THIAGO CORNI FERREIRA

THE EFFECT OF A CARBON PERMIT MARKET ON THE SIZE OF STABLE COALITIONS OF AN INTERNATIONAL ENVIRONMENTAL AGREEMENT ON CLIMATE CHANGE – A DIFFERENTIAL GAME APPROACH

Dissertation submitted to the Applied Economics Graduate Program of the Universidade Federal de Viçosa in partial fulfillment of the requirements for the degree of *Magister Scientiae*.

Advisor: Viviani Silva Lírio
Co-advisors: Leonardo Bornacki de Mattos
Ian Michael Trotter

VIÇOSA – MINAS GERAIS
2022

Ficha catalográfica elaborada pela Biblioteca Central da Universidade Federal de Viçosa - Campus Viçosa

T

Ferreira, Thiago Corni, 1991-

F383e
2022

The effect of a carbon permit market on the size of stable coalitions of an International Environmental Agreement on climate change: a differential game approach / Thiago Corni Ferreira. – Viçosa, MG, 2022.

1 dissertação eletrônica (61 f.): il. (algumas color.).

Texto em inglês.

Inclui apêndice.

Orientador: Viviani Silva Lírio.

Dissertação (mestrado) - Universidade Federal de Viçosa, Departamento de Economia Rural, 2022.

Referências bibliográficas: f. 59-61.

DOI: https://doi.org/10.47328/ufvbbt.2022.241

Modo de acesso: World Wide Web.

1. Tratados comerciais. 2. Mercado de emissão de carbono - Cooperação internacional - Modelos matemáticos. 3. Jogos diferenciais. I. Lírio, Viviani Silva, 1971-. II. Universidade Federal de Viçosa. Departamento de Economia Rural. Programa de Pós-Graduação em Economia Aplicada. III. Título.

CDD 22. ed. 382.91

Bibliotecário(a) responsável: Euzebio Luiz Pinto CRB 6/3317

THIAGO CORNI FERREIRA

THE EFFECT OF A CARBON PERMIT MARKET ON THE SIZE OF STABLE COALITIONS OF AN INTERNATIONAL ENVIRONMENTAL AGREEMENT ON CLIMATE CHANGE – A DIFFERENTIAL GAME APPROACH

Dissertation submitted to the Applied Economics Graduate Program of the Universidade Federal de Viçosa in partial fulfillment of the requirements for the degree of *Magister Scientiae*.

APPROVED: February 23, 2022

Assent:

Thiago Corni Ferreira
Author

Viviani Silva Lirio
Advisor

ACKNOWLEDGMENTS

Agradeço a Deus por mais esta etapa cumprida. Gostaria de agradecer à minha mãe, Isabel Cristina Corni, por ter sempre me apoiado nos meus objetivos, minhas mudanças de percurso, indecisões, e eventuais fracassos. Esse apoio incondicional me permitiu chegar ao dia de hoje. Agradeço ao meu pai, Genesio Ferreira, que infelizmente não pode ver onde cheguei mas tenho certeza que estaria orgulhoso pois sempre foi um amante da ciência. Agradeço ao meu amigo Rodrigo de Vasconcellos Viana Medeiros, cuja amizade já perdura por anos e que por indicação dele, vim para a UFV. Agradeço ao meu amigo Gabriel Alberto Pimentel, que embora não tenha participado diretamente desse processo, é um amigo que sei que posso contar pro resto da vida. Agradeço ao meu amigo Williams, que tentou botar juízo na minha cabeça na hora de escolher o tema desta dissertação e no fim ele estava certo. Agradeço à professora e minha orientadora Viviani Silva Lírio, que sem sua orientação eu não teria chegado até aqui, essa dissertação não existiria. Ajudou-me no meu período mais turbulento dentro do programa, dispôs-se a me ajudar em todos os momentos, abdicando até mesmo de férias para me atender. Agradeço ao Professor Ian Michael Trotter pelos excelentes comentários que melhoraram e valorizaram meu trabalho. Agradeço ao professor Leonardo que se dispôs a ler e contribuir com meu trabalho, meus artigos, sempre com comentários relevantes. Agradeço à Margarida, secretária do programa, tanto por sua eficiência e por ter torcido por mim. Agradeço a todos os colegas de classe, Igor, Ana, Pedro, João, Lucas, Lucas, Renata, Miriã, Junior, Jayme e Erickson. Agradeço ao Professor Marcelo Braga pelas excelentes aulas de Organização Industrial e pelos comentários na fase de projeto. Agradeço ao professor Denis da Cunha pelos comentários na fase do projeto. E por fim agradeço à UFV, por ter me dado uma segunda oportunidade.

Este estudo foi financiado em parte pela Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Código de Finanças 001.

Ao Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), pela concessão da bolsa de estudos.

ABSTRACT

FERREIRA, Thiago Corni, M.Sc., Universidade Federal de Viçosa, February, 2022. **The effect of a carbon permit market on the size of stable coalitions of an International Environmental Agreement on climate change – A differential game approach.** Advisor: Viviani Silva Lirio. Co-advisors: Ian Michael Trotter and Leonardo Bornacki de Mattos.

In this study an International Environmental Agreement with the presence of a market for pollution permits is analyzed. A two-stage differential game is formulated, where in the first stage agents decide whether to joint or not the agreement and in the second stage agents negotiate over the level of emissions. The non-cooperative approach is adopted, termed the self-enforcing approach, which takes in to account the lack of supranational authority who can enforce the full cooperative outcome. The literature using this approach is usually pessimistic regarding the possibility of a large stable number of signatories to the agreement. The aim thus was to introduce the permit market system to investigate if the existence of such market creates any further incentive of agents to form a large coalition. Two such markets were introduced: the regulated market, where signatory of the agreements have access to and the voluntary market, which non-signatories have access to. Our results were positive in the sense that the presence of permit market, a larger number of signatories is achieved when compared with the baseline case without the permit market system. However, it is shown that if the cap set on signatories is too stringent then the incentive diminishes. This result have been corroborated with some robustness checks performed by varying relevant parameters, which also have highlighted some limitations in the model. It is also shown by calculating the steady-state of the stock of pollution that cooperation among agents is fundamental given that the larger the number of signatories the smaller is the steady-state of the stock of pollution.

Keywords: International Environmental Agreements. Climate Change. Carbon Markets. Differential Games.

RESUMO

FERREIRA, Thiago Corni, M.Sc., Universidade Federal de Viçosa, fevereiro de 2022. **O efeito de um mercado de licenças de carbono no tamanho das coalizões estáveis de um Acordo Ambiental Internacional sobre mudanças climáticas – Uma abordagem de jogo diferenciais**. Orientadora: Viviani Silva Lirio. Coorientadores: Ian Michael Trotter e Leonardo Bornacki de Mattos.

Neste estudo analisa-se um Acordo Ambiental Internacional com a presença de um mercado de licenças de poluição. Formula-se um jogo diferencial em dois estágios onde no primeiro estágio os agentes decidem se vão ou não aderir ao acordo e no segundo estágio os agentes negociam o nível de emissões. Adota-se a abordagem não-cooperativa, chamada de abordagem "self-enforcing", que leva em conta a falta de autoridade supranacional que possa impor o resultado com cooperação total entre os agentes. A literatura que utiliza essa abordagem costuma ser pessimista em relação à possibilidade de um grande número estável de signatários do acordo. O objetivo, portanto, foi introduzir o sistema de mercado de licenças ambientais para investigar se a existência de tal mercado cria algum incentivo adicional de agentes para formar uma grande coalizão. Introduz-se dois mercados: o mercado regulado, ao qual os signatários dos acordos têm acesso e o mercado voluntário, ao qual os não signatários têm acesso. Os resultados foram positivos no sentido de que com a presença do mercado de licenças, um maior número de signatários é alcançado quando comparado com o caso base, sem o sistema de mercado de licenças. No entanto, mostra-se que, se o limite estabelecido para os signatários for muito rigoroso, o incentivo diminui. Este resultado foi corroborado com algumas verificações de robustez realizadas por diversos parâmetros relevantes, que também destacaram algumas limitações do modelo. Mostra-se, calculando o estado estacionário do estoque de poluição, que a cooperação entre os agentes é fundamental, visto que quanto maior o número de signatários, menor é o estado estacionário do estoque de poluição.

Palavras-chave: Acordos Ambientais Internacionais. Mudanças Climáticas. Mercados de Carbono. Jogos Diferenciais.

LIST OF FIGURES

Figure 1: Graphical illustration of the basic IEA model..26
Figure 2: The baseline calibration...46
Figure 3:Weak agreement calibration...49
Figure 4: Stringent agreement calibration...50
Figure 5: Steady-state comparison..51
Figure 6: A1...55
Figure 7: A2...55
Figure 8: A3...56
Figure 9: A4...56
Figure 10: A5...56
Figure 11: A6...57
Figure 12: A8...57
Figure 13: A7...57
Figure 14: A9...57

LIST OF TABLES

Table 1:The baseline calibration..44
Table 2:Weak agreement calibration...47
Table 3:Stringent agreement calibration..49

SUMMARY

1 – INTRODUCTION....................9
1.1 – Initial Considerations....................9
1.2 The problem and its importance....................10
1.3. Hypothesis....................15
1.4. Objectives....................15
1.4.1. Specific Objectives....................15
2 – HISTORICAL OVERVIEW....................17
3. CONCEPTUAL FRAMEWORK....................22
3.1. Climate Negotiations....................22
3.2 Emission Trading....................22
3.3 IEA Games....................23
4 – METHODOLOGY....................28
4.1 Differential games....................28
5 – RESULTS AND DISCUSSION....................32
5.1 – Analytical model....................32
5.1.1 The Self-enforcing game....................33
5.1.2 – Emission Game....................34
5.1.3 – The participation game....................41
5.1.4 – Baseline results....................44
5.1.5 – Results with a permit system....................47
5.1.6 – Limitations of the model....................52
6 – CLOSING REMARKS AND FUTURE RESEARCH DIRECTIONS....................53
APPENDIX A – Robustness Checks....................55
BIBLIOGRAPHIC REFERENCES....................59

1 – Introduction

1.1 – Initial Considerations

International environmental cooperation is of surmounting importance, and increasingly. Climate change caused by global warming is a real threat on the incoming decades. The emissions of greenhouse gases is considered the main source of anthropogenic influence on the natural environment. The main greenhouse gases are: carbon dioxide, methane, nitrous oxide and flourinated gases. As a result of industrial activity, energy generation and agricultural activity, these gases are emitted and stay in the atmosphere thereby “trapping” solar radiation emitted from the sun and raising temperatures. At the same time that emission of greenhouse gases is what sustain the modern lifestyle.

The IPCC (Intergovernmental Panel for Climate Change) in the 2014 synthesis report is clear by stating: “Human influence on the climate system is clear, and recent anthropogenic emissions of greenhouse gases are the highest in history. Recent climate changes have had widespread impacts on human and natural systems.” (IPCC, 2014, p. 2). Moreover: “Warming of the climate system is unequivocal, and since the 1950s, many of the observed changes are unprecedented over decades to millennia. The atmosphere and ocean have warmed, the amounts of snow and ice have diminished, and sea level has risen*.”* (IPCC, 2014, p. 2).

The harmful effects of climate change can categorized as: increases in heat waves, floods, sea-level rise, ocean acidification, which impacts marine life and the chemical composition and behavior of the oceans. Increases in droughts, desertification and its impacts on agriculture and food security is also highlighted (IPCC, 2014)

These climate effects in addition to change the overall behavior of our natural system, impacts human welfare and pose significant challenges in some areas. The most salient impacts on human welfare is through human health, food security and economic losses caused by extreme weather events. According to Murphy et al (2018) since 1980 there have been recorded 178 extreme events, each event with a cost of 1 billion dollars. The trend in those types of events being an increasing one. The AR-5 reports that by 2030 it is more than likely than not that crop production will experience a negative impact of 5%. The impacts, however, are not homogeneous, with developing areas suffering a greater damage. Still in agriculture, an increase in 4ºC in average temperature can cause a reduction of 68% in production of beans for the sub-Saharan Africa. (IPCC, 2014, p. 506).

The impacts of climate change have not gone unnoticed by economists. Research has been done on this important subject. For example At a theoretical level, Fankhauser and Tol (2005) develop a growth model and finds that climate change will always reduce the rate of capital accumulation and hence growth.

Measuring overall impacts of climate change on growth is not a settled issue and the estimates vary. According to Eboli et al (2010) climate change could reduce global GDP in 0,3%. In a survey, Tol (2018) gives several estimates, and they vary between 0.1% and -3.0% for an increase in temperature of 2.5º C. However, the impact in developing regions are more pronounced. Using a computable general equilibrium model Kompas (2018), finds that for a 3ºC increase in temperature, developing countries can have a reduction in GDP around 10%.

Accessing the impacts of anthropogenic emission of carbon dioxide in the atmosphere is not easy, particularly due to the fact that the atmosphere is a nonlinear dynamical system. In these kinds of systems, subtle changes can cause dramatic changes in behavior. This is also a cause for caution because the risk for abrupt changes and irreversible behavior of the climate system is possible. As Lenton (2021) observes, in the presence of what are called tipping points, the most severe aspects of climate change can occur much sooner than expected, so the pressing nature of addressing this important issue becomes more prominent as time passes.

