PAPER

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IOP Publishing | Royal Swedish Academy of Sciences Physica Scripta

Phys. Scr. **90** (2015) 035105 (6pp) doi:10.1088/0031-8949/90/3/035105

Quantum properties of a which-way detector

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Received 4 August 2014, revised 1 December 2014
Accepted for publication 8 December 2014
Published 5 February 2015

Abstract

We explore quantum properties of a which-way detector using three versions of an idealized two slit arrangement. First, we derive complementarity relations for the detector; second, we show how the 'experiment' may be altered in such a way that using a single-position measurement on the screen, we can obtain quantum erasure. Finally, we show how to construct a superposition of 'wave' and 'particle' components.

Keywords: quantum information, quantum eraser, complementarity

1. Introduction

Wave-particle duality is one of the most counterintuitive features of quantum theory. In double-slit experiments, this problematic feature becomes explicit by the fact that if, in an experimental apparatus, the information about which slit the quanton (in the sense of [1, 2]) has crossed (which-way information) is available, there are no interference fringes on the screen (particle behavior); however, if the which-way information is not available one can see an interference pattern (wave behavior). In a famous debate between Bohr and Einstein, at the Solvay conference (1927) [3], they introduced a gedanken experiment that consists of double slit experiment with a movable slit placed before the double slit, the recoiling slit experiment. A quanton is first sent through the movable slit, after it crosses the double slit and then it is recorded on the screen. Therefore, observations of the movable slit position, after the interaction with the quanton, can give us information about which slit (of the double-slit apparatus) the quanton has crossed. The apparent difficulty imposed by this apparatus is that one could have both particle and wave behavior in the same experiment in contradiction with the wave-particle duality. However, this apparent difficulty was solved by Bohr, who pointed out that a careful analysis of the movable slit would require the inclusion of uncertainty relations of its position and momentum, which would add random phases in the quantons path, and consequentially it would make the interference pattern vanish. Bohr's arguments in favor of the wave-particle duality were based on the uncertainty principle.

During the 1980s, entanglement began to play an important role in the analysis of interferometric systems. In references [4–6], the authors show that the entanglement between an interferometric quanton and a meter (included in the quantum description) can destroy the interference pattern. The interaction with a probe system makes the which-way information available, and it is sufficient to wash out the interference pattern. According to the authors, entanglement is determinant in this phenomena and it is not necessary to call upon Heisenberg's uncertainty principle, as it was in the early discussions between Einstein and Bohr. A debate about the role of entanglement and uncertainty relations had begun [7–11]. It was also shown that the which-way information available in the entangled state can be erased, and consequently the interference pattern recovered. Experimental observations of the quantum eraser have been reported in several quantum systems [12–17].

The inclusion of the meter in quantum description of another famous experiment, known as the Wheeler's delayed choice experiment [18], which also provided new investigations on the wave-particle duality. Recently, several quantum delayed-choice experiments were proposed [19] and performed [20–23]. Such experiments are based on the substitution of a 'classical beam splitter' by a quantum system that can be prepared in a superposition state of being present or absent. A Mach–Zender interferometer is considered; therefore, the wave or particle behavior observed depends on the presence or absence of the second beam splitter. In Wheeler's delayed choice experiment, a classical apparatus controls the presence or absence of the second beam splitter after the insertion of the quanton in the apparatus. In the

recent version, the quantum delayed-choice experiment, a quantum system plays the role of the second beam splitter, and it allows for the construction a superposition of wave and particle behavior'.

In this paper, we explore the complementarity relations of the measuring apparatus in the context of quantum erasure and quantum delayed choice. In the first section, we study a two-slit experiment already proposed in [24]. Now, considering the complementarity relations of the detection system, two cavities, one before each slit, where the crossing atom leaves a photon that taggs the atom path. These two possibilities are then viewed as the two interferometric alternatives of the apparatus. In this context, the atomic center of the mass degree of freedom will store which-way information of the cavities. Furthermore, we show that a single measurement of the atomic position on the screen may generate a perfectly balanced superposition of the cavities state. Moreover, we propose a gedanken experiment where the final cavities state may be interpreted as superposition of 'wave-like' and 'particle-like' states.

