value of Θ (poor/wealthy government preference) has been used to make experiment, so usually does not have a fixed value. However the default value adopted for a neutral fiscal policy would be 0.50 as we see in the work of Swarbrick (2012), initially we also adopted this value, and then later we use varying values for this parameter to test how government favoritism alters fiscal policy.

Following the real business cycle literature, a stochastic disturbance for a labor-augmenting productivity factor is incorporated in the firm production function as mentioned in section 3.2.3. This disturbance follows an AR(1) process that was estimated[2] using Brazilian data. The estimate is showing in Table 3.2.

Table 3.2: Estimated shock of Productivity

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.000732	0.003774	0.193879	0.8469
ρ_a	0.872672	0.063281	13.79039	0.0000
R^2	0.754139			
σ_a^2	0.030193			

Source: Research results.

The value of σ^2 is set to match the standard deviation of annual HP-filtered log-GDP for Brazil in the 1947-2011 period. The estimated coefficient presented in Table 3.2 is statistically significant. The value found here is similar to that found by Bender-Filho (2011), who estimates the value of $\rho = 0.8061$ and $\sigma = 0.0369$ for Brazil in the 1980-2009 period.

Using the same procedure, we also estimate stochastic disturbance for government spending that follows an AR(1) process. We employ the same series used to calibrated the parameter G, that is, the tax burden as a proportion of GDP for the period from 1947 to 2015, as published by IBGE. We present this estimate in Table 3.3.

As we can see, the estimated coefficient is statistically significant, with values of $\rho = 0.7597$ and $\sigma^2 = 0.058$. Santos (2016) find and assume $\rho_g = 0.70$ and $\sigma^2 = 0.01762$ for the period from 2000 to 2013, which are little lower than our estimates.

[2]The application of the HP-filtered and the model estimates were made in software **R** Core Team.

Table 3.3: Estimated shock of government spending

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.000249	0.007286	0.034217	0.9728
ρ_a	0.759740	0.082694	9.187409	0.0000
R^2	0.576527			
σ_G^2	0.058284			

Source: Research results.

3.5 Numerical Simulations

After finding the analytical solution to our model, we compute the numerical solution of the problem of optimal taxation for the Brazilian economy. We perform a few simulations to increase our understanding of the relationship between the fiscal variables and the rest of the economy. The optimal tax on capital has been shown in section 3.3.1 to be equal to zero in the steady state.

3.5.1 Effects of a Productivity Shock

We start with the analysis of a productivity shock, of the form estimated in Table 3.2, by presenting the response of the optimal taxes. The calibrated model is also presented in appendix 3.F. After calculating the steady state, we calculated the impulse response of the aggregate variables following a shock in productivity, which can be considered a supply shock, since the shock is implemented in equation 3.8. We start by presenting the reactions of the optimal taxation rates (τ_t^H, τ_t^K), which are shown in Figure 3.1.

Following a productivity shock, which in itself leads to an increase in the stock of capital, the optimal capital income tax rate increases in the first few periods after the productivity shock, then after approximate 10 periods it decreases below the original rate and then slowly recovers again.

Figure 3.1 actually shows a large initial drop in the labor income tax rate, which then returns rather quickly to the original state. However, since the government is required to have a balanced budget in our model, the decrease in labor income tax can be offset by an increase on capital, which explain the opposite behavior of the two taxes (τ^K, τ^H). The results shown in Figure 3.1 present exactly this behavior. The decrease of tax on labor has a huge beneficial effect on the income of the poor, since this is their only source of income.

The consumer reactions to the productivity shock show different behavior amongst the agents, mainly because of the different endowments and different tax rates on capital and labor income, as it reflects the arguments in the utility functions of the respective agents. Figure 3.2

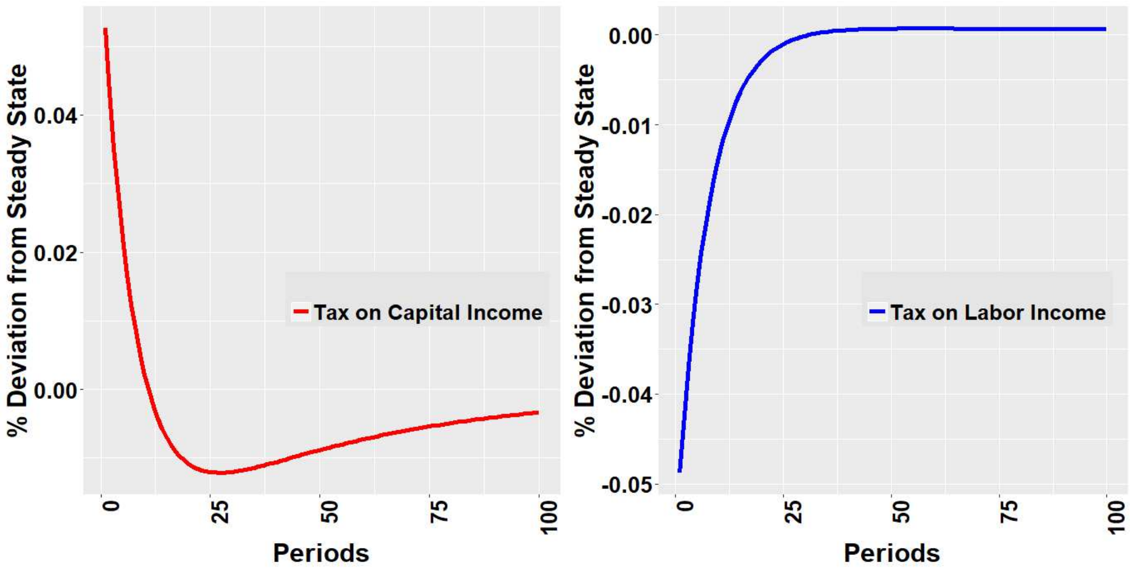

Figure 3.1: Reactions of the optimal tax rates to shocks in Productivity a_t.
Source: Own' Elaboration.

shows the reactions of the consumption of the two agents.

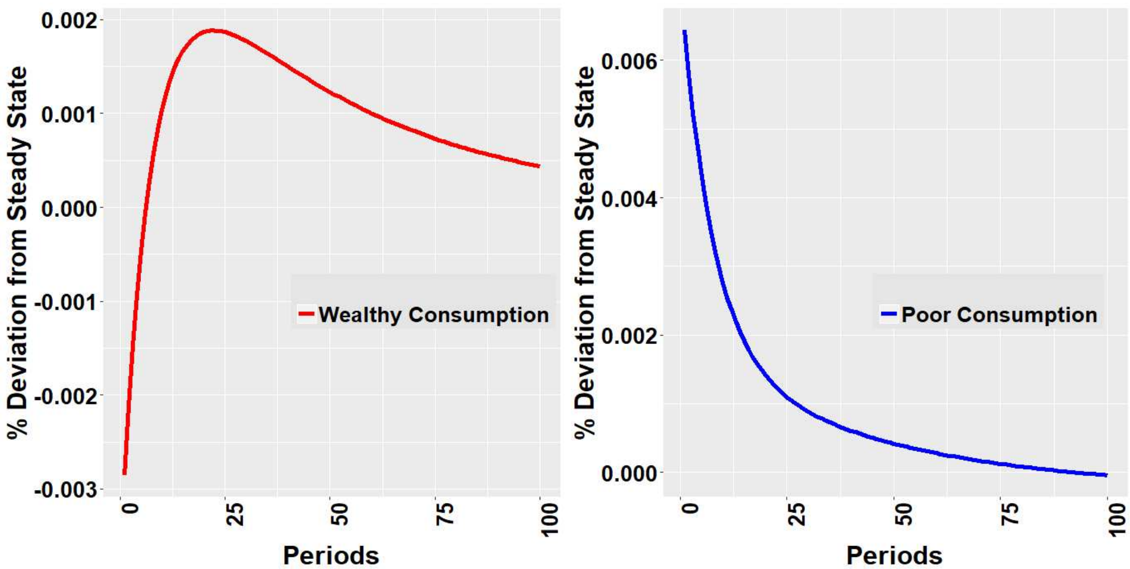

Figure 3.2: Impulse response functions of consumption following productivity shock.
Source: Own' Elaboration.

The level of consumption is closely linked to the taxes on labor and capital. As the tax on labor income falls more heavily on the poor, the consumption of this agent increases dramatically after the productivity shock, then returns slowly to normal level. Also, the income of the poor agent increases as a direct result of the productivity shock, and consume is the only thing he can do with the income. The wealthy agent takes advantage of the productivity shock to increase his stock of capital, despite the increase of tax on capital income. As a consequence of choosing to accumulate more capital the consumption decreases for a few periods, but the higher level of capital stock allows the wealthy agent to increase the consumption above normal after about five periods, and then it returns slowly to normal level.

It is worth mentioning that the fluctuations in the consumption of both agents are large in magnitude, presenting evidence that consumption has a large response to fluctuations caused by temporary technological shocks (supply shocks). In addition, the high level of inequality in Brazil modifies the model considerably and seems to change the response of consumption when compared to countries with less inequality. In countries with lower inequality ($\phi \leq 0.50$) the consumption of all individuals increases, although wealthy agents increase consumption more slowly. This behavior can be explained by the wealthy agents saving initially, in order to increase the capital stock more rapidly.

From Figures 3.1 and 3.2 we can infer that the poor agent takes advantage of higher income provide by lower level of tax on labor income to increase his consumption and increase productivity, especially because in the model we assume the productivity factor is labor-augmenting. At the same time, the wealthy agent diminishes his level of consumption and chooses to increase the stock of capital (K^W).

Figure 3.3 shows the reactions of the income transfer, the labor supply and the capital stock to a productivity shock. The income transfer shows an increase initially, then returns slowly to normal levels, showing again that given a policy of income transfer a productivity shock is beneficial to the poor. The higher level of income transfers is possible in part because the government has a bigger budget from an increase of the taxes on capital income and labor income. In the case of tax on labor income even though the tax rate decreased, the absolute amount may have increased due to the higher productivity.

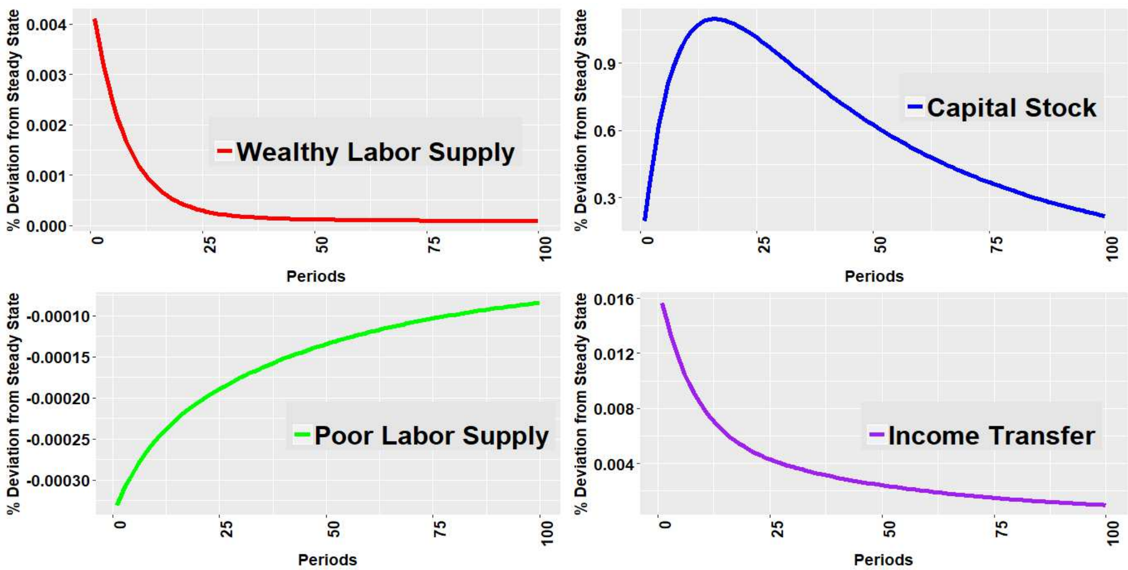

Figure 3.3: Aggregate reactions to shocks in Productivity a_t.
Source: Own' Elaboration.

As we can see, the wealthy agent takes advantage of the productivity shocks to increase capital stock, while at the same time also increasing the labor supply. After a steep initial growth in the capital stock, it decreases until it reaches the steady state. The labor supply of the poor agent starts initially at a lower level, because this agent chose consumption and this means choosing work, since he needs to work to consume. But since his (gross) income is higher, he can afford both to work less (more leisure) and to consume more

After the productivity shock the output (Y) increases, as expected. The wage rate (ω) also increases as result of the lower level of labor supply from the poor agent and more important, as a direct result of the increased productivity. The investment (I) presents a similar behavior as the output, showing in part why the this type of shock creates economic growth. Finally, the interest rate (R) grows abruptly and then slowly returns to steady state, providing the initial incentive for the wealthy agent to increase the capital stock.

From the theoretical point of view, we can argue that the substitution effect is smaller than the wealth effect. The higher level of the interest rate is big enough to compensate the increase of the tax on capital income. This effect could lead to countries with high inequality, such as Brazil, to the so-called middle income trap. We can characterize the middle income trap as the economic development situation, where a middle income country, due to few economic challenges will get stuck at that level. For more details on this subject see Kharas and Kohli (2011). This indication is also reinforced by the increase in the unemployment rate in the years 2014 to 2016.

3.5.2 Effects of a Shock in Government Spending

Now we can turn our attention to a government spending shock, which can be interpreted as a demand shock. As in the case of the technological shock, our objective is to analyze the fluctuations caused by this type of shock. We use the values in Table 3.3 to simulate the government spending. The first result presented is the optimal taxation rates, as shown in Figure 3.4.

The reactions of tax on labor income and capital income are very similar to those of the productivity shock. The results show that a supply shock and a demand shock have similar effects on the optimal fiscal policy. The government should finance the additional spending by taxing capital. Following the spending shock, the consumption responses of the two types of agents are very different, as shown in Figure 3.5.

The consumption of the poor agent falls vertiginously following the demand shock, and thereafter rises slowly towards the steady state. The consumption of the wealthy agent, however, increases initially, before dropping below the original level and then slowly returning to the steady state level. This result is quite different from the result of the production shock (ρ_{a_t}), which exhibited the opposite behavior. This may be evidence that a demand shock is more beneficial to the wealthy agent than a supply shock, which seems to benefit the consumption of the poor agent more (Figure 3.2). The main difference between the two types of shocks is that a productivity shock implies a higher income for everyone (wealthy and poor), whereas a shock

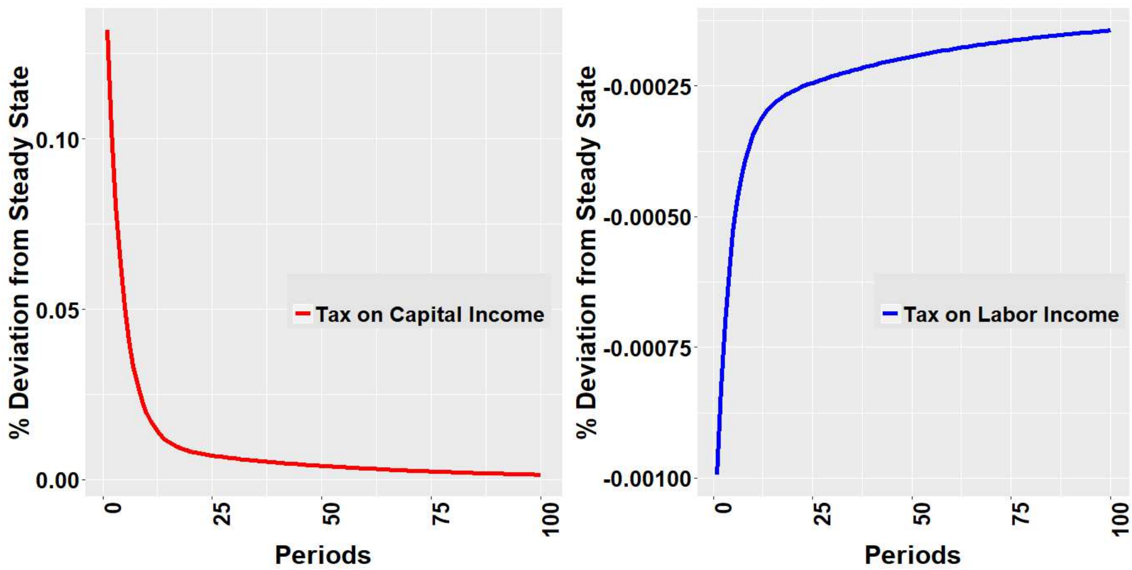

Figure 3.4: Reactions of the optimal tax rates to shocks in government spending g_t
Source: Own' Elaboration.

in government spending implies a lower disposable income for the households. In the first case poor an wealthy agents are better off, while in the second case only the wealthy households can shift consumption across time to mitigate the effects of the shock, so they are better off than poor agents.