In a special report, the IPCC has raised the objective of keeping the average surface temperature of the planet below 1,5º C from the pre-industrial average temperature level in order to avoid the most serious effects of climate change. The costs of mitigation, however, have to drastic. According to the Special Report, the IPCC claims that emission would have to be reduced about 45% from the 2010 levels and reach net 0% emission by 2050 (IPCC, 2018, p. 12). This is a goal that is highly unlikely to be met without cooperation. This is why international environmental agreements on climate change are important. To stop climate change coordination among countries is vital if the problem of climate change and its impact are to be faced adequately.

1.2 The problem and its importance

How can political inaction be the norm when the science points to a catastrophic scenario if carbon dioxide and other greenhouse gases continue to be emitted at the current levels? To an economist, the problem at hand is a case of a prisoner's dilemma. This is a situation where cooperation would yield a higher benefit, in our case, a better and sustainable environment, but the incentive to deviate and behave non-cooperatively is present.

There have been, so far, two major attempts to curb climate change caused by global warming: The Kyoto protocol and more recently, the Paris Agreement. Both of them recognize that climate change cannot be adequately dealt with if countries unilaterally take actions, efforts must be coordinated.

The year 1992 was important in the history of climate change negotiations, the United Nations Conference on Environment and Development (UNCED) was held in Rio de Janeiro. In the conference it was signed the United Nations Framework Convention on Climate Change (UNFCCC), which was a treaty designed to stop the dangerous effects of climate change. The result of the framework as solidified in 1997 with the creation of the Kyoto protocol, which was signed by 192 countries. The main feature of that agreement was that developed nations would carry a higher burden of the mitigation efforts, while developing nations were not obliged to cut emissions. One important mechanism established in the protocol to ensure that cost-effective mechanism were to be in place was the set up of a carbon market. It is a cap and trade mechanism, meaning that, if any country need to exceed its "cap" given by rules of the agreement it can by permits in a market and therefore internalize the costs of excess emissions.

The Kyoto protocol, although has been somewhat effective in reducing carbon emissions (Grunewald and Martinez-Zarzoso, 2015), as of now, the amount of carbon dioxide in the atmosphere has reach record high levels. Barret and Stavins (2003) point to some flaws of the Kyoto protocol on the imbalance between the emission target set by the protocol between the developed and developing nations. The authors suggest that developing nations should also cut emissions and contribute. By not doing so, the incentive of developed nations to defect from the agreement was greater. The United State signed but did not ratify the agreement and countries such as Canada and Japan withdrew from the second commitment period.

It is also important to notice that although the emissions targets set in the Kyoto protocol were legally binding, international relations scholars such as Popovski (2018) point to the problem of enforcement of international law. Generally, international law can be considered “hard” or “soft”. Hard law is legally binding, while soft law is not. A treaty such as the Kyoto Protocol, enforces countries which have ratified the protocol to make legal adjustment in their domestic law to abide to the protocol. Soft laws are general recommendations and aspirations and have no power to enforce behavior. As Popovski (2018) argues, some legal realist suggest that no international law is in fact hard, since there is no supranational government who can enforce such laws.

The problem of the lack of supranational government and the enforcement of hard laws is an important aspect of our approach. In fact, if the rules of treaties could be enforced somehow, there wouldn’t even be a problem to begin with, countries would simply implement environmental rules and the problem would be solved. The Kyoto protocol couldn’t reach its objective because despite massive number of signatories, the countries were sovereign and didn’t abide to the rules.

The question of cooperation among sovereign states is the subject of study of international relations. In that area of study there are some ideas which might important to highlight in order to better grasp the conflict among potential signatories of an environmental agreement.

One of the most influential views is realism. In the realist view of international relations, states are the main actor in an anarchic non-cooperative international scenario. Each state acts according to his own interests and security. An international stable and peaceful environment is reached when a hegemonic country exists, called the hegemon. The hegemon through its power (economic or military) imposes its will and sets an international order. In the hegemonic approach, the Kyoto protocol was not fully effective because the hegemon or hegemonic states found against their interest to comply with the agreement. This unveils the weakness of the supposed “hard” international law to impose strong sanction which go against the interest of the hegemonic states.

While the realist perspective puts a negative view on the possibility of cooperation, relying on the existence of a hegemonic state to dictate an international regime, the very existence of international environmental agreements, the United Nations and the Framework Convention on Climate Change would belong to a different view, the Liberal Institutionalist view. The liberal view poses that cooperation among nations is possible through international mechanisms, dialogue and interaction among sovereign states in a feasible international regime. To construct mechanisms such that nations with conflicting goals can reach a common one is of paramount importance for climate change to be dealt with effectively.

In this research, our approach will be that of game theory. Game theoretical models of environmental agreements are a major tool to analyze these issues. How does game theory maps to what we have been saying so far? First, an IEA game is a model of negotiation, that is, agents negotiate over emission levels. That presupposes an international institution and a legal framework on which such negotiation take place. Since international law has limited power over actions of sovereign countries, the players in our game cannot be forced into signing the agreement, that is, they sign the agreement if it is in their best interest to do so, in game theory language, they behave noncooperatively. Since the game is noncooperative, the size of the coalition is determined in self-enforcing way. And finally, once the agents decide to enter the agreement, their emission level is binding and they fulfill their obligations. We don't consider the possibility of cheating.

In line with the noncooperative, self-enforcing approach, the works of Carraro and Sinicalco (1993) and Barrett (1994), the general conclusion is that only small stable coalition are possible. Barrett (1994) also states that large coalitions are possible only if the pay-off of the cooperative agreement doesn't deviate too much from the noncooperative scenario. Given that the baseline models in the noncooperative case have not given a good prospect for full cooperation, in this research we try to enlarge this class of models by introducing a mechanism that could alter the trade-off that agents face when deciding if they will join an international environmental agreement.

We consider marked-based mechanism. Particularly carbon markets, carbon credits, or pollution permits. With this type of mechanism an agent which exceed his quota of emissions can buy permits in a market to offset his excessive amount of emissions. Such mechanism were present in the Kyoto protocol and is also present in the recently accorded Paris Agreement. Additionally, we posit that there exists two such markets, one defined by the agreement and a voluntary market. In fact those voluntary markets do exist and are growing steadily since 2005, and have reached 63 million credits worth of dollars issued in 2017, (Fearnehough et al,2020). Therefore we can analyze the role that decentralized solutions impact on the size of stable coalitions and the impact on the total level of emissions as well.

Also, it is important to add that most of the literature in IEA games utilizes static or repeated games. The approach that we will follow is different, we are going to model our game as a differential game. The literature with this particular approach is still incipient, therefore, we think it is important to extend the literature with this particular approach. The main advantage of the differential game structure in comparison with the static or repeated approach is that we can take into account the effect that carbon dioxide accumulates in the atmosphere as a stock which slowly decays in time. This important effect is not present in the static game literature, when only flows of emission cause damage. In the differential game approach, the stock of greenhouse gases is the main driver of damages. We believe thus, that differential games add more realism to the phenomena being described. Since it is the stock of pollutants on the atmosphere that matter and cause damage, not the current flow of emissions. Differential games, as will be described latter puts the issue into a system dynamics perspective, which in our view is fundamental in understanding the coupling of human-climate interactions.

Being a more peripheral methodology, not much has been accomplished so far. There are only a small number of papers dealing with IEA games. We can mention the works of Rubio and Casino (2005), Rubio and Ulph (2007), De Zeeuw (2008) and Breton et al (2010). All of these works have analyzed an IEA game in a dynamic setting with different approaches upon which the coalition is formed (the first stage of the game). Rubio and Casino (2005) consider that agents take a once-and-for-all decision to join the agreement. Rubio (2007) seeks to provide a dynamic coalition formation, while De Zeeuw (2008) considers far-sighted stability. Breton et al (2010) considers an evolutionary approach to coalition formations, which agents switch strategies depending on the most successful strategy. We have looked into the literature and could not find more recent papers, we would like to make this explicit. The once-and-for-all approach to coalition formation has yielded small coalitions, namely two, while the approach by Rubio (2007) being dynamic, has sustained a larger number of signatories, it starts with 15 out of 100 but it decreases to 7 as time passes. The evolutionary approach by Breton et al (2010) has been able to sustain partial and full-cooperation, but it is dependent on some range of parameter values, with no cooperation also being a possible outcome.

We use the approach by Rubio and Casino (2005) for the formation of the coalition, that is we take the once-and-for-all approach to the stage one of our game, but we add a market for carbon emission permits and analyze how does the addition of the market mechanism alters the size of self-enforcing coalitions. It is important to realize that the addition of a permit market to the class of dynamic IEA games is not a mere breach in the literature that we hope to fill. Our results could be important for policy making. A well designed structure of incentives is paramount in any policy-making. Therefore part of our objectives is to show, also, under which conditions the permit market creates incentives for countries to join and international environmental agreement. Our research, then, enlarges this class of games and also can provide information for practical reasons.

1.3. Hypothesis

Under a self-enforcing noncooperative game with symmetric countries, a carbon permit market increases the level of participation and decreases participation if the restriction on emissions are too stringent.

1.4. Objectives

To analyze the effect a carbon permit market on the size of stable coalitions of an International Environmental Agreement under a self-enforcing noncooperative game with symmetric countries.

1.4.1. Specific Objectives

i) To derive the optimal strategies, namely the level of emissions, for signatories and non-signatories of the agreement;

ii) Find the size of stable coalitions using the stability criterion;

iii) To explore numerically the model in order to find out the effect of the distribution of initial permits on the size of stable coalitions.

This dissertation is organized as follows: Section 2 gives a historical overview. Section 3 gives the fundamental concepts involved in our study, namely, the process of negotiation, what are emissions trading as a flexible mechanism and a basic structure of IEA games is provided. Section 4 is devoted to a brief presentation of differential games and how the optimal strategies are found. Section 5 presents our model and results and we conclude with section 6 summarizing our finding and pointing to future research directions.

2 – Historical Overview

The purpose of this section is to outline the historical development of the climate change awareness. Although our approach will be more abstract in nature, we do believe that a historical development of the problem aids our understanding and enriches whatever conclusions we reach from our investigation.

Concretely, we will see that the climate change awareness and general preoccupation from politicians and the population is strictly intertwined with the development of climate science. It will be noticed that only after general scientific consensus, which were only possible with the development of climate science, that these matters came into prominence in the political arena. Therefore, the notion that controlling global climate change is dependent on the cooperation of several actors which must coordinate their action through international agreements is a direct product of this historical development. Our outline is heavily based on Bolin (2007)

The mathematician and natural philosopher Joseph Fourier was one of the pioneers of climate science. In a paper published in 1824 he pondered about what was the caused of the mean temperature of the planet. He considered that the heat balance of the planet was the equilibrium between the incoming solar radiation and the outgoing solar radiation, however, in his calculations he reached a temperature which was much lower than the real temperatures observed. This negative result led him to consider that the atmosphere had a role to play in stabilizing the temperature at much higher levels. The atmosphere then, would prevent heat from escaping to outer space and then the temperature would be much higher. This is development of the greenhouse effect.

This idea, according to Bolin (2007) was further developed by John Tyndall. He measured the heat absorption by gases in the atmosphere, including water vapour and carbon dioxide and concluded that variations in the composition of the gases in the atmosphere could induce climatic variations.

Up until 1894 the state of knowledge was largely theoretical and qualitative, it was the Swedish physicist Svante Arrhenius that more quantitative measures were established. Under his leadership a group in the University of Högskola in Stockholm, two hypothesis were contrasted: that the variations in the average temperature were largely a result in variations of incoming solar radiation, and the role that sources and sinks of carbon dioxide could play.

Since Arrhenius was skeptical of the first hypothesis he set out to calculate the variations of temperature given different levels of carbon dioxide and water vapour in the atmosphere. From these studies he found that increases in carbon led to an increase in water vapour, giving a positive feedback between increases in the temperature by accumulation of carbon and further increases led by increases in water vapour in the atmosphere. But his most interesting finding were his calculations of the expected change of surface temperature as a function of the latitude and time given a change in concentration of carbon dioxide in the atmosphere. He calculated, according to Bolin (2007), variations of 0.67, 1.5, 2.0, 3.0 time the prevailing carbon concentration at the time, which he assume it was of 300 ppm. He concluded that the average global surface temperature with a doubling of carbon dioxide in the temperature would be of 5.7º. With this, he concluded that variations of carbon in the atmosphere could be an important and relevant mechanism for climatic variations. The hypothesis that incoming solar radiation was the most important factor was refuted.

It is important to notice that Arrhenius did not consider the possibility that human action could produce global warming, since he believed that the current burning of coal at that time would quickly be dissolved in the sea. But as Bolin (2007) points out, the capacity of the sea to absorb carbon is much smaller than Arrhenius assumed. Also he did not expect the increase in the burning of fossil fuels that would come in the following years. Arrhenius findings were significant but unfortunately due to lack of data, his results were neither confirmed nor rebutted. It was the scientist Charles Edward Keeling, in 1958 who started to measure the amount of carbon dioxide in the atmosphere. As Bolin (2007) explains, Keeling's measures in the late 50's detected an amount of carbon dioxide in the atmosphere of 315 ppm, with an increase of 1.2 Gt (gigatons) per year. Since then, the Keeling curve provides measures of the accumulation of carbon dioxide in the atmosphere and it is one the most telling facts regarding anthropocentric causes of climate change.