2. Which-way detector

2.1. Non-selective position measurements

Let us consider the double-slit experiment with high-Q cavities that work as which-way 'detectors' proposed by Scully *et al* [24]. Two-level atoms prepared in the excited state cross, one at the time, a double-slit apparatus with a high-Q micromaser cavity placed on the entrance of each slit. The atom interacts with the cavity mode M_+ (M_-) before crossing the slit + (−). The interaction time with the mode corresponds to a π pulse, so that the atom leaves the excitation in the corresponding mode. The state vector of the global system after the double-slit is

$$|\psi\rangle = \left(\lambda_+ \left|\psi_+\right\rangle \left|1_+, 0_-\right\rangle + \lambda_- e^{i\phi} \left|\psi_-\right\rangle \left|0_+, 1_-\right\rangle\right) |g\rangle, \quad (1)$$

where $\langle\psi_+|\psi_+\rangle = \langle\psi_-|\psi_-\rangle = 1$, $\langle\psi_+|\psi_-\rangle = \langle\psi_-|\psi_+\rangle = e^{-\frac{d^2}{4b^2}}$, $\lambda_+^2 + \lambda_-^2 = 1$, $|\psi_+\rangle$ ($|\psi_-\rangle$) are state vectors in the center-of-mass coordinate subsystem corresponding to the atom crossing slit + (−), d is the distance between the center of the slits, and b is the initial wave-packet width. If the atom crosses the double-slit through cavity + (−), it leaves one excitation on mode M_+ (M_-), and the which-way information of the atom is completely available in the cavity modes subsystem. Therefore, the modes + (−) play the role of a which-way detector. The quality of this detector is characterized by its complementarity relations, as shown in what follows.

In [25] Mohrhoff studied the same system and started a debate in the context of quantum erasers. He argued that when the atoms touch the screen, the measurement of a photon in the resonator no longer corresponds to a which-way measurement of the atomic trajectory. However, in [26], the authors show the contrary and in [27], Mohrhoff agreed and completed the analysis given in [26]. In this paper, we also studied the state of the system after the atomic position measurement on the screen, but for a different purpose. We explore the possibility of performing quantum eraser measurements in the cavities subsystem.

In this analysis, both the particle and the which-way 'detector' are included in the quantum description. The which-way information available in the detectors reduces the visibility of the interference pattern on the screen. The duality between which-way information and visibility is quantified by complementarity relations. In the present section, we reverse the analysis and consider the interferometric properties of the 'detector', i.e., we consider the cavity modes as a two-way interferometer. In this interpretation, the two interferometric alternatives are $|1_+, 0_-\rangle$ and $|0_+, 1_-\rangle$. The which-way information about these interferometric alternatives is available in the center-of-mass coordinate system of the two-level atom.

To make the analysis concrete, we consider the center-of-mass coordinate state of the quanton described by Gaussian wave packets

$$\left|\psi_+\right\rangle = \int \psi_+(x, t) \mid x\rangle \mathrm{d}x \quad (2)$$

$$\left|\psi_-\right\rangle = \int \psi_-(x, t) \mid x\rangle \mathrm{d}x, \quad (3)$$

where

$$\psi_\pm(x, t) = \left[\frac{1}{B(t)\sqrt{\pi}}\right]^{\frac{1}{2}} \times \exp\left[-\frac{\left(x \mp \frac{d}{2} - \frac{b^2 k t}{\tau}\right)^2}{2B^2(t)}\left(1 - \frac{it}{\tau}\right) - \frac{ik^2 b^2 t}{2\tau} + ikx\right],$$

k is the transverse wave number, d is the distance between the center of the slits, b is the initial wave-packet width, $B(t) = b\sqrt{1 + \frac{t^2}{\tau^2}}$, and the scale that characterizes the variation of $B(t)$ is given by $\tau = \frac{mb^2}{\hbar}$, where m is the quanton's mass. We consider a one-dimensional wave packet that corresponds to the assumption that the spread of the wave packet is in the transverse direction to the beam propagation considered classical. This is justified provided the spread in this direction is sufficiently small, what can be achieved by a high enough longitudinal velocity.