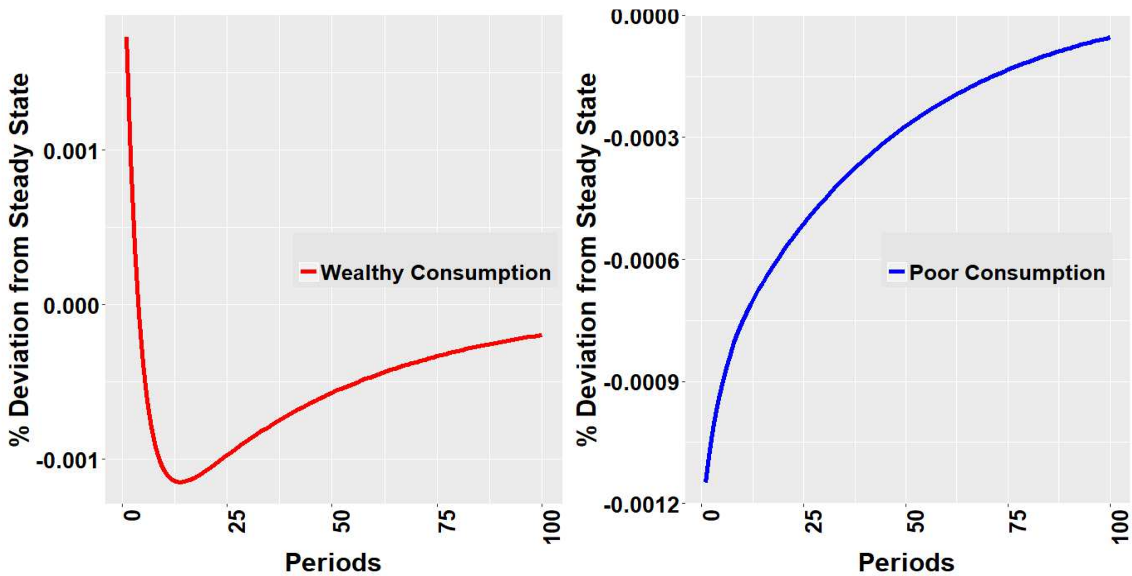

Figure 3.5: Reactions of consumption to shocks in government spending g_t.
Source: Own' Elaboration.

This result can be explained by a lower level of capital stock (negative) and the fact that capital supply is linked to the investment controlled by the wealthy agents. In this scenario the wealthy agent chose accumulate less capital due to the low interest rate and increase his consumption. The government spending inhibits private investment by the *crowding-out* as explain by Ahmed and Miller (2000) and Afonso and Aubyn (2009) .

The effect of the government spending shock on the other macroeconomic variables is presented in Figure 3.6. An important result is the presence of the *crowding-out* effect, where increases in government spending drive down private sector investment. When the government increases its spending, it can lead to a substantial impact on the interest rate (which also increases). The labor supply of both agents increases, although the magnitude of both responses is small.

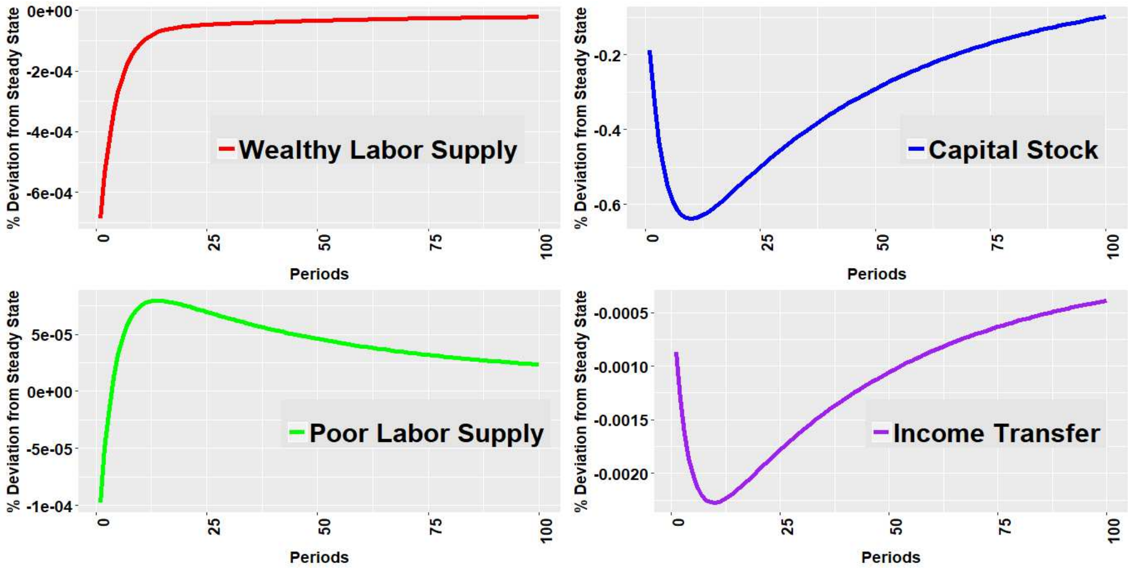

Figure 3.6: Reactions to shocks in government spending g_t.
Source: Own' Elaboration.

Similar results can be found in other studies, such as in Giavazzi and McMahon (2012) who show that the effects of changes in government spending in the short run may come at the expense of neglecting the wealth channel, and as a consequence the output, as well as the investment, decreases. More importantly, the income transfer (T^P) seems to decrease and this effect has a harmful impact on the poor agent.

When compared to the model with a single representative agent, the model with heterogeneous agents presented here has some analytical advantages, mainly by introducing inequality. The results show that the optimal fiscal policy depends, in large part, on the level of income inequality of the country.

3.5.3 Limit of Taxation

We start by testing the limits of taxation by changing the favoritism of the government towards of the poor agent. Our objective in this section is to test how fiscal policy (τ^K, τ^H, T^P) behaves when the government increases the parameter Θ, which means that the utility of the poor has more weight in equation (3.19).

When $\Theta \leq 0.50$, the model was unstable and a steady state could not be found computationally, in other words, Octave was not able to provide any kind of result. One possible explanation for this fact is the high income inequality presented by the Brazilian economy. Thus, for the stability of the model a favoritism towards of the poor agent is necessary, so the fiscal policy has to be more pro poor, which makes the condition $\Theta \geq 0.50$ necessary for the computation the model.

Figure 3.7 shows the behavior of the optimal fiscal policy, represented by the tax on capital income, tax on labor income and government transfers, as the favoritism of the government changes towards the poor agent. Initially, we compute the model with $\Theta = 0.50$ and plot the steady state values, then we do the same for the values $\Theta = 0.62$, $\Theta = 0.70$, $\Theta = 0.81$ and $\Theta = 0.99$ the extreme of pro-poor policy[3].

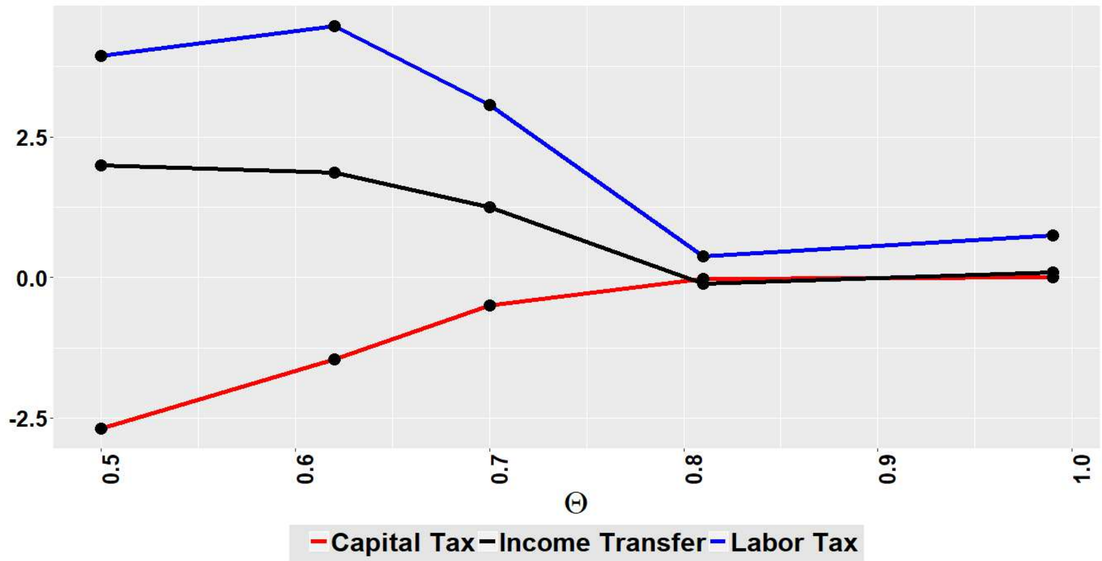

Figure 3.7: Effect of changing preference (Θ) toward poor agent.
Source: Own' Elaboration.

The high level of income inequality in Brazil causes a pro-poor government to perform large income transfers. Since the transfer does not come for free, the government has to drastically increase the tax on labor income. The high level of tax on labor income causes another problem, namely a decrease in the labor supply, which leads to a decrease in the level of production and economic growth. The solution to maintaining the economic growth is to encourage an increase in the capital stock by providing subsidies, as a way to compensate for the decrease in labor supply.

In fact, as explained in section 3.2, and explained abundantly in the microeconomic literature, so that the output remains on the production possibility frontier the marginal rate of transformation has to be maintained, so the decrease of one input has to be compensated by

[3]The values of Θ are arbitrarily chosen, just to simulate a increases in the government favoritism towards the poor.

an increase of the other so that it can remain on the same production frontier and maintain the equilibrium. This create substitution effect, which is showing in our model, by the exchange between labor and capital.

As shown in Figure 3.7, the government grants a subsidy on capital that decreases as the government favoritism of the poor agent increases, which also decreases the the income transfers. This result in particular makes perfect sense, since in our model the poor are not allowed to hold capital, the government makes the wealthy hold capital "on behalf of" the poor (by incentivizing extra capital with subsidies), and then performs the transfer to compensate the poor, that is, to give them "their share" of the returns of capital. Only that it needs to do this via labor income tax aimed (mainly) at the wealthy, even though it cannot differentiate in this respect.

3.5.4 Tax Reform

Fiscal policy affects other macroeconomic variables – mainly aggregate demand and the level of economic activity, savings and investment and the distribution of income – primarily through two different mechanisms. Firstly, the government can impact the economy by increasing or decreasing their spending in various sectors. Secondly, the government can impact the economy by changing the level and composition of taxation.

After finding the analytical solution of the model, we experiment with increases in the labor income tax to see how this will affect the agents' welfare. As pointed out by Domeij and Heathcote (2004), changing the balance of these taxes is likely to incur significant distributional changes.

To simulate the tax reform, we use the same model as in section 3.3, and set the capital income tax according to the interest on shareholders' return of stocks traded on São Paulo Stock Exchange BM&FBOVESPA, which is 15%. Following Santos (2016), we set the labor income tax rate to 27%. Subsequently, we simulated an increase of 5 percentage points on the labor income tax rate τ^H. The results are shown in Figure 3.8.

As expected, the poor agents suffer from the increase in labor income tax, because this is their only source of income. On the other hand, the rich suffer less because they can increase their capital stock to offset the loss of labor income. This result has already been discussed in the literature, as we can see in Domeij and Heathcote (2004), who find that whilst long run welfare can be expected to increase following a reduction in the capital tax rate, the short run cost from the increased labor taxes are too heavy a price to pay.

As result of an increase in the labor income tax, the hours worked decreases, because the increased tax reduces the *net* return of labor ($(1-\tau_t^H)\omega_t$). Because of the Cobb-Douglas production function that we assume and the proprieties of diminishing returns in each specific

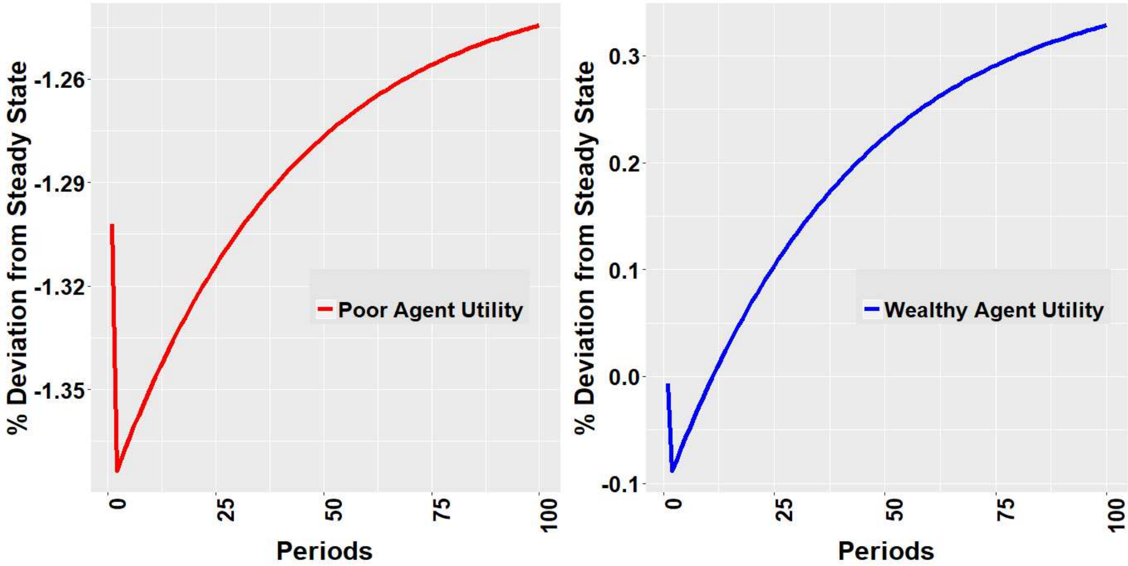

Figure 3.8: Reactions in welfare of a change of 5% p.p. in the tax of labor income.
Source: Own' Elaboration.

factor of this equation, it's very possible that you might actually find that the gross return, ω_t, actually increases.

The increase in welfare, after the initial decrease comes from the increase in the stock of capital. What happens then is a substitution between the inputs of production, increasing the input of higher marginal product return (capital) and decreasing the lower marginal product return labor. It is not necessary analyze the increase on tax on capital income, because, as was shown in section 3.3, any positive amount will decrease the production in steady state.

3.5.5 Effects of Reducing Poverty

From our theoretical model, we can assume that the proportion of poor households is a very important factor to consider when formulating fiscal policies. In fact, a large part of the differences between the fiscal policies of developed and developing countries is a result of the different levels of income inequality. In accordance with economic theory, the model shows that income inequality can affect the optimal fiscal policy and other macroeconomic variables.

We simulate an extreme change in inequality, with a large reduction in the parameter ϕ, which represents the proportion of poor households in Brazil. ϕ was previously calibrated to the value $\phi = 0.89$ in section 3.4. We now reduce the value of this parameter to $\phi = 0.10$, and perform a new set of simulations of the model. Figure 3.9 presents the results for the fiscal policies and the output of the economy.