In the 1960's global research efforts to study the atmosphere began to take place. According to Bolin (2007), the launch of the satellite TIROS-1 marked a turning point in global research efforts, since the use of satellite technology should be explored collaboratively. With the new technology, expanded research efforts were initiated to better understand and forecast weather patterns. As Bolin (2007) and also Powell (2011) explain, the period after World War II was ripe for science, funding was plentiful and the cold war also created a fruitful environment for the development of science. Climatology in general benefited in that period and research initiatives like the GARP (Global Atmospheric Research Program) was created by the WMO (World Metereological Organization) jointly with the ICSU (International Counsil of Scientific Unions) and IUGG (International Union of Geodesy and Geophysics). The GARP program concentrated on understanding the physical process in the trophosphere and the stratosphere that were essential to understand the general circulation of the atmosphere and improve weather forecasting. Research efforts also benefited from the advanced in computing power that occurred in that period, general circulation models could be analyzed computationally, thus improving our understanding of the atmosphere. As Powell (2011) mentions, the original question that Arrhenius posed was revived in 1967 and reached the conclusion that doubling the amount of carbon dioxide in the atmosphere could increase mean level temperature by 2º C, and another study in 1975 concluded an increase of 3.5ºC. Jules Charney, a scientist at MIT then reviewed the literature pointed to the fact that increasing the amount of carbon dioxide in the atmosphere increased mean level temperature to a significant degree in all of the models.

Given the scientific developments that occurred throughout the 1960's and 1970's it was only a matter of time before the climate issue reached the political arena. One of the earliest assessment of the danger of the human interference on the environment happened in 1972 in Stockholm, in a conference organized by the UN. Although most the discussions on the conference was local pollution, it led to the creation of the UNEP (United Nations Environmental Program) which would play a significant role in future years. Also, a statement was issued that human interference on the climate system should be watched and better evaluated. Along with the efforts by the GARP program to summarize and advance the scientific knowledge the climate awareness began to gain momentum not only to the scientific community but to the political sphere.

In 1979, the First World Climate Conference took place in Geneva as a joint effort by the UN, WMO, UNEP, FAO (Food and Agriculture Organization), UNESCO (United Nations Educational, Scientific and Cultural Organization), WHO (World Health Organization), ICSU and other scientific organizations. The conference resulted in a declaration for all countries to take notice of the human influence on the climate, to use the existing and advance knowledge in that area and create strategies to avoid climate damages adverse to human well-being Zillman (2009). That statement generated a series of conferences on climate change: Villach

Conference (1985), Toronto Conference (1988), Ottawa Conference (1989) and many others, see Powell (2011).

Right after the discovery of the depletion of the Ozone layer and the approval of the Montreal Protocol in 1987, Bolin (2007) argues that something similar should be done about climate change. In the same year, the WMO and the UNEP agreed that an intergovernmental assessment panel on climate change should be formed. In 1988 a meeting in Geneva gathered members of 28 countries, and the structure of the working groups of the IPCC were defined. The general guideline was to produce a report on the human influence on climate change. Working group I had the task of gathering and surveying the available knowledge on climate change. The working group II should assess the impacts of climate change in the socio-economic sphere. The working group III had to propose response strategies. (Bolin, 2007, p. 50)

According to Powell (2011), the first assessment report was very conservative, it pointed out to the fact that the earth was indeed getting warmer but it could not state precisely the cause. The general conclusion was the same conclusion that Arrhenius reached several years back. That is, if carbon dioxide continues to increase the average temperature could increase severely (Powell, 2007, p. 47).

The First Assessment of the IPCC created the opportunity for the first Earth Summit, in Rio de Janeiro in 1992. The FAR (First Assessment Report) was used as the scientific input in the elaboration of the UN Framework Convention on Climate Change (UNFCCC). The convention was signed in 1992 by 165 countries, in 1994 it was ratified by the 50th country and the convention went into force, in 1997 the Kyoto protocol was adopted (Oberthür and Ott, 1999).

The most important article in the framework convention is the article 2, which reads as:

> "The ultimate objective of this Convention and any related legal instruments that the Conference of the Parties may adopt is to achieve, in accordance with the relevant provisions of the Convention, stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system. Such a level should be achieved within a time-frame sufficient to allow ecosystems to adapt naturally to climate change, to ensure that food production is not threatened and to enable economic development to proceed in a sustainable manner." (UNITED NATIONS, 1992, p. 9)

Of course that, in order to fulfill the objectives in article 2, a credible assessment of what would be a "dangerous anthropogenic intereference with the climate system" is needed, which level of emission would result in which level of damages. Since then, the IPCC has become the most credible source for such assessment. It has published, as of now, five assessment reports and it is currently working on its sixth assessment. The general conclusions of each assessment has been towards greater certainty on the anthropogenic influence on the environment. Bolin (2007) argues that without the efforts of a comprehensive assessment of the available scientific knowledge, the chances of a global effort such as the UNFCCC would have been very dim. The Kyoto protocol, whether it has achieved or failed in reaching the objectives of article 2 of the framework convention was an important step in creating a global arena for discussion on this important issue.

We have tried to paint in broad strokes in this section the development of the global awareness to climate change. From early developments in atmospheric sciences, the mechanism of the greenhouse effect and the role of greenhouse gases in average temperature, to global research efforts through various organizations and finally the reaching of the scientific conclusions to the international political agenda. It is clear that the scientific development is key in raising our concerns on the dangers of climate change and how cooperation among nations in vital.

3. Conceptual Framework

3.1. Climate Negotiations

How do countries actually negotiate? In this section we present a brief explanation.

Climate negotiations have taken place under the United Nations Framework Convention for Climate Change (UNFCCC), a framework convention is a form of preliminary treaty, that is, it has no legal force, but countries that sign the convention agree with the principles and general dispositions there stated. It consists of general forms of governance, institutional mechanism and policy guidelines. A protocol is a modification of the treaty and consists of legally binding commitments. (Kutney, 2014, p. 1)

Before ratifying the agreements, meetings take place where potential signatories negotiate over the concrete measure to be taken. The Intergovernmental Negotiating Committee (INC) is formed where discussion over the preliminary framework convention are taken. After the completion and signature of the convention, the Conference of the Parties (COP) are held where modifications on the guidelines of the framework convention take place, in that period many modifications, conflict and resolution of conflict emerges, so the process can be long. According to Kutney (2014), each meeting is divided in to two session, the administrative body go through the design of actual policies to be taken and in the second session high-level ministers and head of state meet. When an agreement is reached, the protocol becomes part of the framework convention, and countries effectively sign the agreement. The protocol becomes legally binding when countries ratify the protocol. This process takes place domestically within the parliament or equivalent of each country.

3.2 Emission Trading

The Kyoto protocol created flexible mechanisms to aid countries to meet their obligations to cut emissions. One of these mechanism, described in the article 17 of the agreement, was the establishment of a market for tradable pollution permits. The so called Annex B countries could buy carbon permits if they needed to exceed their quota of emissions. This is a market based solution which allowed countries to better manage their plans to meet their targets accorded in the framework convention. By placing a price on a ton of carbon, for example, the real cost of pollution can be partly dealt with, since the country that emits pollutants must compensate for the damage imposed on other countries.

Being a cap-and-trade mechanism, the cap is usually set according to the targeted level for emissions reduction. If an agreement has stricter rules than the amount of pollution permits issued should naturally be smaller, if the agreement is weaker, in the sense of allowing higher emission levels, than the supply of permits should be higher. As Ibikunle and Gregoriou (2018) argue, while regulators cannot decide the price of permits, they can influence its supply and thus influence the price of permits.

Following the Kyoto protocol, one of the biggest emission trading schemes is the European Union Emissions Trading Scheme (EU ETS). According to Chevallier (2012) the market was divided into three phases, where in phase I and II the allowances were distributed according to the granfathering principle, that is, the distribution of permits would be according to historical level of emissions of each country, with countries which have historically emitted more receiving more permits. In phase III, permits would auctioned off. In the first phase 2,2 billion allowances were distributed, 2,08 in phase II. With a price of 20 euros per allowance, it generated a market of 40 billion euros per year.

The pollution permit system is but one of the flexible mechanism implemented by the Kyoto protocol. Pollution permits is known as an allowance-based transaction system, that is, firms are given the legal right to pollute a certain amount. Countries can also offset their carbon emission through joint implementation and the clean development mechanism (Bayon et al, 2009). They can be considered project-based mechanism where a country by financing carbon reduction projects in a different country generate carbon credits from which it can use or sell it in a market.

Alongside the mechanisms created by the Kyoto protocol, there is a voluntary carbon market. The voluntary carbon market creates carbon offset through projects approved by independent organizations who certify that submitted projects qualify standards of emission reduction and thus creates a certificate that can be sold or bought in the voluntary market. (Bayon et al, 2009).

3.3 IEA Games

We now translate everything we have been saying so far in a more technical language. First, the problem we seek to solve is the provision of a public good. In our case, a cleaner environment (carbon dioxide an other greenhouse gases are essential to life on Earth, however, when they accumulate too much they can cause harm, in that sense we call them pollutants). The provision of a public good suffer inherently from a problem: the incentive to free ride. Free riding occurs because one agent can benefit from the effort others.

Externalities are costs or benefits enjoyed by a third party outside a transaction or an interaction among agents. In our case, transboundary pollution is a negative externality. The emission of a country will benefit the country in question at a cost to all other countries. At the same time, a country that employ resources to cut emission will benefit not only himself, but all others around him. The fact is that the provision of public good and externalities are very closed linked subjects. A positive externality is a public good, while a negative externality is a public bad. The under provision of good positive externalities exists due to lack of property rights.

The way that a typical society solves the free-rider problem and produces public goods is by the government to produce them at an optimal level and tax its citizens to fund the production of those goods. This solution is not available to deal effectively with transboundary externalities, since there is no international authority who can enforce the production of such good and taxation to finance its production.

An International Environmental Agreement attempts to coordinate the action of agents to supply an adequate level of this global public good (emission levels mitigation). Because no country can be forced to sign the agreement, a basic feature of an IEA is that any coalition must be self-enforcing. That is, it must be on the agent self-interest to join the agreement. In this research the self-enforcing approach will be followed.

The self-enforcing IEA game is a two-stage game where in the first stage agents decide to join or not the agreement and in the second stage the emission levels are decided. The game is solved by backwards-induction.

The self-enforcing coalition is found by applying the internal and external stability condition. At this stage countries evaluate the benefit generated by being a signatory versus being a non-signatory. If the welfare generated by signing the agreement is greater than not signing it, the agent signs the agreement, and when the benefit of one more country signing the agreement is smaller than not signing it, the agent does not sign the agreement. This condition was first used by d'Aspremont et al (1983) to analyze the stability of a cartel and was later used by Barrett (1994), and Carraro and Sinicalco (1993) to analyze the stability of IEAs.

In order to get more intuition about the problem at hand, we can formalize a game as follows:

A game is a tuple such as $\Gamma=(N,\{X_i\}_{i\in N},\{H_i\}_{i\in N})$:

Where

N = the number of players;

X_i = the set of strategies for player i;

H_i = the pay-off function for player i.

We can build a strategy space which is the domain to the pay-off function:

$$X=\prod_{i=1}^{N} X_i$$

This is the cartesian product of each strategy set of the players.

And the pay-off function is given by:

$$H:\prod_{i=1}^{N} X_i \rightarrow \mathbb{R}$$

A simple IEA game can be exemplified by:

A benefit function:

$$B_i(e_i)=ae_i$$

A cost function:

$$C(e_i)=\frac{be^2}{2}$$

There for the net benefits are given by

$$\pi_i=B_i(\sum e_i)-C(e_i)$$

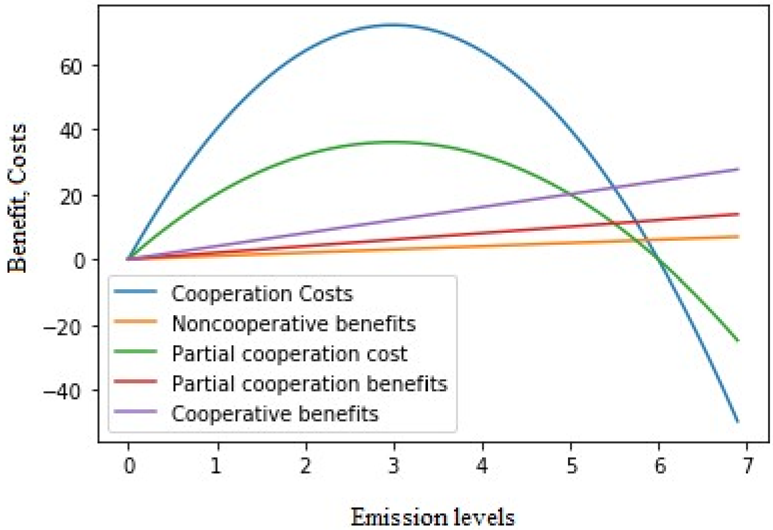
If countries choose to cooperate then they maximize their joint net benefits:

$$\sum_{i=1}^{N} \pi_i$$

If they don't cooperate they maximize π_i

We will not bother to solve this model, the reader is referred to Barrett (1994). But we can present a graph which illustrates the intuition behind the solution:

Figure 1:Graphical illustration of the basic IEA model



Source: elaborated by the author.