If we ignore the atomic position measurements on the screen and trace over the continuous variable degree of freedom, we have the reduced state:

$$\rho_S = \left(\lambda_+^2 \left|1_+, 0_-\right\rangle\left\langle 1_+, 0_-\right| + \lambda_-^2 \left|0_+, 1_-\right\rangle\left\langle 0_+, 1_-\right| + \lambda_+\lambda_- e^{i\phi - \frac{d^2}{4b^2}} \left|0_+, 1_-\right\rangle\left\langle 1_+, 0_-\right| + c.\,c.\right) |g\rangle\langle g|, \quad (4)$$

where $Tr\,(\rho_S) = \lambda_+^2 + \lambda_-^2 = 1$.

Using the definitions for visibility (V) and predictability (P) for a two-level system [28] we have:

$$P = \left|\lambda_+^2 - \lambda_-^2\right|, \tag{5}$$

$$V = 2\left|\lambda_+\lambda_-\right| e^{-\frac{d^2}{4b^2}}. \tag{6}$$

A well-known result is that the visibility and predictability are related by the inequality $V^2 + P^2 < 1$ when the interferometric system is not in a pure state. In reference [28] the authors show that the missing quantity that turns the inequality into an equality is the entanglement. Here, we calculate the linear entropy, which quantifies the entanglement between the center of mass and cavity degrees of freedom, and obtain

$$S = 2\lambda_+^2\lambda_-^2\left(1 - e^{-\frac{d^2}{2b^2}}\right), \tag{7}$$

and show that $P^2 + V^2 + 2S = 1$.

The linear entropy S quantifies the which-way information available in the detector. Note that the vality of the model requires that the overlap between the two center of mass Gaussian states is small enough, i.e., $e^{-d^2/(4b^2)} \ll 1$, yielding a neglible visibility. In this case, it is ensured that the particle that goes through the slit + (−) only interacted with mode M_+ (M_-).

We can define the distinguishability of the detector (associated with its quality) as [29]

$$D = \sqrt{P^2 + 2S} = \sqrt{1 - V^2} \approx 1. \tag{8}$$

This can be illustrated by two extreme situations. Let us assume that $S = 0$. In this case, we can be sure that the quanton crossed slit + (−), recorded by the detector final state $|1_+, 0_-\rangle$ ($|0_+, 1_-\rangle$). When $P = 0$, i.e., S is maximum, the detector will be found in a statistical mixture, which means that the quanton crossed the slits in a maximally coherent superposition of states $|\psi_+\rangle$ and $|\psi_-\rangle$ and are therefore completely entangled with the detector.

2.2. Selective position measurement (quantum eraser)

In this subsection, we consider selective measurements of the center-of-mass position on a (distant) screen and show that some of these measurements correspond to a quantum erasure process for the detector subsystem, in the sense that it will be left in a coherent superposition. Therefore, it will necessarily have a nonvanishing visibility. The common suport of the wave packets, which increases in time, decreases the 'quality' of the which-way information encoded in the center-of-mass system. In the region where the two Gaussian states are superimposed, a measurement of the atomic position on the screen gives ambiguous information about the interferometric alternatives ($|1_+, 0_-\rangle$ and $|0_+, 1_-\rangle$). Therefore, this superposition allows us to perform measurements (on the observable X) that increase the visibility of the interferometric subsystem. These measurements can be interpreted as quantum eraser measurements because they erase the which-way information and increase the visibility in the modes of the subsystem.