The level of income transfer drops significantly when the proportion of the poor has been reduced, almost ceasing to exist. We can interpret this result as evidence that as inequality is reduced, the need to transfer income no longer exists. Such result is also found by Swarbrick

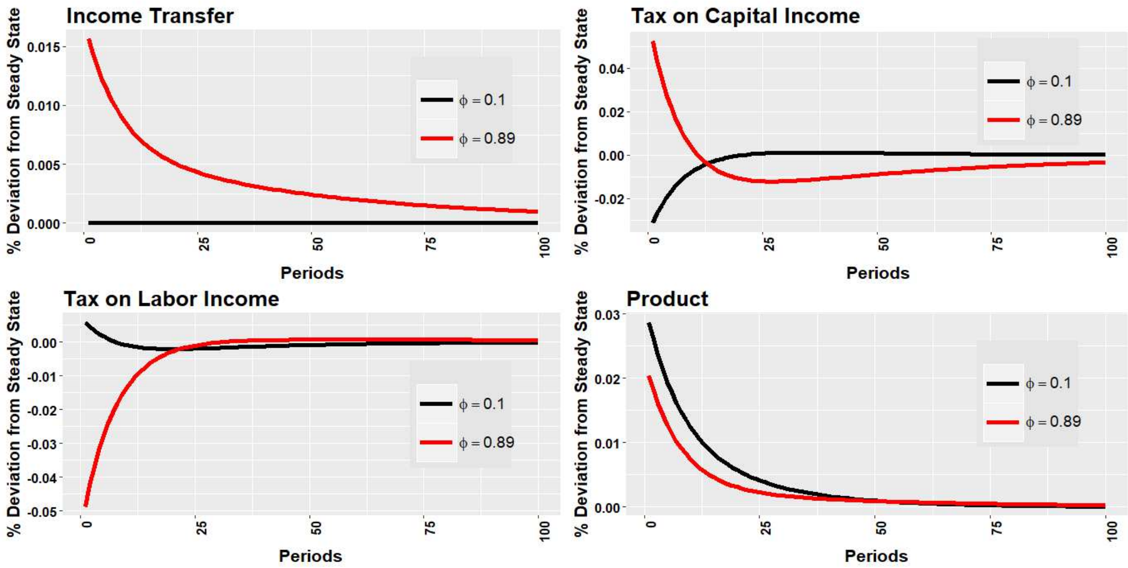

Figure 3.9: Reactions of productivity shocks a_t by reducing inequality.
Source: Own' Elaboration.

(2012), who use $\phi = 0.50$ and show that government transfers to the poor agent to be close to zero over the business cycle.

Taxes show a completely different behavior in the two scenarios, demonstrating that inequality has a direct impact on optimal fiscal policy. An economy with a lower level of inequality has an optimal fiscal policy which is less susceptible to shocks, as we can see by the magnitude of the reactions to the productivity shock. While in the first scenario, $\phi = 0.89$, the government reacts by increasing the tax on capital income and reducing the tax on labor income, in the second scenario the opposite behavior is shown, although in this case to a lesser extent.

As we can see, the reactions of output to productivity shocks changes significantly when the level of inequality changes. As expected, a reduction in the inequality increases the output, supporting the hypothesis that reducing inequality can lead to greater economic growth.

3.6 Conclusions and Remarks

The purpose of this paper is to analyze the optimal level of taxation for the Brazilian economy with heterogeneous agents over the real business cycles. To achieve this goal, we calibrated a DSGE model to the Brazilian economy and simulated two types of shocks: a productivity shock and a shock in government spending.

The analytical solution of the proposed model shows that, given the Ramsey problem, it is optimal not to tax the capital in the steady-state. The numerical solution confirms this result, indicating that – in the long run – this result is valid theoretically and numerically.

The taxes on the productivity shock has opposite behavior with tax on capital income increasing and on labor income decreasing. The taxes reactions has a direct effect on the consumption of both agents, which seems that poor agent take more advantages.

The results of the simulations show that the optimal tax rates on capital income and labor income respond to a shock in government spending in opposite ways, mainly because we assume that government has a balance budget. In addition we find evidence of the existence of a crowding-out effect.

When we compare the two types of shock, the wealthy agent is in a better situation in both, in part because he is able to smooth consumption over time and in part because of the increase in productivity. The poor agent seems to benefit only in the case of a supply shock. Another important finding, is that the composition of the government budget changes as to the favoritism of the government ranges from rich to poor.

The simulations show that taxes react differently depending on the level of income inequality, although the fiscal policy of income transfer has limits, when the government has to drastically increase the tax on labor income. A fiscal policy that favors the poor must be accompanied by a high level of tax on labor income, which leads to a decrease in economic growth. The solution to maintaining the economic growth is to encourage an increase in the capital stock by providing subsidies. We also present evidence that reducing poverty can increase output, eliminating the necessity of transfers and reducing considerably the fluctuations of tax revenue.

Appendix

3.A Household Solution

Based on the equations (3.1) and (3.2) we can construct the Lagrange of the problem of maximization of the poor agent:

$$\mathscr{L} = ln(C_t^P) + \xi ln(1 - H_t^P) + \lambda_t^P \left[(1 - \tau^H)\omega_t H_t + T_t^P - C_t^P \right] \tag{3.22}$$

$$\frac{\partial \mathscr{L}^P}{\partial C_t^P} \; : \; \frac{1}{C_t^P} - \lambda_t^P = 0 \tag{3.23}$$

$$\frac{\partial \mathscr{L}^P}{\partial H_t^P} \; : \; \xi \frac{1}{(1 - H_t^P)} + \lambda_t^P (1 - \tau_t^H)\omega_t = 0 \tag{3.24}$$

The Lagrangian is constructed to solve the wealthy household optimization problem of maximizing.

$$\begin{aligned} \mathscr{L} = \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ \beta^t \left[ln(C_t^W) + \xi ln(1 - H_t^W) \right] \right. \\ + \quad & \left. \lambda_t^W \left[(1 - \tau_t^H)\omega_t H_t^W + \left[1 + (1 - \tau_t^K)R_t - \delta \right] K_t^W - K_{t+1} - C_t^W \right] \right\} \end{aligned} \tag{3.25}$$

$$\frac{\partial \mathscr{L}^W}{\partial C_t^W} \; : \; \beta^t \left(\frac{1}{C_t^W} \right) - \lambda_t^W = 0 \tag{3.26}$$

$$\frac{\partial \mathscr{L}^W}{\partial H_t^W} \; : \; \beta^t \left(\xi \frac{1}{(1 - H_t^P)} \right) + \lambda_t^W (1 - \tau_t^H)\omega_t = 0 \tag{3.27}$$

$$\frac{\partial \mathscr{L}^W}{\partial K_{t+1}^W} \; : \; \lambda_{t+1}^W \left[1 + (1 - \tau_t^K)R_t - \delta \right] - \lambda_t^W = 0 \tag{3.28}$$

3.B Equilibrium Equations

$$\frac{1}{\beta} = \left[1+(1-\tau^K)R-\delta\right] \tag{3.29}$$

$$(1-\tau^H)\omega = \xi\left[\frac{C_t^W}{(1-H_t^W)}\right] \tag{3.30}$$

$$C+\delta K^W = (1-\tau^H)\omega H+(1-\tau^K)RK^W \tag{3.31}$$

$$(1-\tau^H)\omega = \xi\left[\frac{C_t^P}{(1-H_t^P)}\right] \tag{3.32}$$

$$C = (1-\tau^H)\omega H^P+T^P \tag{3.33}$$

$$G+T = \tau^H\omega H+\tau^K RK \tag{3.34}$$

$$\omega_t = \alpha\left(\frac{H_t}{K_t}\right)^{(1-\alpha)} \tag{3.35}$$

$$R_t = (1-\alpha)\left(\frac{H_t}{K_t}\right)^{\alpha} \tag{3.36}$$

3.C Ramsey Solution

The Ramsey problem in Lagrangian form is build in the equation where the objective function is constrained by the household (poor and wealthy) first order conditions, the household budget constraints and the government budget constraint, therefore we have:

$$\frac{\partial\mathscr{L}}{\partial C_t^W}:(1-\Theta)\left[\frac{1}{C_t^W}\right]-\lambda_t^W-\Phi_t^W\left[\frac{1}{(C_t^W)^2}\right](1-\tau_t^H)\omega_t+\psi_t\left[\frac{1}{(C_t^W)^2}\right] + \psi_t\beta^2\left[\frac{1}{(C_t^W)^2}\right]\left[(1-\tau_{t+1}^K)R_{t+1}+1-\delta\right]=0 \tag{3.37}$$

$$\frac{\partial\mathscr{L}}{\partial H_t^W}:(1-\Theta)\left(\xi\frac{1}{(1-H_t^W)}\right)+\Psi_t\left[(1-\phi)\omega_t-(1-\tau_t^H)(1-\phi)\omega_t\right] + \lambda_t^W\left[(1-\tau_t^H)\omega_t\right]+\Phi_t^W\left[\frac{\xi'\left(\frac{1}{(1-H_t^W)}\right)}{(1-H_t^W)^2}\right]=0 \tag{3.38}$$

$$\frac{\partial \mathscr{L}}{\partial K^W_{t+1}} \quad : \quad \beta \Psi_{t+1}\left[F_{K^W} - (1-\phi)(1-\tau^K_t)R_t\right]$$
$$+\beta \lambda^W_{t+1}\left[1+(1-\tau^K_t)R_t - \delta\right] - \lambda^W_t = 0 \tag{3.39}$$

$$\frac{\partial \mathscr{L}}{\partial C^P_t} : \Theta\left(\frac{1}{C^P_t}\right) - \lambda^P_t + \Phi^P_t\left[\frac{1}{\left(C^P_t\right)^2}\right](1-\tau^H_t)\omega_t = 0 \tag{3.40}$$

$$\frac{\partial \mathscr{L}}{\partial H^P_t} : \Theta\left[\xi \frac{1}{(1-H^P_t)}\right] \qquad +\Psi_t\left[\phi\,\omega_t - (1-\tau^H_t)\omega_t \phi\right]$$
$$+\lambda^P_t\left[(1-\tau^H_t)\omega_t\right] + \Phi^P_t\left[\frac{\xi'\left(\frac{1}{(1-H^P_t)}\right)}{(1-H^P_t)^2}\right] = 0 \tag{3.41}$$

$$\frac{\partial \mathscr{L}}{\partial \tau^H_t} : \Psi_t \omega_t H_t - \lambda^W_t \omega_t H^W_t - \lambda^P_t \omega_t H^P_t + \Phi^W_t\left(\frac{1}{C^W_t}\right)\omega_t + \Phi^P_t\left(\frac{1}{C^P_t}\right)\omega_t = 0 \tag{3.42}$$

$$\frac{\partial \mathscr{L}}{\partial \tau^K_t} : \Psi_t R_t K_t - \lambda^W_t R_t K^W_t + \psi_t \beta\left[\frac{1}{C^W_{t+1}}\right]R_t = 0 \tag{3.43}$$

$$\frac{\partial \mathscr{L}}{\partial T^P_t} : \lambda^P_t - \Psi_t \phi = 0 \tag{3.44}$$

3.D Response to Shocks

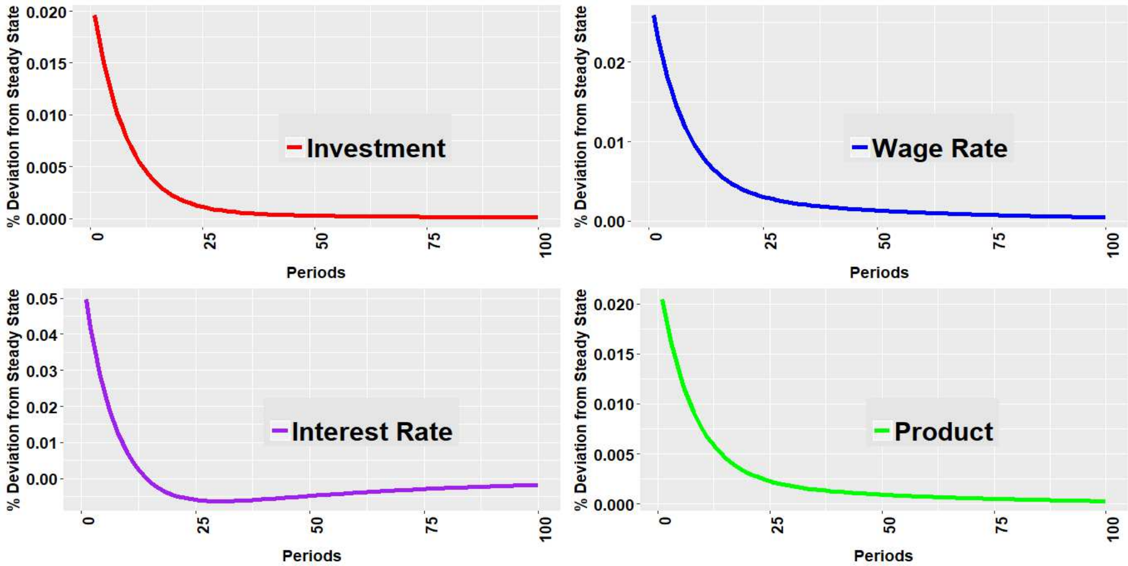

Figure 3.D.1: Macroeconomics impulse response following productivity shock a_t.
Source: Own' Elaboration.

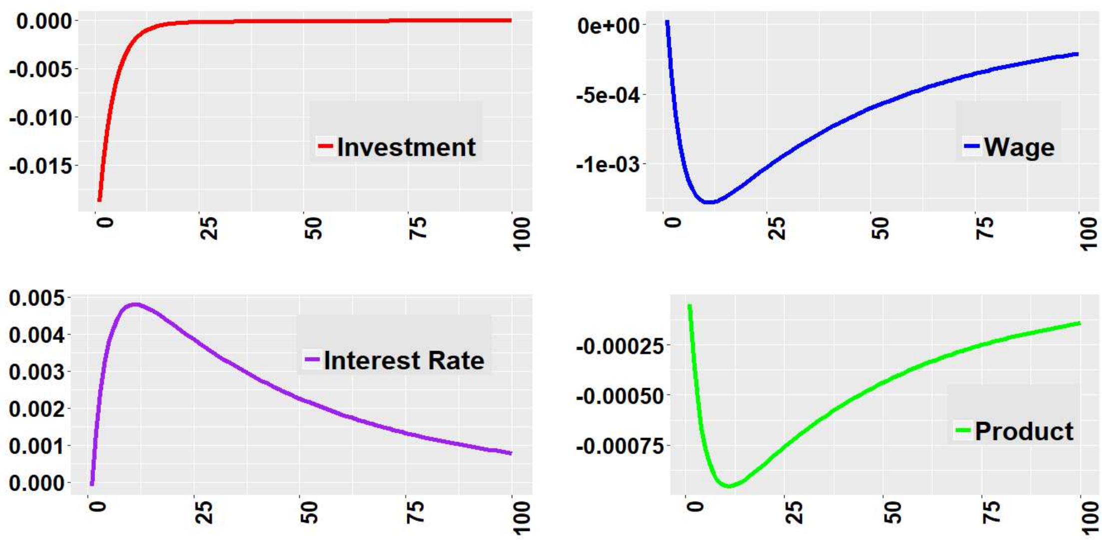

Figure 3.D.2: Macroeconomics impulse response following following government speeding shock g_t.
Source: Own' Elaboration.