The inverted parabola represents the cost function. Each country equates their marginal cost to the marginal benefits of total emissions. This happens when the distance between the cost and benefits yields the higher vertical distance, or when the tangent of the cost function is parallel to the benefit function. We can see above that cooperation always yields the highest emissions abatement, but if agents deviate, that is, equation their total marginal benefit to their individual marginal cost, they can free-ride. Partial cooperation sits in between, yielding a lower benefit than the total cooperative outcome but a higher level of abatement than the non-cooperative outcome.

In the participation game players decide if they join the agreement or not according to the following rule:

$$\pi_S(n*) \geq \pi_{NS}(n*-1)$$
$$\pi_{NS}(n*) \geq \pi_S(n*+1)$$

That is, an agent compares his pay-off if he participates in the agreement with *n* signatories against his pay-off if he defects from the agreement. Similarly, a defector compares compares his pay-off with *N-n* non-signatories against the pay-off he would receive in the agreement with *n+1* signatories. In general a stable coalition will exists with *n* signatories, where *n* is less than *N* (the total number of players)

4 – Methodology

4.1 Differential games

In order to test our hypothesis and stated objectives we have to make recourse to a solution concept for our game. Below we describe the following: the structure of an optimal control problem and a theorem that guarantees that the strategies the players select is indeed optimal. We give a proof of a theorem that states that if a value function exists such that it solves the Hamilton-Jacobi-Bellman equation then the pay-off of any given player is maximized.

In game theory, players select a strategy from their strategy set that yields the highest payoff. Differential games are not different in this sense, but the structure of the game differs. Differential games make use of optimal control theory, so a strategy is the player control input into the system from which the player can choose from a set of admissible functions in the control set. The other players' control inputs also influences a given player's pay-off, as does the state variable. The state equation is usually given by a differential equation, and the utility is given by a functional, a functional according to Gelfand (2000, p. 1) is a mapping from functions, or trajectories to real numbers.

To understand differential games, first we have to show the structure of an optimal control problem. Consider the following:

$$\begin{gathered} J(u(.)) = \max_{u} \left\{ \int_{t_0}^{T} g[s, x(t), u(t)]\, dt + q(x(T)) \right\} \\ s.t \\ \dot{x} = f(t, x(t), u(t)) \quad x(0) = x_0 \end{gathered} \tag{I}$$

This optimal control problem can be solved using the technique of dynamic programming.

The dynamic programming approach uses the Hamilton-Jacobi-Bellman equation:

$$-V_t(t,x)=\max_u\{g[t,x,u]+V_x(t,x)f[t,x,u]\} \quad \text{(II)}$$

Then, according to Petrosyan and Zenkevich (2016), the following theorem holds:

Theorem 1 (Petrosyan and Zenkevich (2016, p. 402): A set of controls $u^*(t)=\phi(t,x)$ constitutes an optimal solution to the control problem (I) if there exists a continuously differentiable function $V(t,x)$ defined on $[t_0,T]\times\mathbb{R}^m\rightarrow\mathbb{R}$ and satisfying the following bellman equation:

$$\begin{gathered}-V(t,x)_t=\max_u\{g[t,x,\phi^*(t,x)]+V_x(t,x)f[t,x,\phi^*(t,x)]\}\\ V(T,x)=q(x)\end{gathered} \quad \text{(III)}$$

Proof: The proof of theorem 1 goes as follows:

Define the value function $V(t,x)$ to be the maximized value of the control problem:

$$V(t,x)=\max_u\left\{\int_{t_0}^{T} g[t,x(t),u(t)]dt+q(x(T))\right\}$$

$$V(t,x)=\left\{\int_{t_0}^{T} g[t,x^*(t),u^*(t)]dt+q(x^*(T))\right\}$$

$$V(T,x^*)=q(x^*)$$

And satisfying the state equation in (I) as well. Consider another set of controls $u(t)$ which are feasible from the control set U and solves the corresponding state trajectory but does not hold with equality the bellman equation, then we have:

$$\begin{gathered}g(t,x,u)+V_x(t,x)f(t,x,u)+V_t(x,t)\le 0\\ g(t,x^*,u^*)+V_x(t,x^*)f(t,x^*,u^*)+V_t(t,x^*)=0\end{gathered} \quad \text{(IV)}$$

Upon integrating the two expressions in (IV), it yields:

$$\int_{t_0}^{T} g(t,x(t),u(t))dt+q(x(T))\le\int_{t_0}^{T} g(t,x^*(t),u^*(t))dt+q(x(T)) \quad \text{(V)}$$

That proves that $u^*(t)=\phi^*(x,t)$ is an optimal strategy.

In infinite-horizon optimal control, the terminal condition is replaced by $V(x,t)=0$ when $t \rightarrow \infty$. We can modify rewrite the optimal control in the following way:

$$W(x)=\int_{t}^{T} e^{-r(t-t_0)} g(x(t),u(t))dt \quad \text{(VI)}$$

And:

$$V(x,t)=e^{-r(t-t_0)}W(x)$$
$$V_t(x,t)=-r e^{-r(t-t_0)}W(x)$$
$$V_x(x,t)=e^{-r(t-t_0)}W_x(x)$$

Substituting these results in the bellman equation and multiplying by $e^{r(t-t_0)}$, yields the HJB-equation for the infinite-horizon optimal control:

$$rW(x)=\max_u \{g(x,u)+W_x(f(x,u))\} \quad \text{(VII)}$$

Now that we have presented the optimal control problem and the Hamilton-Jacobi-Bellman equation which gives the optimal solution to the optimal control problem. We can present a differential game as strategic version of optimal control problem we have presented.

A differential game is modeled as a simultaneous system of optimal control problems as given by (I), the decision of each player is given by:

$$\begin{gathered} J_i(u(t),x(t))=\max_{u_i}\left\{\int_{t_0}^{\infty} e^{-rt} g[s,x(t),u(t)_1,u(t)_2,u_i(t)\ldots,u(t)_n]dt\right\} \\ s.t \\ \dot{x}=f(x(t),u(t)_1,u(t)_2,\ldots,u(t)_n) \quad x(0)=x_0 \end{gathered} \quad \text{(VIII)}$$

That is, each player maximizes his pay-off considering that every other player is playing their optimal response. A Nash-equilibrium in a differential game is defined as set of controls $\gamma^*=(\phi_1^*(x),\phi_2^*(x),\ldots,\phi_n^*(x))$, such that the following holds (Basar et al., 2018, p. 68):

$$J(\gamma^*)\geq J([\gamma_i,\gamma^*_{-i}]) \quad \text{(IX)}$$

Where the set Γ_i is the strategy set of each player, that is the space of feasible controls. The definition (IX) follows readily from the theorem 1 that we have presented. That is, one holds constant the strategy of every other player and apply theorem 1. The result follows.

After we have found our optimal strategies, solving the stage 1 of our game is straight forward. We have to substitute our optimal strategies in objective functional of each player and integrate. We then apply the stability condition that we have presented in the conceptual framework. After carrying out this procedure we can fully derive conclusion and see if our hypothesis holds or not.

5 – Results and Discussion

5.1 – Analytical model

Now we effectively present our model. We consider N countries indexed by $i \in \{1, 2, \ldots, N\} \subset \mathbb{N}$ each country has amount of production $Y_i = Q(E_{it})$ which is a function of its levels of emissions at any given time and it's given by its technology $Q(.)$ which is shared by all countries and each country has a fixed amount of endowments. The technology is assumed to be strictly concave and satisfy $Q(0)=0$. Given that countries derive benefit from the consumption of Y , we assume the follow benefit function:

$$B(t) = aE_i(t) - \frac{1}{2} E_i(t)^2 \qquad (1)$$

Where $a > 0$ is a parameter that measure the marginal benefit from a unit of emissions. Also $B'(t) > 0$ and $B''(t) < 0$. This particular form for the benefit function is widely adopted in the literature due to its nice properties when applied to optimal control problems, virtually every work in this area which derive analytical solution adopts this particular form, see Dockner and Long (1993), Rubio and Casino (2005), Bretton et al (2010) and Li (2013). This form for the benefit function also has the property of diminish marginal returns to the emission level, which is realistic and desirable.

We also assume that countries suffer from environmental damage from a global stock of pollution P . Therefore, the damage function is:

$$D(t) = \gamma P(t) \qquad (2)$$

Where $\gamma > 0$ and $D'(t) > 0$, that is damages are increasing in the level of pollution stock. A damage function that is linear in the stock of pollution is also adopted by Bretton et al (2010) and Li (2013), but for a quadratic damage function the game is also solvable, but the calculations are more involved and according to Breton et al (2010) without significant difference in results, so we adopt a linear damage function.

The stock of pollution $P(t)$ evolves according to the following law of motion:

$$\dot{P}=\sum_{i=1}^{N} E_i - kP$$
$$P(0)=P_0 \, given \tag{3}$$

Where:

$\dot{P}=dP/dt$ = derivative of P with respect to time;

$\sum_{i=1}^{N} E_i(t)$ = the sum of emissions from all countries;

k = the rate of decay of pollution;

P_0 = initial stock of pollution;

Equation (3) is a linear differential equation, it provides the link of the emission levels of all countries with the stock of pollution. It is a way of coupling the economic activity of countries with the environmental degradation, that is, as countries emit more pollution, that pollution accumulates in the atmosphere according to (3). The particular form for the evolution of the stock of pollution is an extremely simple one but also popular in the literature. As Bretton et al (2010) points out, it cannot grasp the occurrence of catastrophic events, but for tractability of the model is a popular way of modeling.

We can now define the net welfare function for a country i, as:

$$W_i(E_i(t), P(t)) = aE_i(t) - \frac{1}{2}E_i(t)^2 - \gamma P(t)$$
$$\forall i \in \{1, 2, \ldots, N\} \tag{4}$$

5.1.1 The Self-enforcing game

The self-enforcing game, as we have already stated, is a two stage game. Where in the first stage each country decide if it will join the agreement or not, and in the second stage it decides upon its emission levels. We solve the game by backwards-induction, that is, we solve first for the second stage and the for the first stage.

Just for ease of exposition, we now drop the time argument from each function, but be aware that both emissions and the stock of pollution are functions with respect to time.

5.1.2 – Emission Game

The second stage is the emission game, and proceed as follows. We suppose that as a result of the first stage a coalition of *n* signatory countries has been formed. Then the signatories play a non-cooperatively against the signatories. Both the signatories and the non-signatories maximize their welfare according to the infinite-horizon optimal control problem:

$$\sum_{i=1}^{n} W_S = \sum_{i=1}^{n} \int_0^{\infty} e^{-rt} \left(aE_i - \frac{1}{2} E_i^2 - \gamma P \right) dt \quad (5)$$

$$W_{NS} = \int_0^{\infty} e^{-rt} \left(aE_j - \frac{1}{2} E_j^2 - \gamma P \right) dt \quad (6)$$

Where:

W_S is the total discounted welfare of a signatory;

W_{NS} is the total discounted welfare of a non-signatory;

r is the discount rate;

And the differential equations in (3), becomes

$$\dot{P} = \sum_{i=1}^{N} E_i + \sum_{j=1}^{N-n} E_j - kP \quad (7)$$

Notice that the signatories maximize their joint welfare. That is, they act cooperatively among themselves to reduce emissions while playing non-cooperatively against the non-signatories.

Now, consider that the signatories by signing the agreement are imposed an emission target from which if they exceed that target they must buy pollution permits in a regulated market. Analogously, the non-signatories countries have access to the voluntary carbon market, but since they do not have to comply with any emission target, we assume that they impose on themselves a cap, which can be determined by their environmental awareness. Also we can suppose alternatively that each non-signatory country have their own environmental laws and an internal permit market. Anyways, we assume the existence of two permit markets, one for each type of agent, signatory and non-signatory. We also assume that the price of permits is given exogenously, which is a strong assumption to make, but we can argue for it by assuming that no country can significantly alter the market price, or each government posts its price for the permits, so the permits could work as a sort of carbon tax. In our model if a given country stays above its quota it could be interpreted as a tax to be paid, if it stays below, a tax rebate or subsidy. The assumption of an exogenous price has been used by Li (2013) and Chang et al (2018).Adding the permit markets, our problem becomes:

$$\sum_{i=1}^{n} W_S = \sum_{i=1}^{n} \int_0^{\infty} e^{-rt} \left(aE_i - \frac{1}{2} E_i^2 - \gamma P - \rho_1 (E_i - E_S) \right) dt \quad (8)$$

$$W_{NS} = \int_0^{\infty} e^{-rt} \left(aE_j - \frac{1}{2} E_j^2 - \gamma P - \rho_2 (E_j - E_{NS}) \right) dt \quad (9)$$

The maximization in (8) and (9) are subject to (7).

Where:

ρ_1 is the price of a ton of carbon dioxide in the regulated market;

ρ_2 is the price of a ton of carbon dioxide in the voluntary market;

E_{S0} is the initial quota distributed by the signatories of the agreement;

E_{NS0} is a self-imposed cap on emissions or a limit set by the domestic environmental law of a non-signatory.

Notice that:

If $E_i - E_{S0} > 0$, then the signatories have exceed their quota and then must buy permits.