Let us consider that a measurement of the atomic position on the screen is performed and the eigenvalue x is obtained. After the measurement, the cavity modes state vector is given by:

$$|\psi(x, t)\rangle = \lambda_+\psi_+(x, t)\,|1_+, 0_-\rangle + \lambda_- e^{i\phi}\psi_-(x, t)\,|0_+, 1_-\rangle, \tag{9}$$

with $\langle\psi_+|\psi_+\rangle = \langle\psi_-|\psi_-\rangle = 1$.

To study the consequences of the atomic position measurements on the interferometric system, let us consider the quantitative complementarity relation introduced in [28]

$$V_x^2 + K_x^2 \leqslant 1. \tag{10}$$

To introduce such the inequality the authors consider a bipartite system composed by the interferometric system, with $|+\rangle$ and $|-\rangle$ as interferometric alternatives, and a which-way detector. X is an observable in the which-way detector subsystem and x is one eigenvalue of X. The inequality (10) is a quantitative complementarity relation for the interferometric system after the measurement of X with the result x. V_x is the 'conditioned visibility' that depends on the choice of the measured observable (X) and on the obtained eigenvalue x. The 'conditioned visibility' is calculated as the visibility in the state vector after the measurement. The 'conditioned which-way knowledge' K_x reflects the *a posteriori* which-way knowledge (after the measurement) and is given by $K_x = |p(+|x) - p(-|x)|$, where $p(+|x)$ ($p(-|x)$) is the conditioned probability that the interferometric system took the alternative $|+\rangle$ ($|-\rangle$), conditioned that the eigenvalue x has been obtained.

In the present system, the observable measured in the which-way detector subsystem is the atomic center-of-mass coordinated on the screen (X) with eigenvalues represented by x. We calculate the conditioned visibility (V_x) and knowledge (K_x)

$$V_x = \frac{2\left|\lambda_+\right|\left|\lambda_-\right|}{\left|\lambda_-^2(\cosh\delta + \sinh\delta) + \lambda_+^2(\cosh\delta - \sinh\delta)\right|} \tag{11}$$

$$K_x = \frac{\left|\lambda_-^2(\cosh\delta + \sinh\delta) - \lambda_+^2(\cosh\delta - \sinh\delta)\right|}{\left|\lambda_-^2(\cosh\delta + \sinh\delta) + \lambda_+^2(\cosh\delta - \sinh\delta)\right|} \tag{12}$$

where $\delta = \frac{d\tau\left(b^2kt - x\tau\right)}{b^2(t^2 + \tau^2)}$.

The quantitative complementarity relation is then

$$V_x^2 + K_x^2 = 1, \tag{13}$$

which corresponds to the inequality (10). The quantities V_x and K_x depends on the measured eigenvalue x, but it also depends on t/τ. Let us consider that $\lambda_+ = \lambda_- = 1/\sqrt{2}$. In this

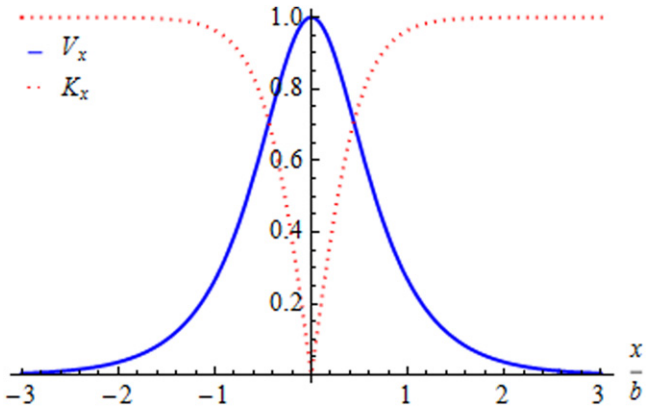

Figure 1. Conditional visibility as a function of the position on the screen, with $\lambda_+ = \lambda_- = 1/\sqrt{2}$, $k = 0$, $d/b = 4$, $t/\tau = 1$.