3.E Dynare Code Tax Reform

```
//==============================================================================
// 1 Part - Declaration of endogenous and exogenous variables.
//==============================================================================
var I C H K WW RR TTAU tauK tauH KR HRHR HPHP CRCR CPCP tau Y HR HP cr
        cp g a W R;
varexo epsA epsG;
```

```
//==============================================================================
// 2 Part - Declaration of parameters.
//==============================================================================
parameters
delta
beta
chi
theta
alpha
psi
Theta
gamma
G
TTAU
TAU;
```

```
//==============================================================================
// 3 Part - Parameter calibration for Brazil.
//==============================================================================
delta = 0.025;
alpha = 0.65;
beta = 0.98;
chi = 1;
theta = 0.89989;
gamma = 0.50;
G = 0.3266;
TTAU = 0;
TAU = 0;
```

```
//==============================================================================
// Parameters use to calculate steady-state
//==============================================================================
psi = ((1+beta*delta-beta)/(beta*(1-tauK)*(1-alpha)))^(1/alpha);
Theta = (1+chi*G) / ((1-tauH)*alpha*psi^(alpha-1) + chi*psi^(alpha-1)
        - (delta*chi/psi));
```

```
//===============================================================================
// 4 Part - Implementation of the model in Dynare.
//===============================================================================
model;
#CR = exp(cr);
#FCR = exp(cr(+1));
#CP = exp(cp);
HR = 1 - chi*CR/((1-tauH)*W);
1/CR = beta*(1+(1-tauK)*R-delta)*1/FCR;
HP = 1 - chi*CP/((1-tauH)*W);
CR + KR = (1-tauH)*W*HR + (1+(1-tauK)*R-delta)*KR(-1);
CP = (1-tauH)*W*HP+TAU;
W = alpha*(H/K(-1))^(alpha-1);
R = (1-alpha)*(H/K(-1))^alpha;
G = tauH*W*H + tauK*R*K(-1) - TTAU;
Y = (H)^alpha*K(-1)^(1-alpha);
I = K - (1-delta)*K(-1);
C = theta*CP + (1-theta)*CR;
H = theta*HP + (1-theta)*HR;
K = (1-theta)*KR;
UP = ln(CP) + ln(1-HP);
UR = ln(CR) + ln(1-HR);
CCP = exp(cp);
CCR = exp(cr);
end;
```

```
//===============================================================================
// 5 Part - Initial guesses for steady-state computation in Octave.
//===============================================================================
initval;
H = Theta;
K = Theta/psi;
Y = H^alpha*K^(1-alpha);
W = alpha*psi^(alpha-1);
R = (1-alpha)*psi^alpha;
I = (Theta/psi) - (1-delta)*(Theta/psi);
tauK = 0.15;
tauH = 0.27;
HP = 0.5;
HR = 0.5;
end;
steady;
```

```
//==============================================================================
// 6 Part - Simulation using the new tax rate.
//==============================================================================
endval;
tauH = 0.32;
end;
steady;
simul(periods=2000);
```

3.F Dynare Code Ramsey Policy

```
//==============================================================================
// First Part: Declaration of endogenous and exogenous variables.
//==============================================================================
var I C H K WW RR TTAU tauK tauH KR HRHR HPHP CRCR CPCP tau Y HR HP cr
    cp g a W R;
varexo epsA epsG;
```

```
//==============================================================================
// Second Part A: Declaration of Parameters.
//==============================================================================
parameters
delta
beta
chi
theta
rhoA
sigmaA
rhoG
sigmaG
alpha
G
psi
Theta
gamma;
```

```
//==============================================================================
// Second Part B: Parameters Calibration for Brazilian Economy.
//==============================================================================
delta = 0.02500;
alpha =  0.65466;
beta = 0.9776;
chi = 1;
theta = 0.89989;
G = 0.3266;
rhoA = 0.872672;
rhoG = 0.759740;
sigmaA = 0.030193;
sigmaG = 0.058284;
gamma = 0.5000;
//------------------------------------------------------------------------------
// Parameters use to facilitate the computation of Steady-State.
//------------------------------------------------------------------------------
psi = ((1+beta*delta-beta)/(beta*(1-tauK)*(1-alpha)))^(1/alpha);
Theta = (1+chi*G) / ((1-tauH)*alpha*psi^(alpha-1) + chi*psi^(alpha-1)
        - (delta*chi/psi));
```

```
//==============================================================================
// Third Part: Equations of the model.
//==============================================================================
model;
#CR = exp(cr);
#FCR = exp(cr(+1));
#CP = exp(cp);
#TAU = exp(tau);
```

```
HR = 1 - chi*CR/((1-tauH)*W);
1/CR = beta*(1+(1-tauK)*R-delta)*1/FCR;
HP = 1 - chi*CP/((1-tauH)*W);
CR + KR = (1-tauH)*W*HR + (1+(1-tauK)*R-delta)*KR(-1);
CP = (1-tauH)*W*HP+TAU;
W = alpha*(exp(a))^alpha*(H/K(-1))^(alpha-1);
R = (1-alpha)*(exp(a))^alpha*(H/K(-1))^alpha;
TTAU = tauH*W*H + tauK*R*K(-1) - exp(g)*G;
Y = (exp(a))^alpha*(H)^alpha*K(-1)^(1-alpha);
I = K - (1-delta)*K(-1);
TTAU = theta*TAU;
C = theta*CP + (1-theta)*CR;
H = theta*HP + (1-theta)*HR;
K = (1-theta)*KR;
a = rhoA*a(-1)+epsA;
g = rhoG*g(-1)+epsG;
//--------------------------------------------------------------------------------
// Deviation from Steady State.
//--------------------------------------------------------------------------------
WW=W/STEADY_STATE(W);
RR=R/STEADY_STATE(R);
HRHR=HR/STEADY_STATE(HR);
HPHP=HP/STEADY_STATE(HP);
CRCR=CR/STEADY_STATE(CR);
CPCP=CP/STEADY_STATE(CP);
end;
```

```
//==============================================================================
// Fifth Part: Initial guesses for steady-state computation.
//==============================================================================
initval;
tauK = 0;
tauH = 0;
H = Theta;
K = Theta/psi;
Y = H^alpha*K^(1-alpha);
W = alpha*psi^(alpha-1);
R = (1-alpha)*psi^alpha;
I = (Theta/psi) - (1-delta)*(Theta/psi);
TTAU = tauH*alpha*psi^(alpha-1)*Theta
     + tauK*(1-alpha)*psi^alpha*(Theta/psi) - G;
HP = 0.5;
HR = 0.5;
a = 0;
epsA = 0;
g = 0;
epsG = 0;
end;
```

```
//==============================================================================
// Sixth Part: Specification of shocks a and g.
//==============================================================================
shocks;
var epsA = sigmaA^2;
var epsG = sigmaG^2;
end;
ramsey_policy (periods=1000,order=1,planner_discount=0.9776);
planner_objective gamma*(cp+chi*ln(1-HP))+(1-gamma)*(cr+chi*ln(1-HR));
stoch_simul(periods=1000,irf=100);
```

4. Fiscal Policy and Income Inequality in a model with Heterogeneous Agents.

Abstract

Income inequality is one of the main obstacles to economic development. Understanding the dynamics of this social phenomenon is central to understanding economic growth. Thus, based on models that relate fiscal policy, inequality and growth, we propose the following problem: Given the initial distribution of capital, what is the effect of a set of flat-rate taxes, on income inequality? In order to answer this question we build a model that captures the effect of a re-distributive fiscal policy on income inequality. The model we propose is a version of a competitive equilibrium of the basic neoclassical growth model, which incorporates income inequality endogenously and heterogeneous agents: poor and rich, allowing us to understand this problem in a dynamic way. We use the Ramsey problem to determine the optimal sequences for the three types of flat-rate taxes, capital income, labor income and consumption in a non-stochastic economy. The analytical solution found suggests that, in the steady state, the optimal tax on capital should always be zero regardless of the government's favoritism towards particular agents. Also the government should finance the transfers to the poor agent using different combinations of taxes on consumption and labor income.

keywords: Inequality, Fiscal Policy, Ramsey Problem, Economic Growth, Income Distribution.

JEL Codes: D63, E62, O40.

4.1 Introduction

4.1.1 Initial Considerations

Many studies in economic theory have turned their attention to the relationship between inequality and economic growth. It is important to point out that inequality can be harmful to the economy's growth rate. This result has brought social equity to center of the policy debate in macroeconomic theory. Through appropriate use of taxes and transfers, government can correct socially undesirable distributive outcomes arising from market forces. The evidence shows that most developed economies are more effective at this re-distributive function than developing economies (GONI; LOPEZ; SERVEN, 2011).

One of the most known theories in the international literature is the study of Kuznets (1955). This author assumes that in the early process of capital accumulation, the income and wealth distribution becomes unequal, but after sufficient wealth is accumulated, the wealth and income distributions equalize and inequality decreases. The hypothesis proposed by Kuznets indicates that there is a relationship between the level of inequality and economic growth, although this relationship is not linear. Many studies have indicated that initially, in the short run inequality increases with economic growth and, in the long run decreases began a turning point. This effect is attributed by the authors to two factors, wage growth and the decrease in the return an investments.

However, there are other theories used to explain the relationship between inequality and economic growth, but they are different and there is no consensus of how this relationship occurs. Among these theories, those based on the transition between the agricultural and industrial sectors, imperfections in the financial system and technological progress stand out.

The theories presented, while different, show there is strong evidence that economic growth and inequality are related. It is also possible that different economic policies influence the relationship between this two variables. Nevertheless, the way in which this relationship occurs is still controversial, as pointed out by Muinelo-Gallo and Roca-Sagales (2011) the issue of the sign and magnitude of theses effects is an open question. Garcia-Penalosa and Turnovsky (2007) argue that earlier evidence suggested a negative trade-off between growth and inequality, therefore recent studies have tended to support a positive relationship. Shin (2012) shows that empirical evidence in studies of East Asian and South American countries presents a negative relationship between income inequality and economic growth. However in developed countries, such as United States and France this relationship is positive.

More recently, some authors have shown two ways of in which inequality and growth are not in conflict. Chen and Turnovsky (2010) indicate that the growth rate and inequality will depend on the set of forces to which both are reacting. Barro (2000) shows that the positive or negative relationship depends on the development stage of the economy. This author also shows, that the effect of income inequality on economic growth is negative in poor countries, but is positive in rich countries.

So inequality is a barrier to economic growth mainly in developing countries. The inequality can reduce the potential economic growth in the long-run and welfare too. Muinelo and Roca-Sagalés (2011) argue that the distribution of wealth can affect the economic growth rate in the long-run by modifying the size and composition of aggregate demand.

A central concern of this discussion is the role that government policies may play in reducing economic inequalities, and determining the effects on the economic growth rate. A important issues raised in this context is what are the most efficient way to reduce inequality? and also what are the main instruments capable of reducing inequality?

Economic theory suggests that fiscal policy can affect inequality in the long-run. This is because fiscal policy can impact aggregate demand, distribution of wealth and income, in addition to the production capacity of the economy. Thus, the choice of a distributive fiscal policy has become of crucial importance in achieving economic growth and reducing inequality at the same time (MUINELO-GALLO; ROCA-SAGALES, 2013).

One of the ways in which tax policy impacts inequality is presented by Rebelo (1991). He shows that redistribution of income by using tax on income has negative effects on growth because it reduces the gap of income and wealth and consequently reduces growth. Income redistribution financed through taxes on capital income also reduces the incentive to accumulate wealth.

Using an endogenous growth model with elastic labor supply, Garcia-Penalosa and Turnovsky (2007) investigate how different ways of financing an investment impact the income distribution. They show that growth is driven by policies that increase the return on capital. The authors also argue that capital is more unequally distributed than labor, hence, higher returns on capital represent greater income inequality.

There are other authors that argue inequality decreases as a consequence of the tax policy chosen by the median voter, as show' by Muinelo-Gallo and Roca-Sagales (2011, p. 10) "greater fiscal redistribution through distortionary taxes, while it may reduce investment incentives, also decreases social conflict and contributes to greater stability that encourages productive activities and capital accumulation". Therefore, more unequal democratic societies demand redistribution financed by distortionary taxes.

The reduction of inequality and, therefore, a policy of income redistribution may be achieved through fiscal policy. A progressive tax policy, for example, can modify the inequality in the long run. What is the optimal fiscal policy, which, with less distortions on growth, decreases income inequality? the answer to that question is part of the optimal tax theory. The well known Ramsey problem is the study of what amount of tax causes minimum distortion and reduces inefficiency.

The literature on optimal taxation focuses on the effects of fiscal policy on economic growth although, there are also several studies that focus on the impact on other macroeconomic variables such as income inequality. An example of this theory is the seminal study of Chamley (1986), which shows that the optimal tax on capital in the long run should be zero. Though there are other studies, such as conducted by Aiyagari (1995) that show in a model with incomplete insurance markets and borrowing constraints, that optimal capital income tax should be positive, even in the long run.

The effects of fiscal policy considering endogenous saving rates is used by Judd (1985) to show that fiscal policy has a transitory effect on output, and has no permanent effect on economic growth. In an other study, Oh (2013) showed that Real Business Cycle models with only shocks to total factor productivity can not explain the inequality. Furthermore, inequality is moderately countercyclical in United States of America.

Fochezatto and Bagolin (2006) indicate that a progressive tax policy has small effects on income distribution. On the other hand, a policy of income redistribution, made by government transfers trough income taxation, has a significant impact on inequality.

There are few authors that estimate the impact of fiscal policy on inequality. Theses studies focus primarily on econometric estimates using panel data, but the results are mostly not robust, presenting evidence that the size of the impact and its significance depends on the set of control variables used and the initial conditions of the economy.

As stated high inequality can be harmful to the economy's growth rate. Through appropriate use of taxes and transfers, government can correct socially undesirable distributive outcomes arising from market forces. The evidence shows that most developed economies are more effective at this re-distributive function, than developing economies (GONI; LOPEZ; SERVEN, 2011). Through the necessary fiscal reforms, its re-distributive functions effectively. The re-distributive impact of a country's fiscal system is shaped by three factors: (1) The feasible volume of transfers, (2) the incidence of taxation and (3) the incidence of transfers.

Despite its importance, little has been researched about the impact of fiscal policy on income inequality. Thus, investigating the relationship between fiscal policy and income inequality can show important mechanisms through which the latter can be reduced. It also highlights the impact of these variables on economic growth, because more equitable initial allocations lead to a higher growth rate.

There is a lack in the national literature of analyses of the impact of fiscal policy on income inequality, especially studies that analyze the issue dynamically. To understand the linkages between fiscal policy, inequality and economic growth, it is necessary to adopt a structurally, consistent dynamic general equilibrium approach.

Thus, based on models that relate fiscal policy, inequality and growth, we propose the following problem: Given the initial distribution of capital, what is the effect of a set of flat-rate taxation on inequality? It is also important to understand the interaction between the households, firms and government.

Therefore, this work intends to contribute to literature first by presenting a model that captures the effect of a set of fiscal policies on inequality and consequently economic growth. Second, by presenting a model in which inequality is a endogenous variable. This type of analysis is important, because it captures key interactions between agents and the economic system. The model introduced here aims to describe the behavior of the economy as a whole by analyzing the interaction of many microeconomic decisions.

The main objective of this study is to analyze the impact of fiscal policy on inequality and economic growth in a model with heterogeneous agents. The hypotheses are: (i) there is a causal relationship between fiscal policy and income inequality regardless of the type of tax used by the government; (ii) the type of flat-rate tax: consumption, capital and labor income, can simultaneously affect the growth and inequality of the economy in different magnitudes.

4.2 Theoretical Framework

4.2.1 Fiscal Policy and Inequality

The analysis of fiscal policy effects on the economy is usually done in order to study economic growth. One of mains instruments of fiscal policy are changes in the level and composition of taxation. These changes can affect crucial macroeconomic variables, such as: aggregate demand, savings and investment and the distribution of income.

The studies of economic growth theory focus on factors that cause inequality. In this context, inequality can be explained by imperfections in the credit market. There are many studies that explain the distribution of income and consequently inequality, by limited participation in financial markets (GALOR; ZEIRA, 1993; FOCHEZATTO; BAGOLIN, 2006; MOTTA; TIRELLI, 2012).

As pointed out by Mankiw (2000), households with low wealth fail to smooth consumption over time and do not accumulate wealth, on the other hand households with high wealth smooth consumption not only from year to year, but also from generation to generation. This process is capable of generating income inequality, and shows the importance of fiscal policy that redistributes the income, correcting this problem.

In recent literature of optimal taxation, the debate has turned to the role of fiscal policy in distribution issues. Many authors have questioned the adequacy of the representative agent model to analyze these questions. It is considered necessary to use models with heterogeneous agents. Introducing another type of agent in a economic growth model, in special the case of our model with poor and rich agents, does not alter the assumptions usually made in this type of analysis. This type of modeling brings a gain in terms of analytical results, since we differentiate between rich and poor agents, allowing a better understanding of fiscal policy effects on inequality.

However, before we build a model that incorporates these hypotheses it is necessary to build a standard economic growth model according to the literature, to show the changes which we propose in this model and its implications for the mains conclusions. In the standard models of optimal growth theory, where the economy has only one sector, a representative household solves a version of the savings problem with constant returns to scale production function. This function is usually: $F(K,L)$, where L is labor input and K represents physical capital. When the return on capital is tax-free, the gross rate of return is equal to the marginal product of capital, less depreciation.