If $E_i - E_{S0} < 0$, then the signatories have not exceed their quota and then can sell the excess permits

If $E_i - E_{S0} = 0$, then the signatories have matched their quotas and neither buy or sell permits.

The same is valid for the non-signatories.

After rearranging (8) and (9) we obtain:

$$\max_{E_1,E_2,\dots E_n} \sum_{i=1}^{n} W_S = \int_0^{\infty} e^{-rt}[(a-\rho_1)\sum_{i=1}^{n} E_i - \frac{1}{2}\sum_{i=1}^{n} E_i^2 + n\rho_1 E_{S0} - n\gamma P]dt$$
$$s.t\ \dot{P} = \sum_{i=1}^{N} E_i + \sum_{j=1}^{N-n} E_j - kP \qquad (10)$$
$$P(0) = P_0$$
$$\forall i \in \{1,2\dots n\}$$

$$\max_{E_j} W_{NS} = \int_0^{\infty} e^{-rt}[(a-\rho_2)E_j - \frac{1}{2}E_j^2 + \rho_2 E_{NS0} - \gamma P]dt$$
$$s.t\ \dot{P} = \sum_{i=1}^{N} E_i + \sum_{j=1}^{n-N} E_j - kP \qquad (11)$$
$$P(0) = P_0$$
$$\forall j \in \{1,2,\dots,N-n\}$$

To solve the model presented in (10) and (11) we have to set-up the Hamilton-Jacobi-Bellman equation, since we are going to solve for the state-feedback strategies. We, then, arrive at:

$$rV_S(P) = \max_{E1,E2,\dots,En} (a-\rho_1)\sum_{i=1}^{n} E_i - \frac{1}{2}\sum_{i=1}^{n} E_i^2 + n\rho_1 E_{S0} - n\gamma P$$
$$+ V_P{}'(\sum_{i=1}^{n} E_i + \sum_{j=1}^{N-n} E*_i - kP) \qquad (12)$$

For the signatories

$$rV_{NS}(P) = \max_{ej}(a-\rho_2)E_j - \frac{1}{2}E_j^2 + \rho_2 E_{NS0} - \gamma P$$
$$+ V_P{}'(\sum_{i=1}^{n} E*_i + \sum_{j\neq 1}^{N-n} E_j* + E_j - kP) \qquad (13)$$

For non-singatories. The asterisks denote the optimal response of the other player. That is, each seek a control (strategy) that maximize its own pay-off given that the other agents are doing the same.

Now are going to take a step-by-step approach to the solution. Performing the maximization indicated in (12) and (13), yields:

$$0 = a - \rho_1 - E_i + V_S{}'$$

$$0 = a - \rho_2 - E_j + V_{NS}{}'$$

Upon rearranging, we obtain the optimal emissions level for the signatories and non-signatories:

$$E_S = a - \rho_1 + V_S{}' \tag{14}$$

$$E_{NS} = a - \rho_2 + V_{NS}{}' \tag{15}$$

Remember that we have assumed that we are solving the model by backwards induction, therefore we have *n* signatories and *N-n* non-signatories. So we have a system of optimal controls to solve. For each signatory and non-signatory we have a similar problem since the countries are symmetric, hence we can impose symmetry condition and sum (3) and (4) from 1 to n and n+1 to N, respectively. We thus obtain:

$$\sum E_S = n(a - \rho_1 + V_S{}') \tag{16}$$

$$\sum E_{NS} = m(a - \rho_2 + V_{NS}{}') \tag{17}$$

To facilitate our subsequent algebraic manipulations, let us make the following modifications:

$$a - \rho_1 = c_1$$
$$a - \rho_2 = c_2$$
$$N - n = m$$

Now, the next step is to conjecture a value function that optimizes problem (12) and (13). Since the Hamilton-Jacobi-Bellman equation is linear in the variable P, we conjecture that the value function is also linear in P. This is a common approach to solve the HJB partial differential equation in differential games, see Breton (2010), Li (2013) and Basar (2018). We guess the following form for the value function for the signatories:

$$V_S = A_S + B_S P \tag{18}$$

And the following form for the non-signatories:

$$V_{NS} = A_{NS} + B_{NS} P \tag{19}$$

Taking the derivative with respect to P in (18) and (19), we obtain:

$$V_S{}' = B_S \tag{20}$$

$$V_{NS}{}' = B_{NS} \tag{21}$$

Substituting the results (20) in (14) and (16), and substituting (21) in (15) and (17). Again substituting the results of those substitutions in (12) and (13), along with (18) and (19) also in (12) and (13). If that was confusing we are just substituting the maximized values and the conjectured value function into the HJB-equation. We obtain:

$$\begin{gathered} r(A_S + B_S P) = c_1[n(c_1 + B_S)] - \frac{1}{2} n[(c_1 + Bs)]^2 + n\rho_1 E_{S0} - n\gamma P \\ + B_S[n(c_1 + B_S) + m(c_2 + B_{NS}) - kP] \end{gathered} \tag{22}$$

$$\begin{gathered} r(A_{NS} + B_{NS} P) = c_2(c_2 + B_S) - \frac{1}{2}(c_2 + Bs)^2 + \rho_2 E_{NS0} - \gamma P \\ + B_{NS}[n(c_1 + B_S) + m(c_2 + B_{NS}) - kP] \end{gathered} \tag{23}$$

Our next step is to solve for the coefficients A_S , B_S , A_{NS} and B_{NS} , to do that lets consider the HBJ equation for the signatories (22), first. We expanding the quadratic terms, we obtain:

$$rA_S+rB_sP=nc^2+ncB_S-\frac{1}{2}n(c_1^2+2c_1B_S+B_S^2)+n\rho_1E_{S0}-n\gamma P$$
$$+nc_1B_S+nB_s^2+mc_2B_S+mB_SB_{NS}-kP \quad (24)$$

The trick to find the coefficients is to notice that the left-hand side must equal the right-hand side for all values of P. That is equivalent of saying that:

$$X+YP=Z+WP$$

Where:

$$X=rA_S$$
$$Y=rB_SP$$
$$Z=nc_1^2+nc_1B_S-\frac{1}{2}n(c_1^2+2c_1B_S+B_S^2)+n\rho_1E_{S0}$$
$$+nc_1B_S+nB_s^2+mc_2B_S+mB_SB_{NS}$$
$$W=-n\gamma P-kP$$

Making $YP=WP$, yields:

$$rB_SP=-ndP-kB_SP$$
$$rB_S+kB_s=-n\gamma$$

$$B_S=-n\frac{\gamma}{r+k} \quad (25)$$

And making $X=Z$, yields:

$$rA_S=nc_1^2+nc_1B_S-\frac{1}{2}nc_1^2-nc_1B_S-\frac{1}{2}B_S^2+n\rho_1E_{NS0}+nc_1B_S+nB_S^2+mc_2B_S+mB_SB_{NS}$$

After rearranging and making algebraic manipulations, we obtain the coefficient A_S as:

$$A_S=\frac{1}{2r}n(a-\rho_1+B_S)^2+(N-n)\frac{B_S}{r}(a-\rho_2+B_{NS})+n\frac{\rho_1}{r}E_{NS0} \quad (26)$$

Notice that we have substituted back the values defined for *c* and *m*.

For the non-signatories the problem is very similar, so we won't work everything again. We merely state that the coefficients B_{NS} and A_{NS} are given by:

$$B_{NS}=\frac{-\gamma}{r+k} \tag{27}$$

$$A_{NS}=\frac{1}{2r}(a-\rho_2+B_{NS})^2+(N-n-1)\frac{B_{NS}}{r}(B_{NS}+1)+\frac{nB_{NS}}{r}(a-\rho_1+B_S)+\frac{\rho_2}{r}E_{NS0} \tag{28}$$

The value function for the signatories and non-signatories respectively are:

$$V_S=-n\frac{\gamma}{r+k}P+\frac{1}{2r}n\left(a-\rho_1-n\frac{\gamma}{r+k}\right)^2-(N-n)n\frac{\gamma}{r(r+k)}\left(a-\rho_2-\frac{\gamma}{r+k}\right)+n\frac{\rho_1}{r}E_{NS0} \tag{29}$$

$$\begin{aligned}V_{NS}=\frac{-\gamma}{r+k}P+\frac{1}{2r}\left(a-\rho_2-\frac{\gamma}{r+k}P\right)^2-(N-n-1)\frac{\gamma}{r(r+k)}\left(1-\frac{\gamma}{r+k}\right)\\-n\frac{\gamma}{r(r+k)}\left(a-\rho_1-n\frac{\gamma}{r+k}\right)+\frac{\rho_2}{r}E_{NS0}\end{aligned} \tag{30}$$

Now that we have established the optimal emission levels for the signatories and non-signatories and derived the value functions, we have to find the solution to the stock of pollution in equation (3), to do that we have to find the total level of emissions. Summing (16) and (17) and making the adequate substitutions yields:

$$\sum E_{NS}+\sum E_S=(N-n)\left(a-\rho_2-\frac{\gamma}{r+k}\right)+n\left(a-\rho_1-n\frac{\gamma}{r+k}\right)$$

$$\frac{dP}{dt}=(N-n)\left(a-\rho_2-\frac{\gamma}{r+k}\right)+n\left(a-\rho_1-n\frac{\gamma}{r+k}\right)-kP \tag{31}$$

This is a first-order non-homogeneous linear differential equation, whose solution is:

$$P=\frac{A}{k}+C_1e^{-kt} \tag{32}$$

Where:

$A=(N-n)\left(a-\rho_2-\frac{\gamma}{r+k}\right)+n\left(a-\rho_1-n\frac{\gamma}{r+k}\right)$

$C_1=P_0-\frac{A}{k}$

5.1.3 – The participation game

Now that we have derived the optimal emissions and the corresponding optimal trajectory of the emission game, we have to find the size of the self-enforcing coalition. To do that we substitute the optimal emission from the signatories and non-signatories with the optimal trajectory in (10) and (11) and integrate.

Let us recall how those expressions look like.

$$\sum_{i=1}^{n} W_S=\int_0^{\infty} e^{-rt}[(a-\rho_1)\sum_{i=1}^{n} E_i-\frac{1}{2}\sum_{i=1}^{n} E_i^2+n\rho_1 E_{S0}-n\gamma P]dt$$

For the signatories, and:

$$W_{NS}=\int_0^{\infty} e^{-rt}[(a-\rho_2)E_j-\frac{1}{2}E_j^2+\rho_2 E_{NS0}-\gamma P]dt$$

For the non-signatories:

The optimal emissions level are:

$$\sum E_S=n(a-\rho_1-n\frac{\gamma}{(r+k)})$$

For the signatories, and:

$$E_{NS}=a-\rho_2-\frac{\gamma}{(r+k)}$$

For the non-signatories.

Let us first calculate the indefinite integral. Making the corresponding substitutions, yields:

$$\int e^{-rt}[(a-\rho_1)n[(a-\rho_1)-n\frac{\gamma}{(r+k)}]-\frac{1}{2}(a-\rho_1-n\frac{\gamma}{(r+k)})^2+\rho_1 nE_{NS0}-n\gamma P]dt$$

$$\int e^{-rt}[(a-\rho_2)[(a-\rho_2)-\frac{\gamma}{(r+k)}]-\frac{1}{2}(a-\rho_1-\frac{\gamma}{(r+k)})^2+\rho_2 E_{NS0}-dP]dt$$

Those two expression look quite complicated to integrate, but they really aren't. Let us make some modifications:

$$W_S=\int e^{-rt}[M1+m1P]dt \tag{33}$$

$$W_{NS}=\int e^{-rt}[M2+m2P]dt \tag{34}$$

Where:

$M1=(a-\rho_1)n[(a-\rho_1)-n\frac{\gamma}{(r+k)}]-\frac{1}{2}(a-\rho_1-n\frac{\gamma}{(r+k)})^2+\rho_1 nE_{S0}$

$m1=-n\gamma$

$M2=(a-\rho_2)[(a-\rho_2)-\frac{\gamma}{(r+k)}]-\frac{1}{2}(a-\rho_1-\frac{\gamma}{(r+k)})^2+\rho_2 E_{NS0}$

$m2=-\gamma$

Now, substituting the solution to our differential equation in (32), yields

$$W_S=\int e^{-rt}[M1+m1(\frac{A}{k}+C_1e^{-kt})]dt$$

$$W_{NS}=\int e^{-rt}[M2+m2(\frac{A}{k}+C_1e^{-kt})]dt$$

We can improve this by doing:

$$W_S=M1\int_0^\infty e^{-rtdt}+[m1(\frac{A}{k})]\int_0^\infty e^{-rt}dt+m1C_1\int_0^\infty (e^{-(r+k)}t)dt$$

$$W_{NS}=M2\int_0^\infty e^{-rtdt}+[m2(\frac{A}{k})]\int_0^\infty e^{-rt}dt+m2C_1\int_0^\infty (e^{-(r+k)}t)dt$$

The definite integral from zero to infinite is:

$\int_0^\infty e^{-rtdt}=\frac{1}{r}$

$\int_0^\infty e^{-(r+k)t}dt=\frac{1}{(r+k)}$

So the discounted pay-off for the signatories and non-signatories are:

$$W_S = [M1 + m1(\frac{A}{k})]\frac{1}{r} + \frac{m1C_1}{(r+k)} \tag{35}$$

$$W_{NS} = [M2 + m2(\frac{A}{k})]\frac{1}{r} + \frac{m2C_1}{(r+k)} \tag{36}$$

If we substitute back every coefficient, this expression will look too complicated and intractable, so we have to find the self-enforcing size of the coalition numerically.