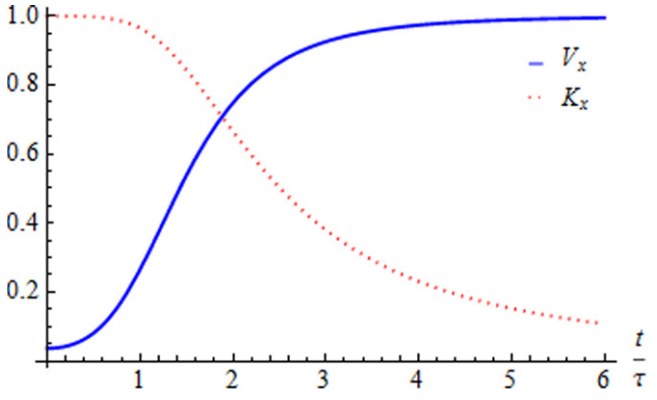

Figure 2. Conditional visibility as a function of the time, with $\lambda_+ = \lambda_- = 1/\sqrt{2}$, $k = 0$, $d/b = 4$, $x/b = 1$.

case, the expression for the conditioned visibility becomes

$$V_x = \text{sech}\left\{\frac{d\tau\left(b^2kt - x\tau\right)}{b^2\left(t^2 + \tau^2\right)}\right\} \tag{14}$$

Equation (14) shows that the values of x associated with the quantum eraser are given by $x = b^2kt/\tau$; notice that the complete quantum erasure occurs where the conditional visibility is maximum. For $k = 0$ the complete quantum erasure occurs for the measurement on the screen $x = 0$, independently of b and time t/τ. However, if $k \neq 0$ and $t/\tau \neq 0$, the system is no more symmetric and the values of x for a complete quantum erasure depend on the time.

First, let us analyze the position x dependence. In figure 1, it is shown as a curve of the conditioned visibility (V_x) and the knowledge (K_x) as a function of x/b for a fixed time of propagation $t/\tau = 1$. We can see that if the atom is measured away from the center of the screen V_x is approximately zero and the K_x is approximately maximum. In these regions of the screen, there is no superposition between the Gaussian wave packets, so there is a high probability to make a right guess about which slit the atom has crossed. On the other hand, if the atom is measured in the center of the screen, the interferometric system is projected onto a state with maximum conditioned visibility; therefore, such measurements work as a quantum eraser. It washes out the conditioned knowledge about the interferometric alternatives. It is interesting to notice that one can observe the continuous variation of V_x, from the maximum to the minimum value, just with measurements of one observable X. Measurements of the same observable X either provide information about which path the atom crossed ($|x|/b \gg 0$) or works as quantum erasure ($|x|/b \approx 0$) [28].

In figure 2, we show the conditioned visibility (V_x) and the knowledge (K_x) as a function of the propagation time t/τ for a fixed position on the screen $x/b = 1$. Notice that for small propagation time, the conditional visibility is close to zero and it increases over time. Therefore, for small propagation times (up to $t/\tau \approx 1$), if the quanton is detected on the screen at position $x/b = 1$, the probability that it has crossed slit + is very high. However, over time, the which-way information of the quanton measured in $x/b = 1$ becomes ambiguous and the conditional visibility increases. In the present system, the same measurement result x works initially as a which-way sorting (up to $t/\tau \approx 1$) and later as a quantum eraser sorting.