According to Sargent and Ljungvist (2000) we have: $R_{t+1} = F_{k,t+1} + (1-\delta)$, where R_{t+1} is gross rate of return, $F_k(k,t+1)$ is the marginal product of capital and δ is the depreciation rate. We use a general utility function $U(C,l)$, where l is the household's leisure and C to represent the consumption level.

$$\max_{C_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(C_t, l_t) \tag{4.1}$$

The utility function is concave and twice differentiable. Using these assumptions Sargent and Ljungvist (2000) build a simple growth model with no taxes. The model proposed by these authors it is a maximization of social planner problem (equation 1). This maximization is subject to a resources constraint as shown in equation (2).

$$C_t + k_{t+1} = f(k_t) \tag{4.2}$$

With these equations we can make the Lagrangian and solve the maximization problem. After solving the maximization problem and finding the First Order Necessary Conditions - FONC, we can find the Euler condition (Equation 4.3) that is:

$$U_{C_t} = \beta U_{C_{t+1}} \left[F_{k_{t+1}} + (1-\delta) \right] \tag{4.3}$$

Where U_{C_t} is the marginal utility of consumption at period t and $U_{C_{t+1}}$ is the marginal utility of consumption at $t+1$. Because of constant returns to scale property, $F_k(K,N) = f'(k)$, where N is the labor demand in this economy, $k = K/N$ and $F(K,N) = Nf(K/N)$. In steady state, with no population growth, where k is constant in time, the equation (4.3) must satisfy:

$$\rho + \delta = f'(k) \tag{4.4}$$

Where $\beta^{-1} = (1+\rho)$. To solve this equation it is necessary to find the value of k known as the augmented golden rule that shows the level of capital-labor ratio of the steady state. Equation (4.4) represents how technology, f, δ and time preference, β, are determinants of the level of capital when both factors are not subject to taxation in the steady state. An important conclusion of the model is that the cross-distribution of wealth and consumption replicates itself over time, so that each individual share always occupies the same position, so there is no mobility. The initial distribution of wealth perpetuates the initial inequality.

Then it is necessary to develop a model where there is a mechanism for changes in inequality, and where this can occur through fiscal policy. We can introduce fiscal polices in this model with a tax of flat-rate marginal rate $\tau_{k,t+1}$. Though if income from capital is taxed at the flat rate marginal rate, then the Euler equation (4.3) becomes:

$$U_{C_t} = \beta U_{C_{t+1}} \left[F_{k_{t+1}}(1 - \tau_{k,t+1}) + (1 - \delta) \right] \tag{4.5}$$

If the flat rate tax on capital is constant, the in the steady state we can find a result similar to the case before. It must satisfy this condition:

$$\rho + \delta = (1 - \tau_k) f'(k) \tag{4.6}$$

We now can conclude that taxing capital diminishes the steady state capital labor/ratio. The equations (4.4) and (4.6) are the most important for studying the dynamic theory of optimal taxation and showing that the optimal is to not tax capital in the long run. Based on this model, Chamley (1986) and Judd (1985) propose a model with flat rate, taxes on labor and capital income at different rates and force the government to finance itself with a stream of flat-rate taxes on capital and labor. This modeling of optimal taxation is known as the Ramsey (1927) problem and studied the behavior and limits of optimal taxation.

Sargent and Ljungvist (2000) pointed out the important role of the Euler equation in determining the limiting tax rate on capital in a non-stochastic economy and therefore, find a solution to the Ramsey (1927) problem. In the Ramsey problem the Euler equation takes a form similar to equation (4.5), therefore the household's Euler equation becomes:

$$W_{C_t} = \beta W_{C_{t+1}} \left[F_{k_{t+1}} + (1 - \delta) \right] \tag{4.7}$$

$$W(C_t, l_t) = U_C(C_t, l_t) + \Phi \left[U_{C_t} C_t + U_{l_t}(1 - l_t) \right] \tag{4.8}$$

Where Φ is a Lagrange multiplier on the government's inter-temporal budget constraint. In the Ramsey problem the taxes must be such that both (4.3) and (4.8) always hold. Thus if a steady state exists, C_t, l_t, k_t, τ_{kt} all converge and the government expenditure sequence converges (CHAMLEY, 1986; JUDD, 1985). It also must satisfy equation (4.4) and (4.6). The main conclusion of these conditions is that, in the optimum $\tau_k = 0$, in other words, the steady state properties of two versions of the consumption Euler equation show that it is optimal not to tax capital asymptotically (SARGENT; LJUNGVIST, 2000).

This conclusion is robust if the government issuing debt or maintain a balanced budget in the sense of Ricardian equivalence in every period. However if the tax system is incomplete, the optimal tax on capital can be different from zero.

On the other hand, Aiyagari (1995) shows that with incomplete insurance markets and borrowing constraints, the optimal capital tax rate is positive in the long run. Jones, Manuelli and Rossi (1997) extend this model to allow the accumulation of human capital, assuming constant returns and a linearly homogeneous function in human capital, they show that a zero tax also applies to income from labor. Chari Lawrence J. Christiano (1994) found that taxing labor causes little fluctuation while taxing private capital cause large fluctuations. There are also studies that show the effects of taxation on consumption (see Mendoza, Milesi-Ferretti and Asea (1997)). Taxing consumption may be distorting with a negative effect on growth when leisure is included in the utility function

An important aspect of the optimal taxation problem introduced above is determining income taxes, which can be regressive, flat, or progressive. Furthermore, there is one type of tax that does not create any distortion, the lump-sum tax. The lump-sum tax does not create distortions because these taxes do not depend on agents decisions.

Although taxing capital in a progressive way is often considered an appropriate instrument for redistribution, Chamley (1986) argues that doing so is not an efficient policy in the long run (SWARBRICK, 2012). The Ramsey problem focuses on how government' maximizes households' welfare subject to raising set revenues through distortionary taxation. We propose a model with three types of flat rate tax: consumption, capital and labor, and the impact of these taxes on income inequality.

4.3 Model with Heterogeneous Agents

In order to study the impacts of fiscal policies on inequality we built a model following the tradition of studies in this area. The model we propose is a version of a competitive equilibrium of the basic neoclassical growth model, which incorporates inequality and heterogeneous agents, allowing us to understanding this problem in a dynamic way.

In an economy described by a representative agent, the household receives income from labor and capital and chooses a path of consumption and capital investment to maximize their utility, the government taxes both labor and capital income and also is subject to an intertemporal budget constraint. Without any market imperfections, it is best not to distort the choices of that consumer, so a lump-sum tax should be implemented. In real economies, lump-sum taxes are rare because this tax falls equally on the rich and poor, placing a greater relative burden on the poor.

The model we built introduces a new factor inequality in an attempt to show how fiscal policy can alter the distribution of wealth in an economy. This model assumes heterogeneous agents, opposing the use of a representative agent as in Sargent and Ljungvist (2000). Krusell and Smith (1998) showed that while aggregate consumption behavior is enough to analyze most macroeconomic variables, in order to study the distributional effects of tax policies it is necessary to introduce different types of agents in the model.

A model with heterogeneous agents should consider the empirical evidence as suggested by Mankiw (2000). He shows that a significant part of the population lives without virtual wealth, spending the majority of their income and saving only a small portion. Meanwhile there is a small portion of the population with high level of wealth. The wealthiest individuals are able to smooth consumption over time and save a large portion of their wealth.

Thus a model with heterogeneous agents must have, two types of agents: poor and rich, the difference between them is the amount of accumulated wealth and savings rate (marginal propensity to save). In fact the difference in the marginal propensity to save is critical to building a model that considers inequality as endogenous. This is one of the main mechanisms by which fiscal policy can affect income inequality. Kaldor (1955) argues that economic growth can affect inequality through the marginal propensity to save of the agents, because the marginal propensity to save is greater for the rich than the poor. Fiscal policy, therefore can change not only the level of wealth of each agent, but also their marginal propensity to save, affecting simultaneously, income inequality and economic growth.

Muinelo-Gallo and Roca-Sagales (2013) indicates that a fiscal policy can redistribute wealth from the rich, who have marginal productivity of investment relatively low. Due to decreasing returns on individual investments, the poor who has marginal productivity of investment relatively high can not invest more than their limited endowments, which would enhance aggregate efficiency and growth and mainly reduce income inequality.

This relationship between agents' wealth and their relative allocation of time to leisure is also important for a re-distributive fiscal policy. Wealthier agents have a lower marginal utility of wealth, they therefore increase leisure, and reduce labor supply. Given their relative capital endowments, this translates to an endogenously determined distribution of income. As we will see, there exist various structural changes through which a re-distributive fiscal policy affects inequality and the economy as a whole, including an increase in productivity and savings.

4.3.1 Households

A single good is produced in the economy, employing labor H_t and capital K_t. The product can be consumed by families, used by the government, or used to increase the stock of capital. Thus, we can describe the resource constraints and technology of this economy as:

$$C_t + G_t + I_t \leq F(K_t, H_t) \tag{4.9}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{4.10}$$

Where $\delta \in (0,1)$ is the rate at which capital depreciates and G_t is an exogenous sequence of government spending, $F(K_t, H_t)$ is a homogeneous linear production function with positive and decreasing marginal products of capital and labor. I_t is the investment, K_t is the capital in the period t and K_{t+1} is the capital in the period $t+1$. It is sometimes appropriate to replace investment from (4.9) and express the restriction of resources and technology as a single equation:

$$C_t + G_t + K_{t+1} = F(K_t, H_t) + (1-\delta)K_t \tag{4.11}$$

The investment equation also represents the way the transition takes place in this economy, in other words, how capital is accumulated from one period to another. This function is known as the law of motion of capital.

To understand the inequality, we construct a model with two types of agents: poor and rich. The literature on fiscal policy and income inequality usually supposes that the poor (P) consume all their income each period and do not keep savings and consequently wealth. The rich (W) are owners of the capital and are able to smooth consumption between periods, as can be seen in the models of Mankiw (2000), Swarbrick (2012). The model proposed by Judd (1985), for example, has two classes of agents, one class are workers that do not save and an other class called capitalists, who do not work. There is no redistribution in the limit of all these models. Therefore, in these models, inequality is exogenously given.

Assumption 4.3.1. *There is an imperfection in the capital market that makes poor households have lower access to this market. This imperfection in the capital market generates a difference between the level of capital stock between rich and poor. This difference is represented by the level of capital stock and their respective returns and the marginal propensity to save between rich and poor.*

Different from the previous studies, such as Mankiw (2000), Gali, Salido and Valles

(2007). We suppose in this work that both agents can own physical capital and that there is a difference γ in the return to savings for the rich and the poor agent, therefore:

$$R_t^P = R_t^W(1-\gamma) \tag{4.12}$$

Where R_t^P is the return for the poor agent and R_t^W is the marginal product of capital. We also assume $\overline{K}$ to be the minimum level of capital to become a rich agent. So poor agents can also save and accumulate wealth. This difference in the model allows social mobility, in the form in which poor agents can become rich and rich agents may become poor. This change enables analyzing inequality as an endogenous variable in the model, allowing the study of the effects of fiscal policy on inequality.

Lemma 4.3.1. *If $\gamma = 0$, so we have from equation (4.12) that, $R_t^P = R_t^W$, this implies that rich and poor agents have the same return on capital and there would be no imperfections in the capital market, as such the problem of maximizing household utility at the rich and the poor could be treated as a representative agent problem.*

In fact, this type of model has been the most common in the literature about optimal taxation, as we can observe in the studies conducted by Chamley (1986), Aiyagari (1995) and Jones, Manuelli and Rossi (1997). This type of analysis is relevant when analyzing the effects of taxation on economic growth. Lemma 4.3.1 would be verified if assumption 1 does not happen, which does not appear to be corroborated by the empirical evidence.

Lemma 4.3.2. *If $\gamma = 1$, so we have that equation (4.12) becomes: $R_t^P = 0$, this implies in a model that poor agents do not have physical capital. So we have a model with heterogeneous agents, where the poor agent holds no wealth (in the form of rule-of-thumb) and rich agents own all the capital.*

This type of model is considered to have heterogeneous agents, and has been explored more recently in the literature of optimal taxation with a focus on income distribution as we can observe in the studies conducted by Judd (1985), Mankiw (2000) and Swarbrick (2012).

Since the model has two types of agents, it is necessary to define the proportion of the population that corresponds to the poor θ_t which must satisfy the following condition $\theta_t < 1$, so the percentage of the population which will be rich is $(1-\theta_t)$. The initial condition of θ_t constitutes an aggregate state variable at the beginning of economy, but becomes an endogenous variable in the following periods and is determined by fiscal policy in the previous period and the agents decisions.

4.3.2 Rich Agent

In this economy, the rich agent lives infinitely and receives income from providing labor and capital to the representative firm and chooses the amount of consumption and capital investment. Therefore in this problem the rich households want to maximize their utility, subject to a budget constraint:

$$\max_{\{C_t^W, H_t^W, I_t^W, K_{t+1}^W\}} \sum_{t=0}^{\infty} \beta^t U^W(C_t^W, 1-H_t^W) \qquad \beta \in (0,1) \tag{4.13}$$

$$C_t^W \geq 0, \quad K_{t+1}^W \geq 0, \quad H_t^W \in [0,1]$$

$$\text{Given} \quad K_0^W,$$

where C_t^W is the consumption of wealthier households and H_t^W is the hours dedicated to work for wealthier agent in period t, discounted by at inter-temporal rate $0 < \beta < 1$. We will use $1 - H_t^W$ to represent the hours spent on leisure. The households maximize utility subject to the budget constraint:

$$\begin{aligned}(1+\tau_t^{C,W})C_t^W + I_t^W = \quad & (1-\tau_t^{H,W})\omega_t^W H_t^W \\ & +(1-\tau_t^{K,W})R_t^W K_t^W, \qquad \text{if} \quad K_t \geq \overline{K}\end{aligned} \tag{4.14}$$

where C_t^W and ω_t^W is consumption and wage in period t, respectively, R_t^W is the rent rate of capital, K_t^W is the capital stock and δ is depreciation rate of capital. There are three forms of taxation: $\tau_t^{C,W}$, $\tau_t^{H,W}$ and $\tau_t^{K,W}$ on consumption, labor and capital, respectively.

In addition to the budget constraint for the rich agent, there is the capital stock accumulation function, given the variables K_t^W and K_{t+1}^W, such that the capital stock is given by:

$$K_{t+1}^W = I_t^W + (1-\delta)K_t^W \tag{4.15}$$

where I_t is the investment in period t and δ is the depreciation rate. This equation represents a law of motion. This equation can be rewritten to represent the investment I_t^W of the rich agent, so the equation (4.15) becomes $I_t^W = K_{t+1}^W - (1-\delta)K_t^W$.

Definition 4.3.1. Rich agents are defined, in this model, as those agents who have a level of capital higher than or equal to the threshold level $\overline{K}$. Rich agents have a higher return on capital, R_t^W and different rates of labor and capital income taxation, $\tau_t^{H,W}, \tau_t^{K,W}$.