The self-enforcing criteria is also a game where agents have two strategies: join the agreement or do not join the agreement. When deciding if joining the agreement is going to be profitable, the reasoning is that, if he is a cooperator, what would be his pay-off if he deviated, that is, if he left the agreement. Similarly, a non-cooperator would compare his pay-off against the pay-off he would receive if the joined the agreement. This reasoning is the following inequalities:

$$\begin{aligned} W_S(n*) &\geq W_{NS}(n*-1) \\ W_{NS}(n*) &\geq W_S(n*+1) \end{aligned} \tag{37}$$

The first inequality is the internal stability condition, the second one is the external stability condition.

To proceed in analyzing our numerical results we consider initially three cases: a baseline case with no permits, a weak treaty with a higher cap level entailing a higher supply of permits and a stronger treaty with lesser supply of pollution permits. In the voluntary market, since we assume that they are smaller, we assume that the supply of permits is always less than the regulated market, but at a smaller price as well.

Our model will be calibrated with numerical parameters. It is important to highlight that those parameters do not reflect any real data. The literature follows this pattern, so we follow it as well. But we need to choose parameter values so as to avoid nonsensical results. When we parameterize our model we want to establish two condition: the level of emissions is always positive, thus avoiding zero or negative emission rates; We want also that the steady state level of pollution to be non negative as well. Those two conditions are necessary in order that our model avoid nonsensical results. Some parameters are directly taken from the literature, namely the discount rate and the rate of decay of pollution are directly taken from

Rubio and Casino (2005), the damage parameter is taken from Breton et al (2010). To make sure our model is well calibrated, we make the remark that when absent the permit market, our model reduces to the model proposed by Rubio and Casino (2005), therefore a baseline result is present to reproduce the results achieved by Rubio and Casino (2005), then we proceed to add the permit market on top of the baseline model, rested assured that we have provided well selected parameters. And to make sure that our results are not random, we will provide some robustness checks, but we relegate those results to an appendix and based on those results discuss some possible limitations of our model.

5.1.4 – Baseline results

Our model parameters are:

N = 10;

a = 200;

γ = 0.3;

ρ_1 = 1.4;

ρ_2 = 0.8;

E_{S0} = 160 (weak agreement), 96 (stringent agreement);

E_{NS0} = 186;

r = 0.025;

k = 0.05;

P0 = 0

The small number for the parameter *N* is to make easier to present the results, and also one can suppose that each value of *n* represent a block of countries that sign the agreement. For the baseline results the values of ρ_1 and ρ_2 are set to zero. The parameters *r* and *k* where inspired by Rubio and Casino(2005). Given these parameters the self-enforcing coalition is:

Table 1:The baseline calibration

n	W_S	W_{NS}
1		4.8608e+05

2*	4.864e+05	4.8736e+05
3*	4.8736e+05	4.8992e+05
4	4.8896e+05	4.9376e+05
5	4.912e+05	4.9888e+05
6	4.9408e+05	5.0528e+05
7	4.976e+05	5.1296e+05
8	5.0176e+05	5.2192e+05
9	5.0656e+05	5.3216e+05
10	5.12e+05	

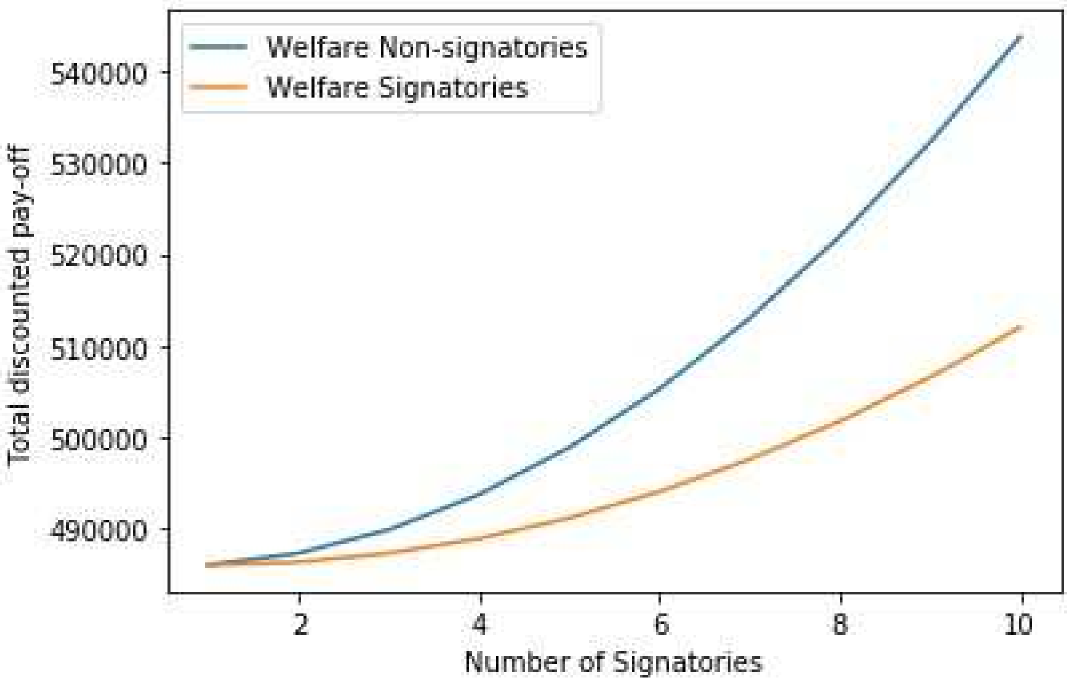
Source: research results.

The number of the stable coalition is given by the stability condition given in (37), we must compare the total discounted pay-off for a signatory at given size of coalition *n* with the total discounted pay-off of a non-signatory at size of a coalition "*n*-1". So we always compare the pay-off at *n,* say 5, and compare to the pay-off of the non-signatory directly below 5, which is 4. We proceed this way until we find an *n* which gives the largest pay-off for the signatories in that, if an additional agent were to join the agreement, his pay-off would decline, so he chooses to be a non-signatory.

The number of the self-enforcing coalition is marked with an asterisk, therefore is two and three. It is possible to inspect that WS(2) = 4.864e+05 and WNS(1) = 4.8608e+05, so WS(2) > WNS(1), so two countries form a bilateral agreement, at WS(3) = WNS(2) marks a point of indifference, but WS(4) < WNS (3), so at most three countries sign the agreement. This is coherent with the results achieve by Barrett (1994) and Rubio and Casino (2005), in fact the baseline case is a replication of those results, so our model has indeed arrived at the same baseline results although through a slightly different approach.

To better illustrate the results, a diagram helps the visualization:

Figure 2: The baseline calibration



Source: research results

The graph above is a visualization of the table we have just presented, each entry in Table 1 is plotted in the graph, so we can see when the pay-off of a signatory is higher when compared to the pay-off of a non-signatory. Remember, we must compare the welfare of for a given number of signatories, say 3, with the pay-off of a non-signatory at 2, this is the correct way of reading the graph. The blue line is the welfare of the non-signatory countries while the orange line is the welfare of the signatory countries. Notice that as the number of signatories increase, the gap between the orange line and the blue line increases, this difference is the size of incentive to free-ride. Also notice that the pay-off from cooperation is increasing, the pay-off from full cooperation is also higher than the pay-off from the fully non-cooperative case. We have illustrated then how the baseline calibration reflects the classical prisoner's dilemma nature of our problem.

5.1.5 – Results with a permit system

Now, lets suppose that a pollution permit system exists. That is, agents will be endowed with an initial amount of pollution permits such that, if they exceed their cap established by their initial endowment they have to buy additional permits in a market at an exogenous price.

To decide how to set the initial endowments, we can consider the emissions at the full cooperative outcome and set how much we would like to reduce from that.

Reminding us that the emission of signatories is given by:

$$E_{NS}=a-\rho_2-n\frac{\gamma}{(r+k)}$$

And setting the previously given parameters, we have that the cooperative level of emission is:

$$E_{NS}=200-(\frac{10(0.3)}{0.025+0.05})=160$$

Now, let us suppose two sort of agreements, a weak and a more stringent one. In the weak agreement agents are distributed pollution permits that are equal to the full cooperative outcome of the game without a permit market, that is, if they pollute more than the full cooperative outcome, agents musty buy permits to compensate for the excess of emissions. In the strong agreement agents are given 60% of the permits of the weak agreement, that is, the incentive to pollute decreases as agents incur in higher costs of emitting pollutants. In the voluntary markets we assume that agents, due to they own environmental concern, set a cap of 5% of their business-as-usual level of emissions. Performing this calculation using the optimal emissions level for the non-signatories we find that their self-imposed cap is equal to 186,2. To work with round numbers we set their cap at 186.

Table 2:Weak agreement calibration

n	W_S	W_{NS}
1	4.852552e+05	4.814152e+05

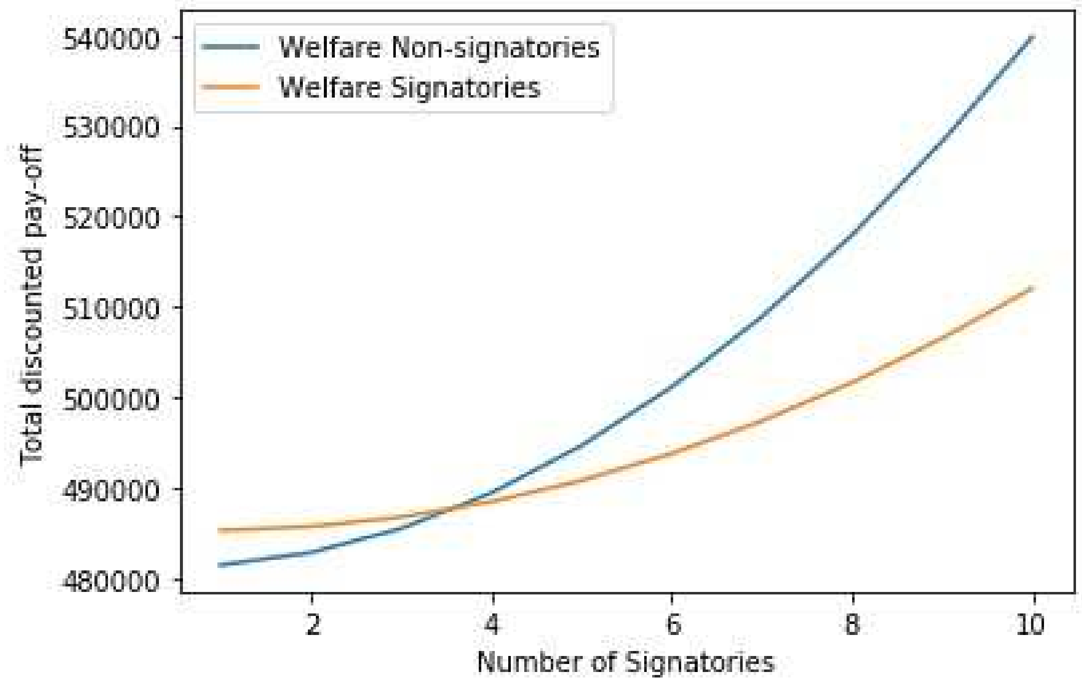
2	4.856711e+05	4.827911e+05
3	4.867271e+05	4.854472e+05
4	4.884232e+05	4.893832e+05
5*	4.907591e+05	4.945991e+05
6	4.937352e+05	5.010952e+05
7	4.973511e+05	5.088711e+05
8	5.016071e+05	5.179272e+05
9	5.065032e+05	5.282631e+05
10	5.120391e+05	5.398791e+05

Source: Research results

As one can observe, the size of the self-enforcing coalition is 5, an improvement over the baseline model. The intuition behind this result is that with the permit system the agents have an incentive to cooperate, as the number of agents in the agreement increase, they jointly cut emissions, by coordinating their action through the cap and the permit price, they are able to approximate their emission levels closer to the full cooperative outcome.

The graph for the model with a permit system is given below.

Figure 3:Weak agreement calibration



Source: research results

Notice that the incentive to defect is still present and we cannot achieve full participation. We must highlight the fact that the size of the self-enforcing coalition is not when the two lines intersect. Remember that an agent compares his pay-off of joining the coalition with the pay-off of a coalition of size *n-1*.