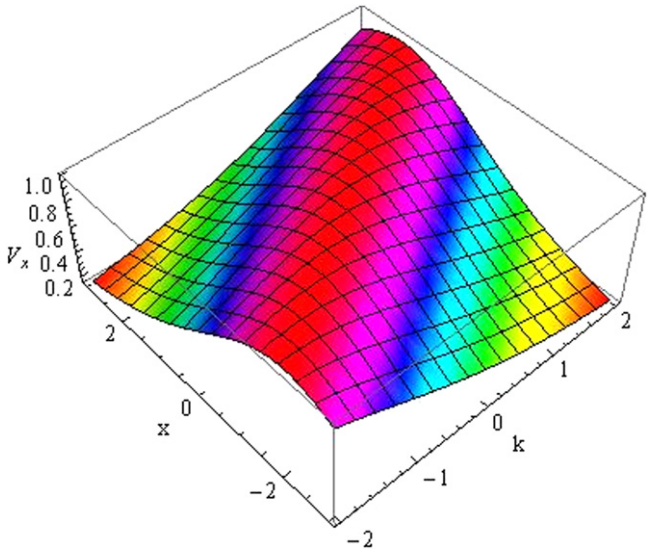

Figure 3. Conditional visibility as a function of position (x) and the transverse wave number (k), with $\lambda_+ = \lambda_- = 1/\sqrt{2}$, $d/b = 4$ and $t/\tau = 1$.

In figure 3, we use the three-dimensional plot when it is clearly seen that there exists a straight line $x = b^2kt/\tau$ where the quantum-conditioned visibility is maximum for fixed $t/\tau = 1$.

3. Wave and particle superposition

We now turn our attention to the construction of an entangled state. The coefficient that is composed by another degree of freedom of one of them is a superposition state ('wave'). On the other hand, the 'particle behavior' is associated with a product state. We propose the construction of this state in the cavity system, but we slightly change the apparatus described in the last section. We consider now a double-slit experiment with only one high-Q cavity, that is placed on the entrance of slit +.

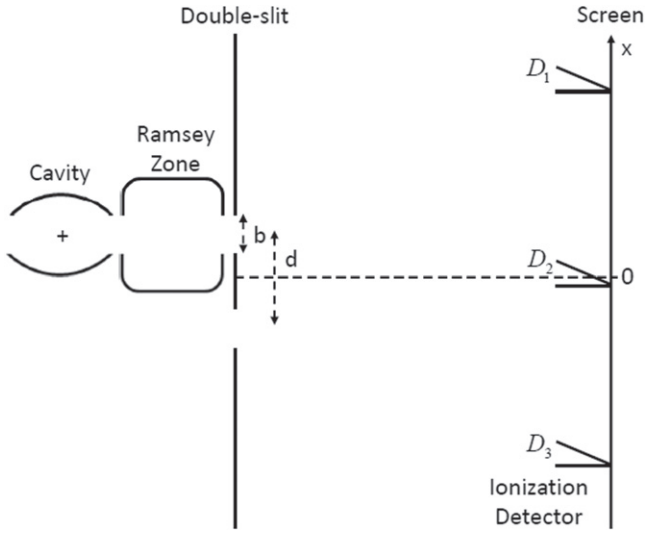

Figure 4. Schematic sketch of the proposed apparatus.

We also include a Ramsey zone that can rotate the internal atomic state next to the high-Q cavity. Therefore, the two-level atoms that cross the slit + interact with mode M_+ inside the high-Q cavity and after interaction with a classical electromagnetic field inside the Ramsey zone. The atom that crosses the slit does not interact with any electromagnetic field.

The mode M_+ is an interferometric system with the two ortogonal 'paths' $|0_+\rangle$ and $|1_+\rangle$. When a state vector of M_+ is in a maximum superposition state of the alternatives $|0_+\rangle$ and $|1_+\rangle$, we can consider that it exhibits wave properties. On the other hand, if M_+ is in a well-defined 'path' state $|0_+\rangle$ or $|1_+\rangle$, it exhibits particle properties. In the present section, we present a scheme to prepare a global state where each Gaussian wave packet is associated to a specific character (wave or particle) of M_+. More specifically, the scheme associates $|\psi_+\rangle$ to the wave character and $|\psi_-\rangle$ to the particle character.