We can replace the investment (I_t^W) equation in the budget constraint of the rich agent to facilitate the derivation of the Lagrangian, which allows us to eliminate an equation and a term of the maximization problem. Based on equations (4.13), (4.14) and (4.15) we can build the Lagrangian function for the rich agent. Therefore, the First Order Necessary Conditions - FONCs for the rich agent are, $\forall \quad t$:

$$\frac{\partial \mathscr{L}^W}{\partial C_t^W} = \beta^t U_C(C_t^W, 1-H_t^W) - \lambda_t^W(1+\tau_t^{C,W}) = 0 \tag{4.16}$$

$$\frac{\partial \mathscr{L}^W}{\partial H_t^W} = -\beta^t U_H(C_t^W, 1-H_t^W) + \lambda_t^W(1-\tau_t^{H,W})\omega_t = 0 \tag{4.17}$$

$$\frac{\partial \mathscr{L}^W}{\partial K_{t+1}^W} = -\lambda_t^W + \lambda_{t+1}^W\Big[(1-\tau_{t+1}^{K,W})R_{t+1} + (1-\delta)\Big] = 0 \tag{4.18}$$

$$\frac{\partial \mathscr{L}^W}{\partial \lambda_t^W} = -(1+\tau_t^{C,W})C_t^W - I_t^W + (1-\tau_t^{H,W})\omega_t H_t^W + (1-\tau_t^{K,W})R_t^W K_t^W = 0 \tag{4.19}$$

Where U_C is the marginal utility of consumption equation (4.16), U_H is the marginal utility of labor (4.17) and λ is the Lagrange multiplier. Isolating λ_t^W in the first two derivatives, equations (4.16) and (4.17), we can find the labor supply:

$$\frac{U_H(C_t^W, 1-H_t^W)}{U_C(C_t^W, 1-H_t^W)} = \frac{(1-\tau_t^{H,W})\omega_t}{(1+\tau_t^{C,W})} \tag{4.20}$$

The labor supply shows how the rich agent trades labor for leisure, and also shows the relation between two types of tax and how wages are important for determining this ratio. Dividing equations (4.16) by (4.17) and substituting in to equation (4.18) we, can find the Euler Equation:

$$\frac{1}{\beta} = \frac{U_C(C_{t+1}^W, 1-H_{t+1}^W)}{U_C(C_t^W, 1-H_t^W)} \frac{(1+\tau_t^{C,W})}{(1+\tau_{t+1}^{C,W})}\Big[(1-\tau_{t+1}^{K,W})R_{t+1}^W + (1-\delta)\Big] \tag{4.21}$$

Equation (4.21) is the well-known Euler Equation. It shows that the inter-temporal marginal rate of substitution between consumption at period t and period $t+1$ equals the relative price between both consumptions. Note that even if consumption tax is constant over time, taxation on capital income distorts the relative price of inter-temporal consumption (CHAMLEY, 1986).

4.3.3 Poor Agent

The poor agent in this economy has a similar maximization problem to the rich agent, that is, maximizing a utility function subject to a budget constraint. Furthermore, the poor agent lives infinitely and receives income from providing labor and capital to the representative firm and a transfer of government. Every period, the poor agent choose the amount of consumption, labor and capital investment.

$$\max_{\{C_t^P, H_t^P, I_t^P, K_{t+1}^P\}} \sum_{t=0}^{\infty} \beta^t U^P(C_t^P, 1-H_t^P) \qquad \beta \in (0,1) \tag{4.22}$$

$$C_t^P \geq 0, \quad K_{t+1}^P \geq 0, \quad H_t^P \in [0,1]$$

$$\text{Given} \quad K_0^P,$$

where the variables are the same as for case of the rich agent and the superscript P indicates the type (Poor) of agent.

Definition 4.3.2. Poor agents are defined, in this model, as those agents who have a level of capital lower than or equal to the threshold level $\overline{K}$ and receives a transfer T_t^P from the government. Poor agents have a lower return of capital, R_t^P and a different scheme of labor and capital income taxation, $\tau_t^{H,P}, \tau_t^{K,P}$.

The poor agent receive income from work and savings, and wants to maximize his utility subject to a budget constraint:

$$\begin{aligned}(1+\tau_t^{C,P})C_t^P + I_t^P = \quad & (1-\tau_t^{H,P})\omega_t^P H_t^P \\ & +(1-\tau_t^{K,P})R_t^P K_t^P + T_t^P, \quad \text{if} \quad K_t < \overline{K},\end{aligned} \tag{4.23}$$

where T_t^P represents the government's transference funded by raising tax from the rich agent. There are three forms of tax that are different from the rich agent: $\tau_t^{C,P}$, $\tau_t^{H,P}$ and $\tau_t^{K,P}$ on consumption, labor and capital respectively. The other variables are the same as for the case of the rich agent. Note that both problems, rich and poor are different only because of fiscal policy and the interest rate received on savings ($\tau_t^{C,W}, \tau_t^{C,P}$, $\tau_t^{H,W}, \tau_t^{H,P}$, $\tau_t^{K,W}, \tau_t^{K,P}$ and $R_t^P = R_t^W[1-\gamma]$). Based on the equations of the poor agent's problem, we can find the FONCs, $\forall t$:

$$\frac{\partial \mathscr{L}^P}{\partial C_t^P} = \beta^t U_C(C_t^P, 1-H_t^P) - \lambda_t^P(1+\tau_t^{C,P}) = 0 \tag{4.24}$$

$$\frac{\partial \mathscr{L}^P}{\partial H_t^P} = -\beta^t U_H(C_t^P, 1 - H_t^P) + \lambda_t^P (1 - \tau_t^{H,P})\omega_t = 0 \tag{4.25}$$

$$\frac{\partial \mathscr{L}^P}{\partial K_{t+1}^P} = -\lambda_t^P + \lambda_{t+1}^P \Big[(1 - \tau_{t+1}^{K,P})R_{t+1}^P + (1 - \delta)\Big] = 0 \tag{4.26}$$

$$\frac{\partial \mathscr{L}^P}{\partial \lambda_t^P} = -(1 + \tau_t^{C,P})C_t^P - I_t^P + (1 - \tau_t^{H,P})\omega_t H_t^P + (1 - \tau_t^{K,P})R_t^P K_t^P + T_t^P = 0 \tag{4.27}$$

From the FONCs we can find the marginal utility of consumption and leisure. Dividing equation (4.24) by (4.25), we can find the labor supply of the poor agent:

$$\frac{U_H(C_t^P, 1 - H_t^P)}{U_C(C_t^P, 1 - H_t^P)} = \frac{(1 - \tau_t^{H,P})\omega_t}{(1 + \tau_t^{C,P})} \tag{4.28}$$

Equation (4.28) shows how the poor agent trades between both goods consumption and leisure at period t. The marginal rate of substitution between consumption and leisure equals to the relative prices between consumption and leisure where the price of consumption good at period t is normalized to one and the wage rate, net of taxation, is the opportunity cost of leisure. Note that as long as taxation on consumption is different from taxation on labor income for poor agents, the taxation distorts the relative price of both goods. Dividing equation (4.24) by (4.25) and substituting in to equation (4.26), we can find the Euler Equation:

$$U_C(C_t^P, 1 - H_t^P) = \beta U_C(C_{t+1}^P, 1 - H_{t+1}^P) \frac{(1 + \tau_t^{C,P})}{(1 + \tau_{t+1}^{C,P})} \Big[(1 - \tau_{t+1}^{K,P})R_{t+1}^P + (1 - \delta)\Big]. \tag{4.29}$$

Equations (4.28) and (4.29) are the counterparts of equations (4.20) and (4.21) for the rich agents. The Euler equation shows the relationship between inter-temporal consumption and all types taxes, return on capital and depreciation.

4.3.4 Firm

The production function of the firm can be assumed as Cobb-Douglas: $Y_t = F(H_t, K_t)$. Usually the marginal products are equal to factor prices, which characterizes a firm with zero profits. A representative firm rents capital and employs labor to produce a consumption good, such that we have the following equation:

$$\Pi_t = AH_t^{\alpha}K_t^{1-\alpha} - \omega_t H_t - R_t K_t \qquad (4.30)$$

where Π_t is the profit, H_t is household labor provided at time t and K_t is the capital stock, ω_t e R_t are wages and capital rental rate respectively. From the First Order Necessary Conditions - FONCs, $\forall t$ we have:

$$\omega_t = \alpha A \left(\frac{H_t}{K_t}\right)^{\alpha-1} \qquad (4.31)$$

$$R_t = (\alpha-1)A\left(\frac{H_t}{K_t}\right)^{\alpha} \qquad (4.32)$$

Equations (4.31) and (4.32) are the first order conditions for the firm's problem, with total factor productivity assumed to be constant. The first equation show that the wage is equal to the marginal product of labor, and the second equation show that the rental rate of capital is equal to the marginal product of capital.

4.3.5 Government

The government's goal is to the maximize welfare of the economy subject to raising set revenues through distortionary taxation.

The government collects taxes on household income from labor and capital to exogenously fund transfers to the poor. The government also collects taxes on consumption. The government is supposed to operate with a balanced budget constraint:

$$\begin{aligned} G_t + T_t^P &= (1-\tau_t^{H,W})\omega_t H_t^W + (1-\tau_t^{H,P})\omega_t H_t^P \\ &+(1-\tau_t^{K,W})R_t^W K_t^W + (1-\tau_t^{K,P})R_t^P K_t^P \\ &+(1-\tau_t^{C,W})C_t^W + (1-\tau_t^{C,P})C_t^P \end{aligned} \qquad (4.33)$$

where $T_t = \theta_t T_t^P$, is the transfer to the poor agent and G_t is spending of government given exogenously. The government tax labor, capital income and consumption of the both agents with: $\tau_t^{H,W}$, $\tau_t^{H,P}$, $\tau_t^{K,W}$, $\tau_t^{K,P}$, $\tau_t^{C,W}$, $\tau_t^{C,P}$, for the rich and poor agent respectively.

4.3.6 Competitive Equilibrium

As the model incorporates two types of agents, it is necessary to define the behavior of the variables in the model. The variables C_t, K_t and H_t are divided between the two types of agents, so the aggregates should be:

$$
\begin{aligned}
C_t &= \theta_t C_t^P + (1-\theta_t) C_t^W && (4.34)\\
H_t &= \theta_t H_t^P + (1-\theta_t) H_t^W && (4.35)\\
K_t &= \theta_t K_t^P + (1-\theta) K_t^W && (4.36)\\
T_t &= \theta T_t^P && (4.37)
\end{aligned}
$$

Every period, the economy is bound by the resource constraint $Y_t = C_t + I_t + G_t$. In the competitive equilibrium, government policy (tax allocations) and prices are set based on the following conditions:

- Families maximize their utility subject to the restrictions for both types of agents.
- Firms maximize profit using capital and labor.
- The government budget is balanced.

The government solves the Ramsey problem to find the allocations, prices and fiscal policy that maximizes social welfare. The policy affects prices $\{R, W\}_{t=0}^{\infty}$, and allocations $\{H^W, H^P, C^W, C^P, K^W, K^P\}_{t=0}^{\infty}$. The equilibrium conditions are used to solve the model, and find the solution of Ramsey's problem.

Assuming there is a steady state condition in which C_t is constant, $C_t = C_{t+1}$, $H_t = H_{t+1}$ and $\tau_t^C = \tau_{t+1}^C$ then we can modify the Euler equation of the rich agent (4.21) so that we have:

$$
(1-\tau^{K,W})R^W + (1+\delta) = \frac{1}{\beta^t} \tag{4.38}
$$

So we have that taxing capital diminishes the steady state for the rich agents. This result is the same as found by the majority of studies done in the literature, is consistent with theory. This famous equation shows how technology, represented by δ, time preferences β and the rate of return of the rich agent's capital R^W which is equal to the marginal product of the rich agent's capital F_K^W, are important to determine the steady state level of capital. Another important point to note is that tax consumption and labor does not affect the steady state.

It is easy to prove that the same result can also be found for the poor agent, so modifying the Euler equation of the poor agent, equation (4.29), we can find.

$$(1-\tau^{K,P})R^P + (1+\delta) = \frac{1}{\beta^t}. \tag{4.39}$$

This equation shows how technology represented by δ, time preference β and the rate of return of the poor agent's capital R^P, which is equal to the marginal product of the poor agent's capital F_K^P are important to determine of steady state level of capital. Notice that even in steady state, we have the condition $R^P = R^W(1-\gamma)$.

Proposition 4.3.1. *Given the differences between rich and poor, we can show that the marginal productivity of the poor agent's capital is greater than that of the rich agent, so we have* $k^P < k^W$.

Demonstration 4.3.1. *The firm considers capital to be* $K = k^P + k^W$*, not differentiating between the capital provided by the rich and the poor agents. This is because the different types of capital(*k_s*) are perfect substitutes. Considering the firm's FONC, we have:*

$$R = F_K(K,H) \tag{4.40}$$

Following the same procedures adopted in the literature we can consider the marginal productivity of capital in terms per capta:

$$R = F_K(K,H) = f_k(k) \quad with \quad k = \frac{K}{H} \tag{4.41}$$

Where $f_k(k)$ *is the marginal product of capital per capta. In equilibrium considering the two types of agents we have that the capital per capta is:*

$$k = \theta k^P + (1-\theta)k^W \tag{4.42}$$

Thus $R = f_k\left[\theta k^P + (1-\theta)k^W\right]$*. Also in equilibrium we have the marginal productivity of capital of each agent is:*

$$f_{k^P} \quad = \quad f_k\frac{\partial k}{\partial k^P} = \theta f_k = \theta R^P \tag{4.43}$$

$$f_{k^W} \quad = \quad f_k\frac{\partial k}{\partial k^W} = (1-\theta)f_k = (1-\theta)R^W \tag{4.44}$$

Where f_k represents the marginal product of capital, $\partial k/\partial k^P$ is the participation of the poor in the marginal product of capital and $\partial k/\partial k^W$ the participation of the rich in the marginal product of capital, that is, θ and $(1-\theta)$, respectively. If we consider that households in the steady state have the same level of capital depreciation $(\delta = 1]$, for simplicity, we assume that there is no taxation and that $\theta \in (0,1)$, so we have:

$$\text{Rich:} \qquad R^W = \frac{1}{\beta} \quad \Rightarrow \quad \frac{f_{k^W}}{1-\theta} = \frac{1}{\beta} \tag{4.45}$$

$$\text{Poor:} \qquad R^P - \gamma = \frac{1}{\beta} \quad \Rightarrow \quad \frac{f_{k^P}}{\theta} - \gamma = \frac{1}{\beta} \tag{4.46}$$

Equating equations (4.45) and (4.46), we have:

$$\frac{f_{k^P}}{\theta} - \gamma = \frac{f_{k^W}}{(1-\theta)} \qquad \text{for} \qquad \theta \in (0,1) \tag{4.47}$$

This result shows that there is a difference in the marginal productivity of capital between agents. For this equality to prevail, we must:

$$\frac{f_{k^P}}{\theta} > \frac{f_{k^W}}{(1-\theta)} \tag{4.48}$$

We conclude that the marginal productivity of the capital of the poor agent is greater than that of the rich agent, this is only possible given the different endowments of capital between them, so that $k^P < k^W$.

4.4 Ramsey Problem

The literature of optimal taxation has the objective of finding a tax system that maximizes a social welfare function of the economy, subject to a set of constraints. In this section, we formulate the problem of taxation dynamically. This approach is known in the literature as Ramsey's problem, and the commonly found solution is called the Ramsey plan.

The government's objective is to maximize households' welfare subject to raising set revenues through distortionary taxation. By creating an ideal fiscal policy, the government takes into account the reactions of consumers and firms to the tax system. The problem is to determine the optimal sequences for the three types of tax, on capital income, labor income and consumption in a non-stochastic economy, using a competitive equilibrium version of the basic neoclassical growth model . We assume that the government has a balanced budget[1].

Definition 4.4.1. A competitive equilibrium is a feasible allocation, as in equation (4.11), a price system $\{R,W\}_{t=0}^{\infty}$, allocations $\{H^W,H^P,C^W,C^P,K^W,K^P\}_{t=0}^{\infty}$ and a government policy such that, i) given the price system and the government policy, the allocation solves both the firm's problem and the household's problem; and ii) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (4.33).

Following the example of Chamley (1986) and Sargent and Ljungvist (2000), we can rewrite the budget constraint of the government in order to facilitate the derivation of the central planner's problem. It consists of using the FONC's of the firm and the equilibrium outcomes in factor markets. We take equation (4.33) and first we add and subtract $\omega_t H_t$ and $R_t K_t$, and therefore have:

$$\begin{aligned} G_t + T_t^P &= F(H_t^W, K_t^W, H_t^P, K_t^P) - (1-\tau_t^H)\omega_t\left[\theta_t H_t^P + (1-\theta_t)H_t^W\right] \\ &-(1-\tau_t^K)R_t\left[\theta_t K_t^P + (1-\theta_t)K_t^W\right] - (1-\tau_t^C)\left[\theta_t C_t^P + (1-\theta_t)C_t^W\right] \end{aligned} \tag{4.49}$$

Based on equation (4.49) and the equations from the FONC's for agents poor and rich, the firm and the Euler conditions, then it is possible to build the problem of Ramsey using an estimation strategy similar to Sargent and Ljungvist (2000). But unlike these authors, we have not included the resource constraint, because if government and household budget constraints hold. this implies that the resource constraint also holds. The Ramsey problem in Lagrangian form is shown below, where the objective function is constrained by the household first order conditions, the household budget constraints and the government budget constraints.

$$\begin{aligned} \mathscr{L} &= \sum_{t=0}^{\infty}\beta^t\Big\{V_t(U_t^W,U_t^P) + \varphi_t\left[Eq.(4.49)\right] + \lambda_t^P\left[Eq.(4.23)\right] + \lambda_t^W\left[Eq.(4.14)\right] \\ &\quad +\phi_t^P\left[Eq.(4.28)\right] + \phi_t^W\left[Eq.(4.20)\right] + \Psi_t^P\left[Eq.(4.29)\right] + \Psi_t^W\left[Eq.(4.21)\right]\Big\} \end{aligned}$$

where V_t is the sum of the utility functions of all agents, weighted by government favoritism for each agent, so we have $\sum_{t=0}^{\infty} V_t = \Upsilon U_t^P + (1-\Upsilon)U_t^W$. Based on the Lagrangian of the Ramsey's problem, it is possible derive the Lagrangian with respect to the following variables: C H, K, τ^C, τ^H, τ^K and T for both agents, also is derived the same variables for periods $t=0$ and $t>1$. All mathematical work and the main results are shown in the appendix. We now can concentrate on finding the optimal tax on capital income. Utilizing the derivative of the

[1]The result find here hold if the the government has debt in each period.