Now we report the results for the strong agreement, that is, with a smaller initial endowment of permits:

Table 3:Stringent agreement calibration

n	W_S	W_{NS}
1	4.818952e+05	4.79495e+05
2	4.823111e+05	4.808711e+05
3	4.833671e+05	4.835272e+05
4*	4.850632e+05	4.874632e+05
5	4.873991e+05	4.926791e+05
6	4.903752e+05	4.991752e+05
7	4.939911e+05	5.069511e+05

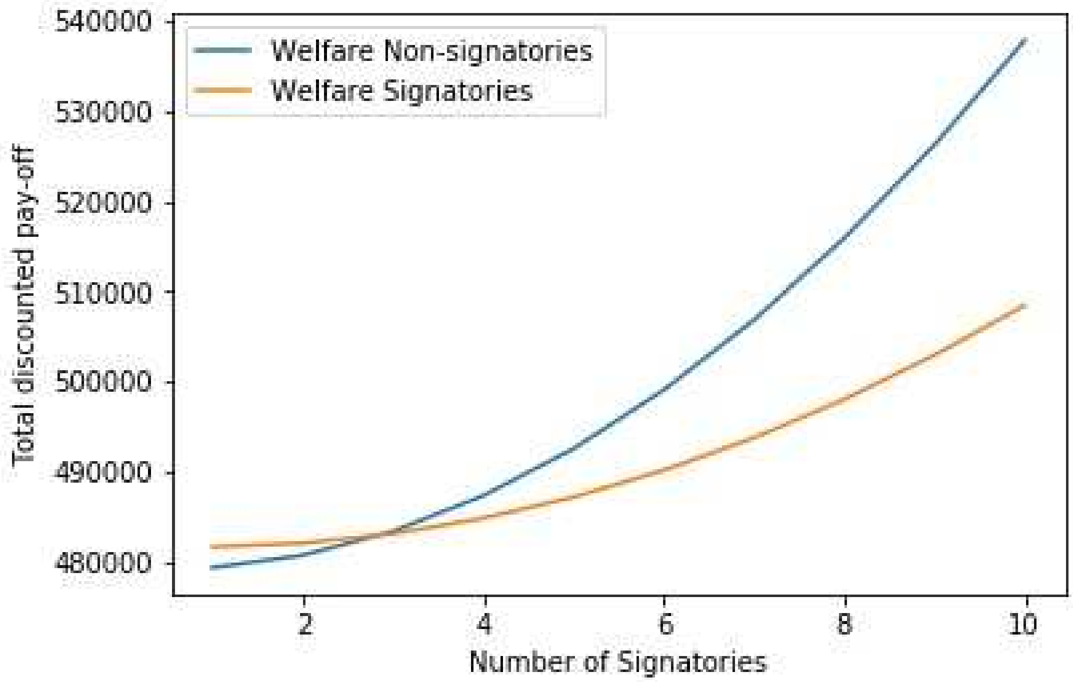
8	4.982471e+05	5.160072e+05
9	5.031432e+05	5.263431e+05
10	5.086791e+05	5.379591e+05

Source: research results

In the strong agreement, the size of the self-enforcing coalition has dropped to 4. This indicates that as the cap on the emission level decreases, countries will find more profitable to deviate from the agreement. Fewer pollution permits penalizes agents when they can in fact stay bellow the emissions cap, we can conclude that, fewer permits will create an incentive to emit more pollutants as opposed to cutting back on emissions and profit from the reduction in pollution emission.

Bellow we show the graph for the strong agreement:

Figure 4:Stringent agreement calibration



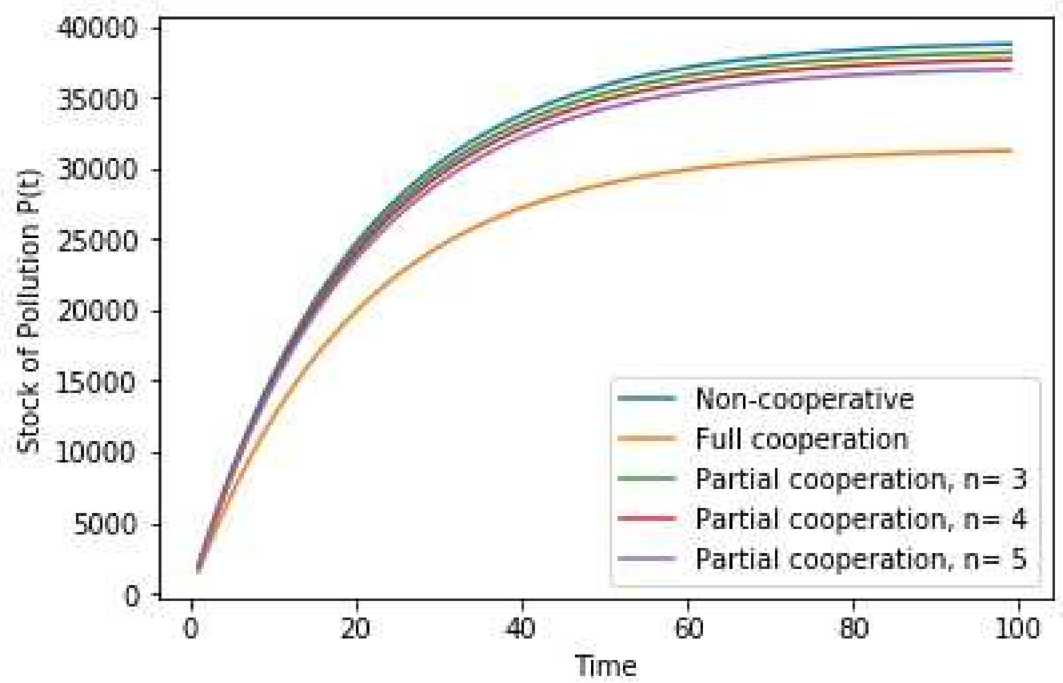
Source: Research results

We again see that the incentive to join the agreement has decreased when compared to the weak agreement but we still could find a higher cooperation level when compared to the baseline model with no permit system.

We also would like to stress that cooperation is fundamental, it is possible to observe that in all cases the welfare from total cooperation is higher than in the non-cooperative case. Also, the steady-state stock of pollution under cooperation is always lower than in the non-cooperative case. So, from our model we can also conclude that it is unlikely that strictly decentralized markets can cope with a global externality problem such as climate change. Cooperation is fundamental along with mechanism that provide the correct market incentives.

Given that we have found the size of the stable self-enforcing coalitions for the baseline case, the weak and strong treaty, as well as the full cooperative outcome, which is just given by the maximum number of players, we can compare the steady-state stock of pollution for the various outcomes. Below, a graph that shows how the steady-state stock of pollution varies for each outcome considered in our numerical explorations:

Figure 5:Steady-state comparison



Source: research results

The different colors of the lines represent a different outcome of the model, we can observe that the smallest level of the stock of pollution occur for the full-cooperative outcome, this strengthens the general conclusion that cooperation is vital to tackle the problem of climate change. The non-cooperative outcome achieves the highest level of stock pollutants, showing how if each player behaves non-cooperatively there is a tendency to over exploit common global natural resources. Although the existence of the voluntary market could reduce somewhat the emission level, the result pales in comparison with the cooperative outcome. We can also observe that partial cooperation diminishes the steady state level of stock pollutant for increasing levels of participation. We can establish then, based on our results the following order relationship

$$SS_{NC} < SS_{PC} < SS_{FC} \tag{38}$$

Where:
SS_{NC} = Non-cooperative steady-state of the pollution stock;
SS_{PC} = Partial cooperation steady-state of the pollution stock;
SS_{FC} = Full cooperation steady-state of the pollution stock.

5.1.6 – Limitations of the model

In this section we would like to discuss the limitation of our model. As the reader will see in the appendix A, our results are sensitive to parameter variations. That is, had we picked different parameters our numerical results would have varied. We would like to stress this limitation insofar as it hampers precise quantitative statements such as “given a permit market system the size of the stable coalition will be five”. Our model cannot provide such results, but we believe that the statement that the weak agreement entails a larger coalition in comparison with the stringent agreement is true. When we vary the parameters the total discounted welfare of signatories and non-signatories vary as well, while some variations, like the damage parameter can negatively influence the size of stable coalitions, parameters such as the discount rate can increase the size of stable coalitions. The results provided in the main body of the text provide then a possible scenario when the parameters are situated inside a specific range.

In order to improve the existing model we recognize that making the price of the permits endogenous could improve the results. If the price of permits responds to higher or lower demand from the players we could gain a more stable set of results. At the same time we believe that we have given a step towards understanding how flexible mechanisms can aid agents to coordinate their actions in order to control emission levels and climate change.

6 – Closing remarks and future research direction

We set for ourselves in this work the task of analyzing the effect of a permit market on the size of stable coalitions in an international environmental agreement game in a dynamic context. Our main hypothesis was that a permit market system would increase the level of participation in the agreement but if the agreement was too stringent on the level of emissions, participation would decrease. To investigate the hypothesis we have solved a two-stage differential game, where we found the optimal emissions level for the players, the optimal trajectory of the pollution stock and we have applied the stability condition in order to find the stable number of signatories of the agreement. The proposition in the hypothesis turned out to be true for our numerical experiments. We have proposed a weak agreement, where the number of permits was higher when compared with the strong agreement. The number of signatories on the weak agreement was 5, while in the strong agreement was 4. This is an improvement when compared to the baseline case, where the permit market in not taken in consideration. The interpretation of this result is that a market mechanism is a useful tool to aid agents to coordinate their action in reducing emissions, when a price is put on the excess emission agents had an additional incentive to approximate their level of emission closer to the full cooperative level. However, if the number of permits was too distant from the full cooperative outcome, the incentive to leave the agreement became present. We can conclude that the design of the permit market is of relevance when proposing an agreement to reduce emission of pollutants. From a policy perspective, then, we can conclude that the number of pollution permits is an important tool to generate enough incentives for countries to join an international environmental agreement. A design which could be considered too ambitious (too few permits), could reduce the number of willing participants when compared to a more modest target for reduction given by the number of permits.

A final conclusion is that cooperation is fundamental, by analyzing the trajectory of the optimal stock of pollution we could verify that the full cooperation led to the lowest level of the stock of pollution, while the full non-cooperative outcome yielded the highest stock of pollution. This imply that, even though we considered that non-signatories had access to a voluntary market for carbon permits the level of emission was still high enough to generate a worse outcome when compared to the full cooperative solution. Partial cooperation reduced the stock of pollution when compared to the full noncooperative outcome but stayed well above the full cooperative solution. This reinforces our conclusion that cooperation is fundamental to deal with global environmental externalities.

For future reseach, there are ready extension to be applied to our model. One can consider for example, endogeneizing the price of permits. A possible formulation would be to consider the price as another state equation of the system. We can consider also a different participation game. In our model we consider that players decide once-and-for-all if they join the agreement or not, and no renegotiation takes place. If the participation game if dynamic itself we can consider how the number of signatories evolve in time. Rubio and Ulph (2007) and Bretton et al (2010) give a mechanism by which this can be modeled. Another possible direction is to consider a different equation for the stock of pollution. The linear differential equation is simplistic and a richer dynamics for the pollution stock would be very useful in analyzing how agents respond to this kind of environment. There is already a literature of the economics of lakes, where multiple steady states are possible, so if something similar is present for the climate the conclusions for the stability of environmental agreements could be quite different. The IEA game can also be extended to arbitrarily complex environment, games could be coupled with growth models, trade models, heterogeneous agents, etc… Those would require, however, more extensive use of computational analysis. The model we have solved in this work is one of the simplest kinds, a linear function of the state variable in the objective functional and a single linear differential equation for the state variable. Even in this setting we could not work out the full implications of the model analytically, so more complex models will be more demanding in computational abilities. We believe this is a challenging area of study but there much room for future contributions on this very relevant topic.

Appendix A – Robustness Checks

The purpose of this appendix is to carry out some robustness checks which involve varying a few parameters to see how our results are sensitive to parameter variations. We want to vary parameters γ, r, ρ_1 to a certain extent to see how the model behaves. In this way we can uncover the limitations of our model and be more sure upon which conditions our conclusions hold. We will only vary those three parameters because parameter "a" has been shown to be very robust to changes, even significant ones, so we will only show the ones which reveal the weakness of our model. We will also only vary the permit price ρ_1, since the changes in relative prices can be analyzed by varying one parameter only. The parameter for the decay of pollution will not be modified since this is given by actual measurements (Rubio and Casino, 2005). In order to not extend too much the number of pages we will present the pictures which contain the same information as the tables presented in our results.

A.1 – Variations in γ

Figure A1 - $\gamma=0.32$ n*=5

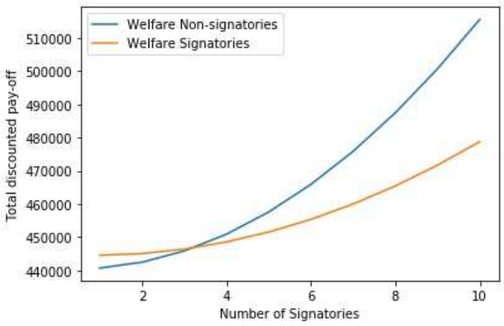
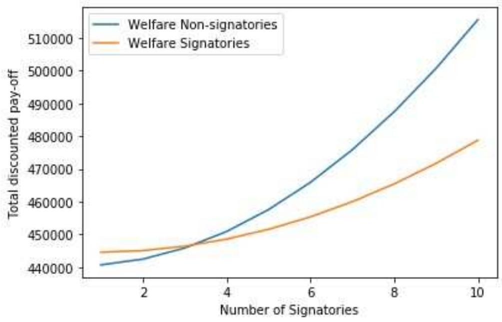


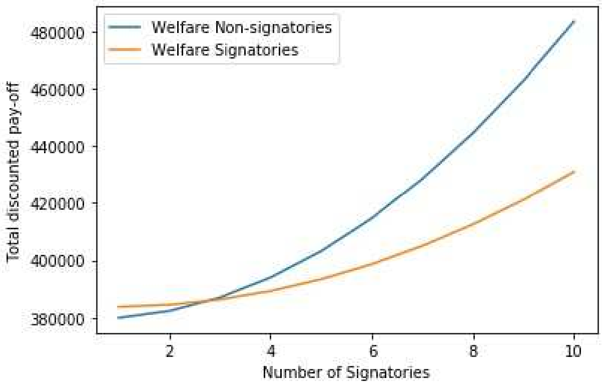
Figure A2 - $\gamma=0.34$ n*=5



Source: Research results

Source: Research results

Figure A3 - $\gamma = 0.4$ n*=4

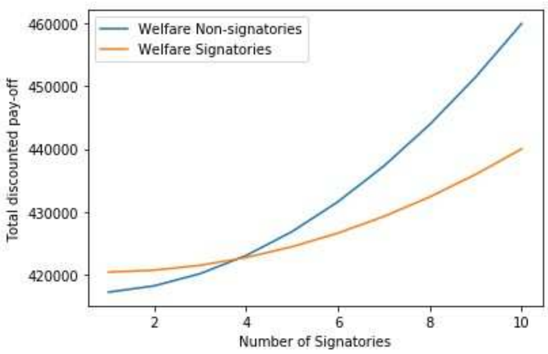


Source: Research results

From the figures above one can see that by increasing the level of damages, the orange line which corresponds to the total discounted welfare level for the signatory countries has a downward displacement. This reveals that the higher the damage caused by the pollution stock the less is the incentive to join the agreement.