We consider that the interaction time between the atom and M_+ corresponds to a $\pi/2$ pulse. The atom that crosses a slit + interacts first with mode M_+ and evolves as

$$|\psi_+\rangle|0_+\rangle|e_+\rangle \rightarrow |\psi_+\rangle\frac{1}{\sqrt{2}}\left(|0_+\rangle|e_+\rangle + |1_+\rangle|g_+\rangle\right). \quad (15)$$

In the Ramsey zone, the atom in ground state evolves as $|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)$ and the excited state as $|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$.

The state vector of the global system after the double-slit is given by

$$\begin{aligned}|\psi\rangle = |\psi_+\rangle\Bigg[&\frac{1}{2\sqrt{2}}\left(|0_+\rangle|e_+\rangle + |0_+\rangle|g_+\rangle\right)\\ &+ \frac{1}{2\sqrt{2}}\left(|1_+\rangle|g_+\rangle - |1_+\rangle|e_+\rangle\right)\Bigg]\\ &+ \frac{1}{\sqrt{2}}|\psi_-\rangle|0_+\rangle|e_+\rangle. \end{aligned} \quad (16)$$

Suppose that it is possible to measure the atomic energy and the atomic position on the screen. To simplify the imaginary apparatus, let us consider three atomic ionization detectors, as shown in figure 4. If we consider only the detections of the excited state $|e\rangle$, we obtain only in the following part of the state 16

$$|\psi_+\rangle\frac{1}{\sqrt{2}}\left(|0_+\rangle - |1_+\rangle\right) + |\psi_-\rangle|0_+\rangle. \quad (17)$$

The Gaussian wave packet $|\psi_+\rangle$ is associated with a maximum visibility state ('wave' state) and the Gaussian wave packet $|\psi_-\rangle$ is associated with a maximum predicability state ('particle' state). Consider that the screen is positioned at a fixed distance. Therefore, the clicks on the detectors shown in figure 4 will be responsible for the preparation of subsystem M_+ on a 'wave' state, 'particle' state or superposition of 'wave' and 'particle' state. If D_1 clicks, the state on the mode M_+ can be written as $\frac{1}{\sqrt{2}}\left(|0_+\rangle - |1_+\rangle\right) = |w\rangle$, because in this region of the space there is no common support between $|\psi_+\rangle$ and $|\psi_-\rangle$, and we can consider that the detected atom had crossed slit +. If D_1 or D_3 clicks, the state of mode M_+ can now be written as $|0_+\rangle = |p\rangle$, because the detected atom had crossed slit −. However, if the atomic ionization detector is placed in a region of the space with a significant common support, we can not guarantee that the detected atom had crossed slit + or −. When detector D_2 (in the center of the screen) clicks, the state on the mode M_+ can be written as $\frac{1}{\sqrt{2}}\left(|0_+\rangle - |1_+\rangle\right) + |0_+\rangle = |w\rangle + |p\rangle$, which corresponds to a superposition of 'wave' and 'particle' state.

In conclusion, we have explored the quantum interferometric properties of subsystems that work as auxiliary (two cavity modes) in the quantum-eraser scheme. We wrote the quantitative complementarity relation for the two-cavity-modes subsystem, and also show that a single-position measurement of the quanton on the screen can work as a quantum eraser and restore the visibility on the two-modes-cavity subsystem. We emphasize that the same measurement result of the quanton's position on the screen can work as a quantum-eraser measurement or a which-way measurement, depending on the relations between the position measured and the propagation time of the particle. Finally, we show a scheme to construct a 'wave' and 'particle' superposition in a cavity-mode subsystem using the measurement of the position on the screen of a two-level atom.

Acknowledgments

The authors acknowledge financial support by CNPq and Fapemig.

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