Lagrangian with respect K_{t+1}^W and K_{t+1}^P, we find the condition of optimal taxation on capital income, τ^K, which for the rich agent is:

$$\beta\varphi^W(1-\theta)\Big[F_K^W-(1-\tau^{K,W})R^W\Big]+\beta\lambda^W\Big[1+(1-\tau^{K,W})R^W+(1-\delta)\Big]=\lambda^W \quad (4.50)$$

It is worth mentioning that despite this result being presented for the rich agent, a similar result can be found for the poor agent:

$$\beta\varphi^P\theta\Big[F_K^P-(1-\tau^{K,P})R^P\Big]+\beta\lambda^P\Big[1+(1-\tau^{K,P})R^P+(1-\delta)\Big]=\lambda^P \quad (4.51)$$

As in Sargent and Ljungvist (2000) and Swarbrick (2012), λ^W and λ^P are interpreted as the marginal social value of goods for the poor and rich agents respectively, and are strictly positive. Furthermore, φ can be interpreted as the marginal value of reducing taxes and is nonnegative. Replacing the Euler equation of the rich agent (4.28) in the steady-state in equation (4.51) and remembering that in the steady-state we have: $\Big[(1-\tau^{K,W})R^W+(1-\delta)\Big]=1/\beta$ and also, $F_K^W=R^W$, from the expression (4.50) we have:

$$\varphi^W(1-\theta)\Big[R^W-(1-\tau^{K,W})R^W\Big]=0 \quad (4.52)$$

Using the same procedure for the poor agent, taking the Euler equation (4.29) replacing in the steady-state and remembering that the steady-state: $\Big[(1-\tau^{K,P})R^P+(1-\delta)\Big]=1/\beta$ and also, $F_K^P=R^P$, we have from the expression (4.51):

$$\varphi^P\theta\Big[R^P-(1-\tau^{K,P})R^P\Big]=0 \quad (4.53)$$

This implies from equations (4.52) and (4.53) that $1-(1-\tau^{K,W})=0$ and $1-(1-\tau^{K,P})=0$, respectively and thus we conclude that $\tau^{K,W}=0$ and $\tau^{K,P}=0$. Thus it can be concluded that the optimal capital income tax is zero in the steady state, a result that is equivalent for the poor and rich agents.

Proposition 4.4.1. *In an economic growth model with heterogeneous agents and a non-stochastic economy, in the steady state the optimal tax rate on income from physical capital should be equal to zero for both type of agents, poor and rich.*

This result does not alter the assumption made in the model that the return (R_t^W) the capital of the rich agent is higher than the poor agent's. This finding is corroborated by previous studies, most notoriously by: Judd (1985), Chamley (1986) and Sargent and Ljungvist (2000), and is consistent with studies of optimal taxation[2].

Note that the result of zero taxation on capital is independent of the government bias to the poor or the rich, represented by parameter Υ in our model, as previously seen the parameter $0 < \Upsilon < 1$ indicates the government's favoritism for the rich or poor agents. So the transfers made by the government to the poor agent should be financed by a combination of flat rate taxes on consumption and labor income.

However, there are a few studies showing different results from proposition 4.4.1 , mainly because under certain assumptions, taxes on capital income would be optimally above zero, such as in Aiyagari (1995) who works with an incomplete market model. Similar results can also be found in other studies, such as Stiglitz (1987) and Jones, Manuelli and Rossi (1997). This type of result is usually found when some kind of good is not taxed. Therefore government spending, in particular the transfer to the poor should be financed by a combination of all taxes.

A similar study was conducted by Giavazzi and McMahon (2012), which presents a model in which, contrary to the theory and other studies, low-income families tend to respond more to fluctuations cutting consumption and working more hours following a government spending shock, for example. This analysis suggests that the wealthiest families react more to fluctuations, although they can smoothen their spending over time.

Having focused on optimal tax policy, we turn our attention turns to the process of change in the composition of taxes on labor and capital. Domeij and Heathcote (2004) argue that combining different types of taxes implies significant distributional changes and this type of combination is used in the context of tax reform, as shown in abundance in the literature.

Swarbrick (2012) shows that changing the government preferences (Υ) did affect fiscal policy as shown. The government becoming more favorable towards the poor caused an increase in transfers to these agents. This was financed mainly by taxing labor but in part by increasing taxes on capital income. Clearly, even in the case of a government who is very biased towards the rule-of-thumb agents, the optimal tax rate on capital income should be close to zero.[3]

Proposition 4.4.2. *Given the proposed model and considering that there is a competitive equilibrium such that $Y^* = F(H_t^W, K_t^W, H_t^P, K_t^P)$, where there are no transfers from the government to the poor agent. A result Pareto optimum can be achieved in particular if an appropriate redistribution of wealth is made so that we have a competitive equilibrium $Y^T = F(H_t^W, K_t^W, H_t^P, K_t^P)$ where the government makes a transfer T_t^P of income to the poor.*

[2]Sargent and Ljungvist (2000) point out that this model can be generalized to a finite amount/number of different classes of agents, N.

[3]It is possible that, as the preference becomes too strong towards the poor, there is political pressure to increase government transfers financed by taxes on capital income. This is a non-optimal policy and causes the model to fail.

Demonstration 4.4.1. *The second welfare theorem states that for any level of Pareto optimal utility, there is a transfer of wealth (T_t^P) satisfying the resource constraint (4.11), such that a competitive equilibrium is achieved after a redistribution of wealth is associated with utility levels prior to the transfer of income.*

Mas-Colell, Whinston and Green (1995) point out that the second theorem states that, out of all possible Pareto optimal outcomes, one can achieve any particular one by enacting a lump-sum wealth redistribution and then letting the market take over. In our model we can define mathematics two equilibrium:

$$\theta_t = \frac{F(H_t^W, K_t^W, H_t^P, K_t^P) - G_t}{\Omega} \tag{4.54}$$

$$\theta_t = \frac{F(H_t^W, K_t^W, H_t^P, K_t^P) - G_t + T_t^P}{\Omega} \tag{4.55}$$

$$\begin{aligned} \Omega &= (1-\tau_t^{H,W})\omega_t H_t^W + (1-\tau_t^{H,P})\omega_t H_t^P + (1-\tau_t^{K,W})R_t^W K_t^W + (1-\tau_t^{K,P})R_t^P K_t^P \\ &\quad + (1-\tau_t^{C,W})C_t^W + (1-\tau_t^{C,P})C_t^P \end{aligned}$$

Equation (4.54) shows the equilibrium condition when there is no transfer from the government to the poor agent and equation (4.55) shows the equilibrium condition when there is transfer.

We can also point out that from the Ramsey problem emerge an existence of a dead weight loss, present by the multiplier (φ) of government budget. This excess burden or allocative inefficiency can be correct by an optimal taxations scheme, which include transfers to the poor agent.

However, as it is possible to see, the results presented in these studies are valid in the long run with the economy in the steady state, and does not explain how macroeconomic variables such as consumption, labor wage *etc*, react during the *Real Business Cycle - RBC*. Also optimal steady-state taxes do not say much about how long it takes to reach the zero tax on capital income and mainly how taxes and re-distributive transfers are set during the transition period.

4.5 Conclusions and Remarks

This paper solves an optimal fiscal policy in a model with agent heterogeneity and inequality. By introducing poor and rich agents, we show that the steady state optimal tax on capital income in the long run should always be zero, regardless of the governments favoritism towards particular agents.

The fiscal policy has two main channels through which to affect the economy, mainly the following macroeconomic variables: aggregate demand and the level of economic activity, savings and investment and the distribution of income. First the government can increase or decrease spending and change the amount of lump-sum transfer' made in this economy. Second, the government can changes level and composition of taxation (flat-rate) on consumption, labor income and capital income. These changes show that fiscal policy can reduce income inequality and that the level of inequality directly impacts economic growth.

The existence of flat-rate taxes makes the economy does not operate at the optimal, because the existence of a dead weight of taxation (φ_t). The analytical solution found in this paper suggests that, in the steady state, there is an optimal tax composition, where the government should be financed by the transfers from different combinations of taxes on consumption and labor income. This tax composition is capable of reducing income inequality and increasing economic growth at the same time.

So not all taxes should be zero in the steady state. This result is usually explained by zero profit conditions. This condition comes from the assumption of linearity in the accumulation technologies. Note that the result of zero taxation on capital is independent of the government bias to the poor or the rich, represented by parameter Υ in our model.

Technological shocks could impact the marginal product of both agents, affecting the incomes from labor and capital and also consumption. Therefore it can be said that technological shocks impact the income inequality in the steady state. For a better understanding of these relationships, it would be necessary to calibrate the model.

Appendix

4.A Equilibrium conditions

Is note necessary to use resource constraint equations (4.11), that is: $C_t + G_t + T_t + K_{t+1} = F(K_t, H_t) + (1-\delta)K_t$ to determine the equilibrium conditions which are used to solve the model, this conditions are:

$$U_H\left(C_t^W, 1-H_t^W\right)\left(1+\tau_t^{C,W}\right) = U_C\left(C_t^W, 1-H_t^W\right)\left(1+\tau_t^{H,W}\right)\omega_t \tag{4.56}$$

$$U_C\left(C_t^P, 1-H_t^P\right)\left(1+\tau_t^{C,P}\right) = U_H\left(C_t^P, 1-H_t^P\right)\left(1+\tau_t^{H,P}\right)\omega_t \tag{4.57}$$

$$\frac{1}{\beta} = \frac{U_C\left(C_{t+1}^P, 1-H_{t+1}^P\right)}{U_C\left(C_t^P, 1-H_t^P\right)}\frac{\left(1+\tau_t^{C,P}\right)}{\left(1+\tau_{t+1}^{C,P}\right)}\left[\left(1-\tau_{t+1}^{K,P}\right)R_{t+1}^P + (1-\delta)\right] \tag{4.58}$$

$$\frac{1}{\beta} = \frac{U_C\left(C_{t+1}^W, 1-H_{t+1}^W\right)}{U_C\left(C_t^W, 1-H_t^W\right)}\frac{\left(1+\tau_t^{C,W}\right)}{\left(1+\tau_{t+1}^{C,W}\right)}\left[\left(1-\tau_{t+1}^{K,W}\right)R_{t+1}^W + (1-\delta)\right] \tag{4.59}$$

$$\left(1+\tau_t^{C,W}\right)C_t + I_t = \left(1-\tau_t^{H,W}\right)\omega_t H_t^W + \left(1-\tau_t^{K,W}\right)R_t^P K_t^W \tag{4.60}$$

$$\left(1+\tau_t^{C,W}\right)C_t + I_t = \left(1-\tau_t^{H,P}\right)\omega_t H_t^W + \left(1-\tau_t^{K,P}\right)R_t^P K_t^P + T_t^P \tag{4.61}$$

$$\begin{aligned} G_t + T_t^P &= F(H_t^W, K_t^W, H_t^P, K_t^P) - (1-\tau_t^H)\omega_t\left[\theta_t H_t^P + (1-\theta_t)H_t^W\right] \\ &-(1-\tau_t^K)R_t\left[\theta_t K_t^P + (1-\theta_t)K_t^W\right] - (1-\tau_t^C)\left[\theta_t C_t^P + (1-\theta_t)C_t^W\right] \end{aligned} \tag{4.62}$$

$$\omega_t = \alpha A \left(\frac{H_t}{K_t}\right)^{\alpha-1} \tag{4.63}$$

$$R_t = (\alpha-1) A \left(\frac{H_t}{K_t}\right)^{\alpha} \tag{4.64}$$

Assuming there is a steady state in which the variables are constant over time, then we have the following equations:

$$U_C\left(1+\tau^{H,W}\right)\omega - U_H\left(1+\tau^{C,W}\right) = 0 \tag{4.65}$$

$$U_C\left(1+\tau^{H,P}\right)\omega - U_H\left(1+\tau^{C,P}\right) = 0 \tag{4.66}$$

$$\left[(1-\tau^{K,W})R^W + (1-\delta)\right] = \frac{1}{\beta} \tag{4.67}$$

$$\left[(1-\tau^{K,P})R^P + (1-\delta)\right] = \frac{1}{\beta} \tag{4.68}$$

$$(1-\tau^{H,W})\omega H + (1-\tau^{K,W})R^W K^W - (1+\tau^{C,W})C - I_t = 0 \tag{4.69}$$

$$(1-\tau^{H,P})\omega H + (1-\tau^{K,P})R^P K^P - (1+\tau^{C,P})C - I_t = 0 \tag{4.70}$$

$$\begin{aligned} F(H^W, K^W, H^P, K^P) - (1-\tau^H)\omega\left[\theta H^P + (1-\theta)H^W\right] - G - T^P \\ -(1-\tau^K)R\left[\theta K^P + (1-\theta)K^W\right] - (1-\tau^C)\left[\theta C^P + (1-\theta)C^W\right] = 0 \end{aligned} \tag{4.71}$$

$$\alpha A \left(\frac{H}{K}\right)^{\alpha-1} = \omega \tag{4.72}$$

$$(\alpha - 1) A \left(\frac{H}{K}\right)^{\alpha} = R \tag{4.73}$$

4.B Derivatives of Ramsey's problem

Based on the equations from maximization problems of both agents, firm and government we can build the Ramsey's problem using the same strategy of Sargent and Ljungvist (2000). First we implement the Lagrangian, where the principal agent maximize the utility of all agents subject to a set of budget constraints.

$$\mathscr{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left[\Upsilon U_t^P + (1-\Upsilon) U_t^W\right] \\ +\varphi_t \left[\begin{array}{c} F\left(H_t^W, K_t^W, H_t^P, K_t^P\right) - (1-\tau_t^H)\omega_t \left(\theta_t H_t^P + (1-\theta_t) H_t^W\right) \\ -(1-\tau_t^K) R_t \left(\theta_t K_t^P + (1-\theta_t) K_t^W\right) \\ -(1-\tau_t^C)\left(\theta_t C_t^P + (1-\theta_t) C_t^W\right) - G_t - T_t^P \end{array} \right] \\ +\lambda_t^P \left[\begin{array}{c} -\left(1+\tau_t^{C,P}\right) C_t^P - I_t^P + \left(1-\tau_t^{H,P}\right)\omega_t H_t^P \\ +\left(1-\tau_t^{K,P}\right) R_t^P K_t^P + T_t^P \end{array} \right] \\ +\lambda_t^W \left[\begin{array}{c} -\left(1+\tau_t^{C,W}\right) C_t^W - I_t^W + \left(1-\tau_t^{H,W}\right)\omega_t H_t^W \\ +\left(1-\tau_t^{K,W}\right) R_t^W K_t^W \end{array} \right] \\ +\phi_t^P \left[U_C \left(1+\tau_t^{H,P}\right)\omega_t - U_H \left(1+\tau_t^{C,P}\right)\right] \\ +\phi_t^W \left[U_C \left(1+\tau_t^{H,W}\right)\omega_t - U_H \left(1+\tau_t^{C,W}\right)\right] \\ +\Psi_t^P \left[\beta U_{C_{t+1}} \frac{\left(1+\tau_t^{C,P}\right)}{\left(1+\tau_{t+1}^{C,P}\right)} \left[\left(1-\tau_{t+1}^{K,P}\right) R_{t+1}^P + (1-\delta)\right] - U_{C_t}\right] \\ +\Psi_t^W \left[\beta U_{C_{t+1}} \frac{\left(1+\tau_t^{C,W}\right)}{\left(1+\tau_{t+1}^{C,W}\right)} \left[\left(1-\tau_{t+1}^{K,W}\right) R_{t+1}^W + (1-\delta)\right] - U_{C_t}\right] \end{array} \right\}$$