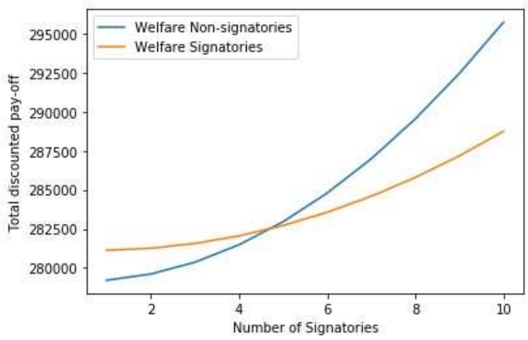
A.2 – Variations in r

Figure A4 - $r = 0.03$ n*=5



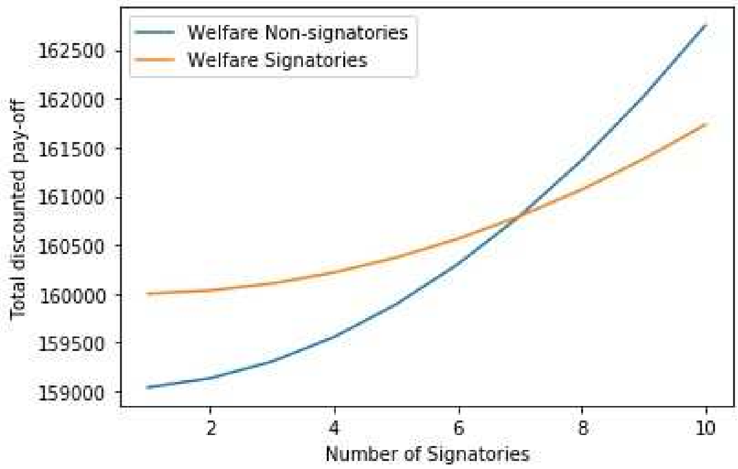
Source: Research results

Figure A5 - $r = 0.05$ n*=6



Source: Research results

Figure A6 - $r = 0.1$ n* = 8

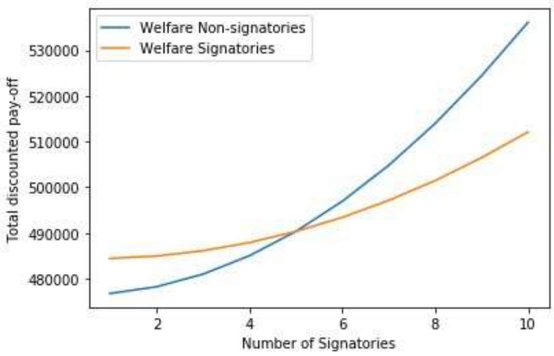


Source: Research results

As it is possible to observe our results are also sensitive to changes in the discount rate. The higher the discount rate, the higher the level of participation in the agreement.

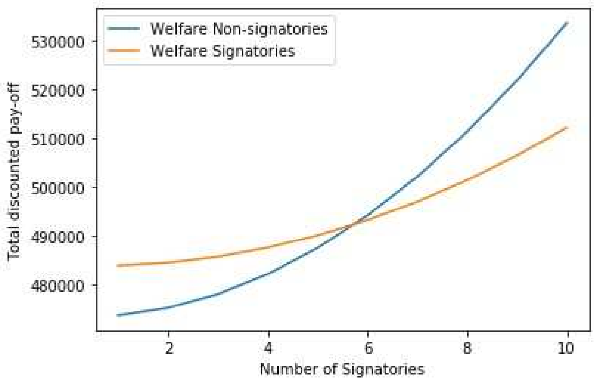
A.3 – Variations in ρ_1

Figure A7 - ρ_1=2.0 n*=7



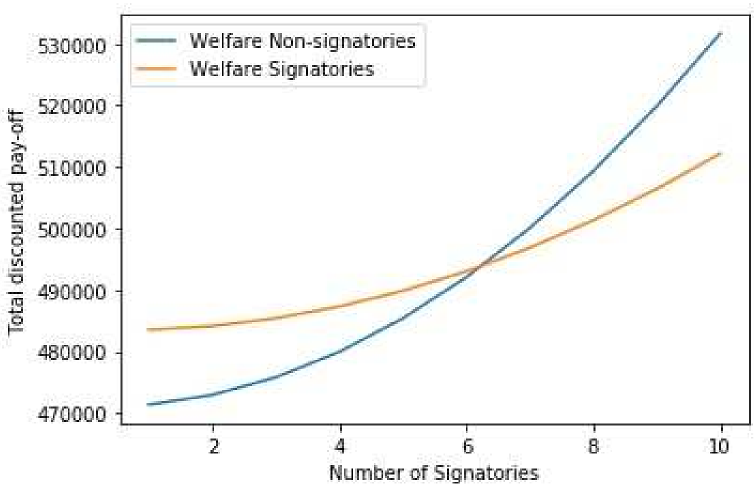
Source: Research results

Figure A8 - ρ_1=2.4 n*=7



Source: Research results

Figure A9 - ρ_1=2.7 n* =8



Source: Research results

We can observe that by increasing the size of the parameter the size of the stable coalition increase. All of the robustness checks have been conducted considering the weak agreement. But identical results were obtained in the stringent agreement, that is, the curves on the above graphs behave exactly the same way by varying each parameter. Our results are then qualitatively robust, that is, the weak agreement will yield a higher number of signatories given a set of parameters when compared to the stringent agreement.

Bibliographic References

ASPREMONT, C. D.; JACQUEMIN, A. On the Stability of Collusive Price Leadership. The Canandian Journal of Economics, v. 16, p. 17-25, fev. 1983

BARRETT, S. Chapter 28 The theory of international environmental agreements in: Handbook of Environmental Economics, v. 3, p. 1458-1493, 2005.

BARRET, S. Self-enforcing International Environmental Agreements. Oxford Economic Papers, v. 46, p. 878-894, out. 1994

Barrett, S., Stavins, R. Increasing Participation and Compliance in International Climate Change Agreements. International Environmental Agreements: Politics, Law and Economic v. 3, p. 349–376, 2003.

BASAR, Tamer; HAURIE, Alain; ZACCOUR, Georges. Nonzero-Sum Differential Games. In: BASAR, Tamer; ZACCOUR, Georges. Handbook of Dynamic Game Theory, 1 ed. Springer, 2018.

BRETON, M.; SBRAGIA, L.; ZACCOUR, G. A dynamic model for international environmental agreements. Environmental and Resource Economics, v. 45, n. 1, p. 25–48, 2010.

BOLIN, Bert. A history of the science and politics of climate change: The role of the intergovernmental panel on climate change. 1 ed, Cambridge University Press, 2007. 9780521880824 DOI: 10.1017/CBO9780511721731.

CALVO, E.; RUBIO, S. J. Dynamic models of international environmental agreements: A differential game approach. International Review of Environmental and Resource Economics, v. 6, n. 4, p. 289–339, 2012.

CARRARO, C.; SINISCALCO, D. Strategies for the international protection of the environment. Journal of Public Economics, v. 52, n. 3, p. 309–328, 1993.

CHEVALLIER, Julien. Econometric Analysis of Carbon Markets. 1 ed. Springer, 2012 DOI: 10.1007/978-94-007-2412-9.

DOCKNER, Engelbert J.; VAN LONG, Ngo. International pollution control: Cooperative versus noncooperative strategiesJournal of Environmental Economics and Management, 1993. DOI: 10.1006/jeem.1993.1023.

EBOLI, Fabio; PARRADO, Ramiro; ROSON, Roberto. Climate-change feedback on economic growth: Explorations with a dynamic general equilibrium model. Environment and Development Economics, *[S. l.]*, v. 15, n. 5, p. 515–533, 2010. DOI: 10.1017/S1355770X10000252.

FANKHAUSER, Samuel; TOL, Richard S. J. On climate change and economic growth. Resource and Energy Economics, *[S. l.]*, v. 27, n. 1, p. 1–17, 2005. DOI: 10.1016/j.reseneeco.2004.03.003.

FISHER, Janet. Voluntary carbon markets. An international business guide to what they are and how they work. [s.l: s.n.]. v. 7 DOI: 10.1080/19438150903314354.

GRUNEWALD, Nicole; MARTINEZ-ZARZOSO, Inmaculada. Did the Kyoto Protocol fail? An evaluation of the effect of the Kyoto Protocol on CO2 emissions. Environment and Development Economics, *[S. l.]*, v. 21, n. 1, p. 1–22, 2016. DOI: 10.1017/S1355770X15000091.

IBIKUNLE, Gbenga; GREGORIOU, Andros. Carbon markets: Microstructure, pricing and policy. 1 ed. Palgrave-Macmillan, 2018. DOI: 10.1007/978-3-319-72847-6.

IPCC, Climate Change 2014 Synthesis Report, 1 ed. Cambridge University Press, 2014.

IPCC. Climate Change 2014 Part A: Global and Sectoral Aspects, 1 ed. Cambridge University Press, 2014.

IPCC. Impacts of 1.5°C of Global Warming on Natural and Human Systems. Special Report, Intergovernmental Panel on Climate Change, *[S. l.]*, n. ISBN 978-92-9169-151-7, p. 175–181, 2018. Disponível em: http://report.ipcc.ch/sr15/pdf/sr15_chapter3.pdf.

KOMPAS, Tom; PHAM, Van Ha; CHE, Tuong Nhu. The Effects of Climate Change on GDP by Country and the Global Economic Gains From Complying With the Paris Climate Accord. Earth's Future, *[S. l.]*, v. 6, n. 8, p. 1153–1173, 2018. DOI: 10.1029/2018EF000922.

KUTNEY, Gerald. Carbon politics and the failure of the kyoto protocol. 1 ed. Routledge, 2014. DOI: 10.4324/9781315851099.

LENTON, Timothy M. Tipping points in the climate system. Weather, *[S. l.]*, v. 76, n. 10, p. 325–326, 2021. DOI: 10.1002/wea.4058.

LI, Shoude. A Differential Game of Transboundary Industrial Pollution with Emission Permits Trading. Journal of Optimization Theory and Applications, *[S. l.]*, v. 163, n. 2, p. 642–659, 2014. DOI: 10.1007/s10957-013-0384-7.

MATHEZ, E.A; SMERDON, J.E, Climate Change: The Science of Global Warming and Our Energy Future, 2 ed. Columbia University Press, 2018.

OBERTHÜR, S. HERMANN, O. The Kyoto Protocol – International Climate Policy for the 21st Century, 1 ed. Springer, 1999.

PETROSYAN, L. ZENKEVICH N. Game Theory, 2 ed. World Scientific, 2016

POPOVSKI, V. The Implementation of the Paris Agreement on Climate Change, 1 ed. Routledge, 2018.

POWELL, J. L. The Inquisition of Climate Science, 1 ed. Columbia University Press, 2011

RUBIO, S. J.; CASINO, B. Self-enforcing international environmental agreements with a stock pollutant. Spanish Economic Review, v. 7, n. 2, p. 89–109, jun. 2005.

RUBIO, S. J.; ULPH, A. An infinite-horizon model of dynamic membership of international environmental agreements. Journal of Environmental Economics and Management, v. 54, n. 3, p. 296–310, nov. 2007.

TOL, Richard S. J. The economic impacts of climate change. Review of Environmental Economics and Policy, *[S. l.]*, v. 12, n. 1, p. 4–25, 2018. DOI: 10.1093/reep/rex027.

UNITED NATIONS. United Nations Framework Convention on Climate Change, 1992

YEUNG, D.W.K; PETROSYAN, L.A, Cooperative Stochastic Differential Games. 1 ed. Springer, 2006.

ZEEUW, A. DE. Dynamic effects on the stability of international environmental agreements. Journal of Environmental Economics and Management, v. 55, n. 2, p. 163–174, mar. 2008.

ZILLMAN, John W. Journal of the World Meteorological Organization Bulletin. The Journal of the World Meteorological Organization, *[S. l.]*, v. 58 (3), n. July, p. 141–150, 2009.