Based on the Lagrangian of the Ramsey's problem, derived in relation to the following variables: C_0^P, H_0^P, K_0^P, C_0^W, H_0^W, K_0^W, τ_{C_0}, τ_{H_0}, τ_{K_0}, and T_0^P, the First Order Necessary Conditions are:

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial C_0^P} = \beta^t \Upsilon U_C^P \quad & - \quad \varphi_0 \theta \left(1-\tau_0^{C,P}\right) - \lambda_0^P \left(1+\tau_0^{C,P}\right) \\ & + \quad \phi_0^P U_{C_0 C_0}^P \left(1+\tau_0^{H,P}\right)\omega_0 - \Psi_0^P U_{C_0 C_0}^P = 0 \end{aligned} \tag{4.74}$$

$$\frac{\partial \mathscr{L}}{\partial H_0^P} = -\beta^t \Upsilon U_H^P \quad + \quad \varphi_0 \left[\theta \omega_0 - \left(1 - \tau_0^{H,P}\right) \omega_0 \theta\right]$$
$$+ \quad \lambda_0^P \left(1 - \tau_t^{H,P}\right) \omega_t - \phi_0^P U_{H_0 H_0}^P \left(1 + \tau_t^{C,P}\right) = 0 \qquad (4.75)$$

$$\frac{\partial \mathscr{L}}{\partial K_0^P} = \varphi_0 \left[\theta R_0^P - \left(1 - \tau_0^{K,P}\right) \theta R_0^P\right] + \lambda_0^P \left[\left(1 - \tau_0^{K,P}\right) R_0^P - \delta\right] = 0 \qquad (4.76)$$

$$\frac{\partial \mathscr{L}}{\partial C_0^W} = (1 - \Upsilon) \beta^t U_C^W - \varphi_0 (1 - \theta) \left(1 - \tau_0^{C,W}\right) - \lambda_0^W \left(1 + \tau_0^{C,W}\right)$$
$$+ \phi_0^W U_{C_0 C_0}^W \left(1 + \tau_0^{H,W}\right) \omega_0 - \Psi_0^W U_{C_0 C_0}^W = 0 \qquad (4.77)$$

$$\frac{\partial \mathscr{L}}{\partial H_0^W} = -(1 - \Upsilon) \beta^t U_H^W + \varphi_0 \left[(1 - \theta) \omega_0 - \left(1 - \tau_0^{H,W}\right) \omega_0 (1 - \theta)\right]$$
$$+ \lambda_0^W \left(1 - \tau_0^{H,W}\right) \omega_t - \phi_0^W U_{H_0 H_0}^W \left(1 + \tau_0^{C,W}\right) = 0 \qquad (4.78)$$

$$\frac{\partial \mathscr{L}}{\partial K_0^W} = \varphi_0 \left[(1 - \theta) R_0^W - \left(1 - \tau_0^{K,W}\right) (1 - \theta) R_0^W\right]$$
$$+ \lambda_0^W \left[\left(1 - \tau_0^{K,W}\right) R_0^W - \delta\right] = 0 \qquad (4.79)$$

$$\frac{\partial \mathscr{L}}{\partial \tau_{C_0}} = \varphi_0 \left[\theta C_0^P + (1 - \theta) C_0^W\right] - \lambda_0^P C_0^P - \lambda_0^W C_0^W + \phi_0^P U_{H_0 H_0}^P + \phi_0^W U_{H_0 H_0}^W$$
$$+ \Psi_0^P U_{C_t C_t} \frac{\left[\left(1 - \tau_t^{K,P}\right) R_t^P + (1 - \delta)\right]}{\left(1 - \tau_t^{C,P}\right)} + \Psi_0^W U_{C_t C_t} \frac{\left[\left(1 - \tau_t^{K,W}\right) R_t^W + (1 - \delta)\right]}{\left(1 - \tau_t^{C,W}\right)} = 0 \qquad (4.80)$$

$$\frac{\partial \mathscr{L}}{\partial \tau_{H_0}} \quad = \quad \varphi_0 \omega_0 H_0 - \lambda_0^W \omega_0 H_0^W - \lambda_0^P \omega_0 H_0^P + \phi_0^W U_C^W \omega_0 + \phi_0^P U_C^W \omega_0 = 0 \qquad (4.81)$$

$$\frac{\partial \mathscr{L}}{\partial \tau_{K_0}} = \varphi_0 R_0 K_0 - \lambda_0^P R_0^P K_0^P - \lambda_0^W R_0^W K_0^W = 0 \tag{4.82}$$

$$\frac{\partial \mathscr{L}}{\partial T_0} = \lambda_0^P - \varphi\theta = 0 \tag{4.83}$$

Based on the Lagrangian of the Ramsey's problem, derived in relation to the following variables in $t > 1$: C_t^W, H_t^W, K_{t+1}^W, C_t^P, H_t^P, K_{t+1}^P, τ_t^C, τ_{H_t}, τ_t^K and T_t the First Order Necessary Conditions are:

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial C_t^W} = \quad & (1-\Upsilon)\beta^t U_{C_t}^W + \varphi_t (1-\theta_t)\left(1-\tau_t^{C,W}\right) - \lambda_t^W \left(1+\tau_t^{C,W}\right) \\ + \quad & \phi_t^W U_{C_t C_t}^W \left(1+\tau_t^{H,W}\right)\omega_t + \Psi_t^W U_{C_t C_t} \\ - \quad & \Psi_{t+1}^W \beta^2 U_{C_t C_t} \frac{\left(1+\tau_t^{C,W}\right)}{\left(1+\tau_{t+1}^{C,W}\right)} \left[\left(1-\tau_{t+1}^{K,W}\right) R_{t+1}^W + (1-\delta)\right] = 0 \end{aligned} \tag{4.84}$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial H_t^W} = \quad & -(1-\Upsilon)\beta^t U_H^W + \varphi_t \left[(1-\theta_t)\omega_t - \left(1-\tau_t^{H,W}\right)\omega_t(1-\theta_t)\right] \\ + \quad & \lambda_t^W \left(1+\tau_t^{H,W}\right)\omega_t + \phi_t^W U_{H_t H_t}^W \left(1+\tau_t^{C,W}\right) = 0 \end{aligned} \tag{4.85}$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial K_{t+1}^W} = \quad & \beta\varphi_{t+1}(1-\theta_t)\left[F_K^W - \left(1-\tau_t^{K,W}\right)R_t^W\right] - \lambda_t^W \\ + \quad & \lambda_{t+1}^W \beta \left[1 + \left(1-\tau_t^{K,W}\right)\right] R_t^W + (1-\delta) = 0 \end{aligned} \tag{4.86}$$

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial C_t^P} = \quad & \Upsilon\beta^t U_{C_t}^P + \varphi_t \theta_t \left(1-\tau_t^{C,P}\right) - \lambda_t^P \left(1+\tau_t^{C,P}\right) \\ + \quad & \phi_t^P U_{C_t C_t}^P \left(1+\tau_t^{H,P}\right)\omega_t + \Psi_t^P U_{C_t C_t} \\ - \quad & \Psi_{t+1}^P \beta^2 U_{C_t C_t} \frac{\left(1+\tau_t^{C,P}\right)}{\left(1+\tau_{t+1}^{C,P}\right)} \left[\left(1-\tau_{t+1}^{K,P}\right) R_{t+1}^P + (1-\delta)\right] = 0 \end{aligned} \tag{4.87}$$

$$\frac{\partial \mathscr{L}}{\partial H_t^P} = -\Upsilon \beta^t U_H^P \quad + \quad \varphi_t \left[\theta_t \omega_t - \left(1-\tau_t^{H,P}\right) \omega_t \theta_t\right] \\ + \quad \lambda_t^P \left(1+\tau_t^{H,P}\right) \omega_t - \phi_t^P U_{H_t H_t}^P \left(1+\tau_t^{C,P}\right) = 0 \qquad (4.88)$$

$$\frac{\partial \mathscr{L}}{\partial K_{t+1}^P} = \qquad \beta \varphi_{t+1} \theta_t \left[F_K^P - \left(1-\tau_t^{K,P}\right) R_t^P\right] - \lambda_t^P \\ + \quad \lambda_{t+1}^P \beta \left[1+\left(1-\tau_t^{K,P}\right)\right] R_t^P + (1-\delta) = 0 \qquad (4.89)$$

$$\frac{\partial \mathscr{L}}{\partial \tau_t^C} = \qquad \varphi_t \left[\theta_t C_t^P + (1-\theta_t) C_t^W\right] - \lambda_t^P C_t^P - \lambda_t^W C_t^W + \phi_t^P U_H^P + \phi_t^W U_H^W \\ + \quad \Psi_{t+1}^P U_{C_t} \beta + \Psi_t^P U_{C_t} \beta \left[\left(1-\tau_t^{K,P}\right) R_t^P + (1-\delta)\right] \\ + \quad \Psi_{t+1}^W U_{C_t} \beta + \Psi_t^W U_{C_t} \beta \left[\left(1-\tau_t^{K,W}\right) R_t^W + (1-\delta)\right] \qquad (4.90)$$

$$\frac{\partial \mathscr{L}}{\partial \tau_{H_t}} = \varphi_t \omega_t H_t \quad - \quad \lambda_t^W \omega_t H_t^W - \lambda_t^P \omega_t H_t^P + \phi_t^W U_C^W \omega_t + \phi_t^P U_C^W \omega_t = 0 \qquad (4.91)$$

$$\frac{\partial \mathscr{L}}{\partial \tau_t^K} \quad = \quad \varphi_t R_t K_t - \lambda_t^P R_t^P K_t^P - \lambda_t^W R_t^W K_t^W + \Psi_t^W \beta U_{C_t}^W R_t + \Psi_t^P \beta U_{C_t}^P R_t = 0 \qquad (4.92)$$

$$\frac{\partial \mathscr{L}}{\partial T_t} \quad = \quad \lambda_t^P - \varphi \theta_t = 0 \qquad (4.93)$$

Based on the equations presented above we now can find the optimal tax on capital K_t, when the economy are in steady state, first to the rich agent W. Taking the equation (4.86) in steady state have the following definition:

$$\beta \varphi^W (1-\theta)\left[F_K^W - \left(1-\tau^{K,W}\right) R^W\right] - \lambda^W + \beta \lambda^W \left[1+(1-\tau^{K,W}) R^W + (1-\delta)\right] = 0 \quad (4.94)$$

$$\varphi^W (1-\theta)\left[F_K^W - (1-\tau^{K,W}) R^W\right] + \lambda^W \left[1+(1-\tau^{K,W}) R^W + (1-\delta)\right] = \frac{1}{\beta} \lambda^W \quad (4.95)$$

Remembering that in steady state expression of Euler's equation (4.67) of the rich agent are: $\Big[(1-\tau^{K,W})R^W+(1-\delta)\Big]=1/\beta$ and also, $F_K^W=R^W$, so from the expression above we have:

$$\begin{aligned}\varphi^W(1-\theta)\Big[F_K^W-(1-\tau^{K,W})R^W\Big] &+ \lambda^W\Big[1+(1-\tau^{K,W})R^W+(1-\delta)\Big]\\ &= \lambda^W\Big[1+(1-\tau^{K,W})R^W+(1-\delta)\Big] \qquad (4.96)\end{aligned}$$

$$\varphi^W(1-\theta)\Big[R^W-(1-\tau^{K,W})R^W\Big]=0 \qquad (4.97)$$

This implies that $1-(1-\tau^{K,W})=0$ and thus we conclude that $\tau^{K,W}=0$ for the rich agent. We now can find the optimal tax on capital K_t, when the economy are in steady state, for the poor agent P. Taking the equation (4.89) in steady state have the following definition:

$$\beta\varphi^P\theta\Big[F_K^P-(1-\tau^{K,P})R^P\Big]-\lambda^P+\beta\lambda^P\Big[1+(1-\tau^{K,P})R^P+(1-\delta)\Big]=0 \qquad (4.98)$$

$$\varphi^P\theta\Big[F_K^P-(1-\tau^{K,P})R^P\Big]+\lambda^P\Big[1+(1-\tau^{K,P})R^P+(1-\delta)\Big]=\frac{1}{\beta}\lambda^P \qquad (4.99)$$

Remembering that in steady state expression of Euler's equation (4.68) of the poor agent are: $\Big[(1-\tau^{K,P})R^P+(1-\delta)\Big]=1/\beta$ and also, $F_K^P=R^P$, so from the expression above we have:

$$\begin{aligned}\varphi^P\theta\Big[F_K^P-(1-\tau^{K,P})R^P\Big] &+ \lambda^P\Big[1+(1-\tau^{K,P})R^P+(1-\delta)\Big]\\ &= \lambda^P\Big[1+(1-\tau^{K,P})R^P+(1-\delta)\Big] \qquad (4.100)\end{aligned}$$

$$\varphi^P\theta\Big[R^P-(1-\tau^{K,P})R^P\Big]=0 \qquad (4.101)$$

This implies that $1-(1-\tau^{K,P})=0$ and thus we conclude that $\tau^{K,P}=0$ for the poor agent.

5. Conclusions and Remarks

Throughout this study we examined the impact of government spending and the tax burden on income inequality and economic growth, as well the optimal taxation over the real business cycle and in the long-run. To achieve this goal, we employed different methodologies both mathematical and econometric at the state and national level in Brazil, as well as a more general theoretical approach.

First we demonstrate the effects of fiscal policy on economic growth and income inequality in the Brazilian states. Based on several panel data models, we present evidence that the tax burden has important effects on economic growth and income inequality, confirming the hypothesis that we posed initially, where this relationship are not linear. The tax burden has a positive effect on economic growth. The analysis of the inequality models also show that tax burden is capable of reducing income inequality. The income inequality also has a negative impact on economic growth.

When the Tax Burden corresponds to 23% of GDP the economic growth is maximum and when the Tax Burden is 19% of GDP the inequality is minimal. By exceeding the limit of 23% the relations between tax burden, economic growth and income inequality reverse. An important finding is that although tax burden has positive effects on growth and inequality, it should be set considering the particularities of each region, especially the level of income.

Second, using dynamic programming, we show how the optimal taxation should be set over the businesses cycle. Our numerical simulations show that in a highly unequal economy the optimal tax rates on capital income and labor income respond to a shock in government spending in opposite ways, if the government has a balanced budget. In addition we find evidence of the existence of a crowding-out effect.

When we compare the two types of shock, the wealthy agent is in a better situation in both, in part because is able to smooth consumption over time and in part because the increase in productivity. The poor agent seems to benefit only in the case of a supply shock. Another important finding is that the composition of the government's budget changes according to the favoritism of the government ranges from rich to poor. Also, we present evidence that reducing poverty can increase product, eliminate the necessity of transfers and reduce considerably the fluctuations of tax revenue.

Third we propose a new model in which the inequality can be analyzed endogenously. In this model, fiscal policy has two main channels through which affect the economy, mainly the following macroeconomic variables: aggregate demand and the level of economic activity, savings and investment, and the distribution of income. Based on the analytical approach, we show that in the steady state, optimal tax on capital should always be zero regardless of the governments favoritism with the fiscal policy, rich or poor.

However in steady state not all taxes should be zero, so the government should finance the transfers by different combinations of taxes on consumption and labor. Technological shocks impact the marginal products of both agents, and consequently their remunerations, which changes the income gap between the agents.

In all models we show evidence that in a country with high level of income inequality such as Brazil, fiscal policy is capable of decreasing on income inequality and is also capable of increasing economic growth. Nevertheless all studies also present evidence of the existence of an optimal fiscal policy, showing the limits of taxation. When these limits are exceeded, the fiscal policy becomes harmful to the economy. Reducing inequality is important for increasing economic growth and reducing fluctuations causes by productivity shocks and government spending.

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